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Transient Heat Transfer Flow in a Channel with Porous Medium and Periodic Suction

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Abstract: In the present paper, we investigate an unsteady heat transfer flow of a viscous, incompressible, electrically conductive fluid through a porous medium with periodic suction and temperature oscillation. The dimensionless governing equations are solved using perturbation technique. The analytical expressions for the velocity, temperature, shear stress and Nusset1 number of the fluid have been obtained. The effects of various parameters on the flow field have been discussed with the help of graphs and tables.

Key words: Heat transfer, periodic suction, porous medium, transient

INTRODUCTION

The study of flows through porous media has become of principal interest in many scientist and engineering applications, such as in the utilization of geothermal energy, high-performance building insulation; heat storage, crude oil extraction, petroleum industries, solid matrix exchangers, chemical catalytic reactor, underground disposal of nuclear waste material and many others. Nield and Bejan (1999) and Pop and Ingham (2001) have made comprehensive reviews of the studies of heat transfer in relation to the above applications. In view of the applications of free convective and heat transfer flows through porous medium under the influence of magnetic field many researchers have studied magneto hydrodynamic free convective heat transfer flow in a porous medium, some of them are Reptis and Perdikis (1985), Ram and Mishra (1976) and Varshney (1979). Umarath *et al*. (2009) investigated unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel. Mansutti *et al*. (1993) have analyzed the steady flows of non Newtonian fluids past a porous plate with suction or injection. MHD unsteady free convection water's memory flow with constant suction and heat Sink was studies by Ramana *et al*. (2007). Mustafa *et al*. (2008) investigated unsteady MHD memory flow with oscillatory suction, variable free stream and heat sources. Das *et al*. (1999) have been studied the transient free convection flow past an infinite vertical plate with periodic temperature variation. Free convective flow through a porous medium between two vertical parallel plates has been studies by Singh (2002).

Pawan *et al*. (2007) investigated unsteady free convection oscillatory couette flow through a porous medium with periodic wall temperature. Effect of suction and injection on MHD three dimensional coquette flow and heat transfer through a porous medium has been studied by Das (2009). Pathak *et al*. (2006) concentrate on effect of radiation on unsteady free convective flow bounded by an oscillating plate with variable wall temperature. Sahim (2008) investigated transient three dimensional flows through a porous medium with transverse permeability oscillating with time. The effect of viscous dissipative heat on three dimensional oscillatory flows with periodic suction velocity has been studied by Sahim (2010). Makinde and Mhone (2005) examined heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mehmood and Ali (2007) extended the work of Makinde and Mhone (2005) by considering the fluid slip at the lower wall. Kumar *et al*. (2010) extended the work of Mehmood and Ali (2007) by employing perturbation technique to the problem.

In this work, we investigate the combined effects of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin fluid through a porous medium subjected to periodic suction and temperature oscillation, and discuss the effect of the parameters involved on the flow, compute the skin frictions at the wall and the rate of heat transfer at the channel.

Mathematical formulation: Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium subjected to periodic suction under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has a small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (x, y), where x lies along the center of the channel, y is the distance measured in the normal section. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

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$$\frac{\partial u'}{\partial t'} - v_0(1+\varepsilon A e^{i\omega' t'})\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + v\frac{\partial^2 u'}{\partial y'} - \frac{v}{K'}u' - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta(T' - T_0') \qquad (1)$$

$$\frac{\partial T'}{\partial t'} - v_0(1+\varepsilon A e^{i\omega' t'})\frac{\partial T'}{\partial y'} = \frac{k}{\partial C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C p}\frac{\partial q'}{\partial y'} \qquad (2)$$

With boundary conditions

$$\left.\begin{aligned} u' &= 0, T' = T_\omega' + (T_\omega' - T_o')\varepsilon e^{i\omega' t'} on y = 1 \\ u' &= 0, T' = T_o' on y = 0 \end{aligned}\right\} \qquad (3)$$

where u is the axial velocity, t is the time, ω is the frequency of the oscillation, T the fluid temperature, P the pressure, g gravitational force, C_p the specific heat at constant pressure, k the thermal conductivity, q the radiative heat flux, β the coefficient of volume expansion, A is a constant and εA<<1,K the porous medium permeability coefficient, B_o the electromagnetic induction,s_e the conductivity of the fluid, ρ the density of the fluid, v is the kinematics viscosity coefficient. It is assumed that both walls temperature T_o, T_ω are high enough to induce radiative heat transfer, v_o is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is toward the plate. Following Makinde and Mhone (2005), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q'}{\partial y'} = 4\alpha^2(T_0' - T') \qquad (4)$$

where a is the mean radiation absorption coefficient. The following dimensionless variables and parameters are introduced:

$$\left.\begin{aligned} &\mathrm{Re} = \frac{Ua}{v}, \bar{x} = \frac{x'}{a}\bar{y} = \frac{y'}{a}, \bar{u} = \frac{u'}{U}, \theta = \frac{T' - T_0'}{T_\omega - T_0}, H^2 = \frac{a^2\sigma_e B_0^2}{\rho v} \\ &\bar{t} = \frac{t'U}{a}, \bar{p} = \frac{aP'}{\rho v U}, Da = \frac{K'}{a^2}, Gr = \frac{g\beta(T_\omega' - T_0')a^2}{vU}, Pe = \frac{Ua\rho C_p}{k} \\ &N^2 = \frac{4a^2a^2}{k}, \lambda = \frac{v_0 a}{v}, \mathrm{Pr} = \frac{v\rho C_p}{k} \\ &\varpi = \frac{\omega' a}{U} \end{aligned}\right\} \qquad (5)$$

where U is the flow mean velocity, the dimensionless governing equations together with appropriate boundary conditions, (neglecting the bars for clarity) can be written as

$$\mathrm{Re}\frac{\partial u}{\partial t} - \lambda(1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2)u + Gr\theta \qquad (6)$$

$$Pe\frac{\partial \theta}{\partial t} - \lambda\,\mathrm{Pr}(1+\varepsilon A e^{i\omega} t)\frac{\partial \theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + N^2\theta \qquad (7)$$

With the boundary conditions

$$\left.\begin{aligned} u &= 0, \theta = 1 + \varepsilon e^{i\omega t} \quad on y = 1 \\ u &= 0, \theta = 0 \quad on y = 0 \end{aligned}\right\} \qquad (8)$$

where Gr is the thermal Grashof number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, D_a is the Darcy number, Pr is the Prandtl number, λ is the suction parameter, and s is the porous medium shape factor.

METHODOLOGY

In order to solve Eq. (6) to (8), we assume the pressure term as

$$-\frac{\partial \rho}{\partial x} = B(1+\varepsilon e^{i\omega t}) \qquad (9)$$

where B is a constant, similarly the velocity and temperature solutions are taken as

$$\left.\begin{aligned} u(y,t) &= B(u_0(y) + \varepsilon u_1(y)e^{i\omega t}), \\ \theta(y,t) &= B(1+\theta_0(Y+)\varepsilon\theta_1(y)e^{i\omega t}) \end{aligned}\right\} \qquad (10)$$

where ω is the frequency of the oscillation. Substituting the above expressions in (9) and (10) into (6), (7) and (8), we obtained

$$\frac{d^2u_0}{dy^2} + \lambda\frac{du_0}{dy} - m_1^2 u_0 = -1 - Gr - Gr\theta_0 \qquad (11)$$

$$\frac{d^2u_1}{dy^2} + \lambda\frac{du_1}{dy} - m_2^2 u_1 = -1 - Gr\theta_1 - \lambda A\frac{du_0}{dy} \qquad (12)$$

$$\frac{d^2\theta_0}{dy^2} + \lambda\,\mathrm{Pr}\frac{d\theta_0}{dy} + N^2\theta_0 = -N^2 \qquad (13)$$

$$\frac{d^2\theta_1}{dy^2} + m_3^2\frac{d\theta_1}{dy} + N^2\theta_1 = -\lambda \Pr A\frac{d\theta_0}{dy} \qquad (14)$$

With the boundary conditions

$$\left.\begin{aligned} u_0 &= 0, \theta_0 = \frac{1}{B} - 1 \quad on\, y = 1 \\ u_0 &= 0, \theta_0 = -1 \quad on\, y = 0 \\ u_1 &= 0, \theta_1 = \frac{1}{B} \quad on\, y = 1 \\ u_1 &= 0, \theta_1 = 0 \quad on\, y = 0 \end{aligned}\right\} \qquad (15)$$

where $m_1 = \sqrt{s^2 + H^2}$, $m_2 = \sqrt{s^2 + H^2 + \mathrm{Re}\, i\omega}$, and $m_3 = \sqrt{\lambda \Pr - Pei\omega}$. Equation (11) to (15) are solved and the solution for fluid temperature and velocity are given as follows:

$$\theta(y,t) = B[A_1 e^{\alpha_1, y} + B_1 e^{-\alpha_2 y} + \varepsilon(A_2 e^{r_1 y} + B_2^{-r_2 y} + D_1 e^{\alpha_1 y} + D_2 e^{-\alpha_2 y})e^{i\omega t}] \qquad (16)$$

$$\begin{aligned} U(y,t) = {} & B[A_3 e^{h_1 y} + B_3 e^{-h_2 y} + D_3 + D_4 e^{\alpha_1 y} + \\ & + D_5 e^{-\alpha_2 y} + \varepsilon(A_4 e^{h1y} + B_4 e^{-h_2 y} + D_6 \\ & + D_7 e^{r_1 y} + D_8 e^{-r_2 y} + D_9 e^{\alpha_1 y} + D_{10} e^{-\alpha_2 y} \\ & + D_{11} y e^{h_1 y} + D_{12} y e^{-h_2 y})e^{i\omega t}] \end{aligned} \qquad (17)$$

The shear stress at the lower wall of the channel is given by

$$\begin{aligned} \tau = \frac{\partial u}{\partial y}\Big|_{y=0} = {} & B[A_3 h_1 - B_3 h_2 + D_4\alpha_1 - D_5\alpha_2 \\ & + \varepsilon(A_4 h_1 - B_4 h_2 + D_7 r_1 - D_8 r_2 + D_9\alpha_1 \\ & - D_{10}\alpha_2 + D_{11} + D_{12})e^{i\omega t}] \end{aligned} \qquad (18)$$

The rate of heat transfer across the channel's wall is given as

$$Nu = \frac{\partial\theta}{\partial y}\Big|_{y=0} = B[A_1\alpha_1 - B_1\alpha_2 + \varepsilon(A_2 r_1 - B_2 r_2 + D_1\alpha_1 - D_2\alpha_2)e^{i\omega t}] \qquad (19)$$

Where A_1, A_2, B_1, B_2, A_3, A_4, B_3, B_4, D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9, D_{10}, D_{11}, and D_{12} are constants, their expressions are not presented here for sake of brevity.

RESULTS AND DISCUSSION

In order to get a physical view of the problem computation are carried out for different values of thermal

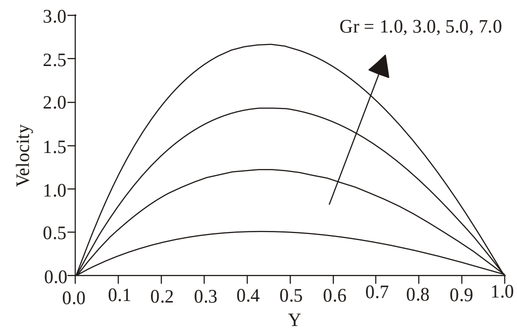

Fig. 1: Velocity profiles for different values of grashof number Gr

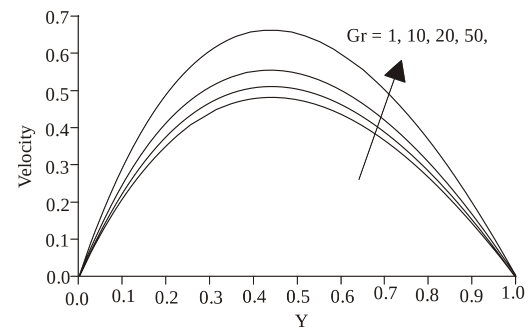

Fig. 2: Velocity profiles for different values of the reynolds number

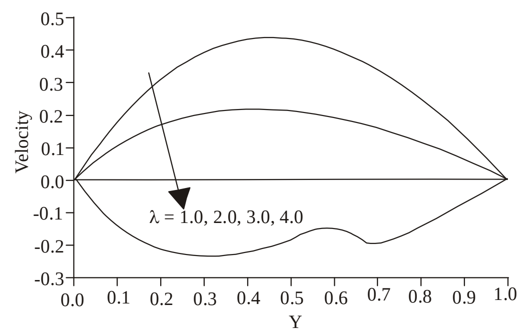

Fig. 3: Velocity profiles for different values of suction parameter

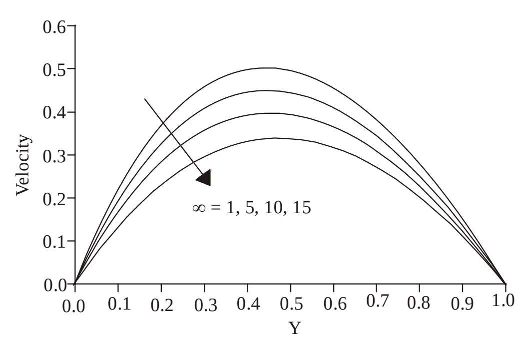

Fig. 4: Velocity profiles for different values of the frequency of the oscillation

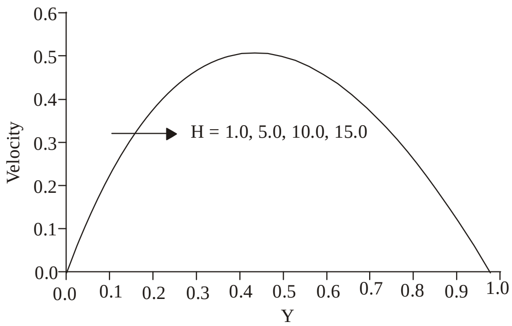

Fig. 5: Velocity profiles for different values of the hartmann number

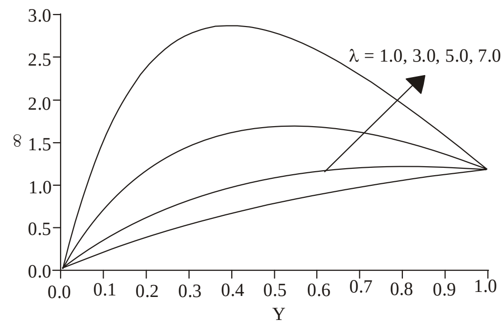

Fig. 8: Temperature profiles for different values of suction parameter

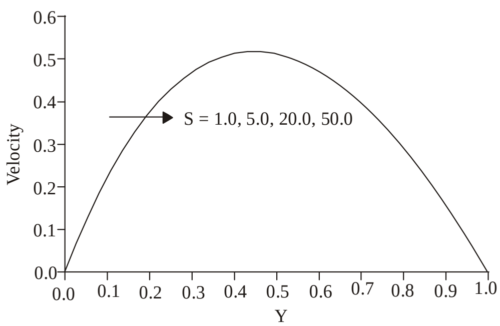

Fig. 6: Velocity profiles for different values of the porosity parameter

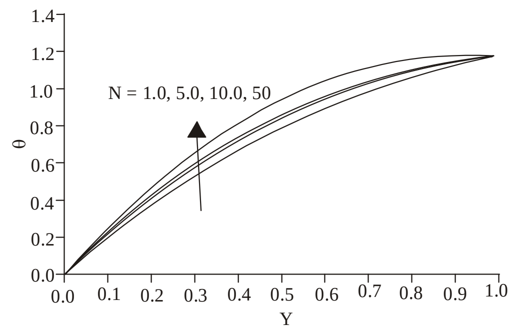

Fig. 9: Temperature profiles for different values of the radiation parameter

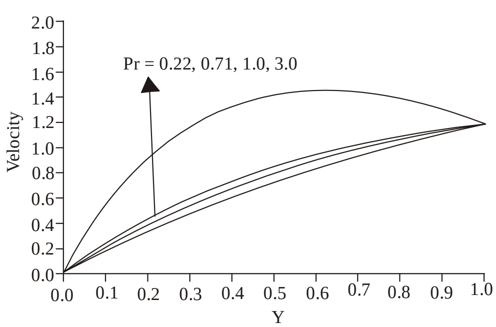

Fig. 7: Temperature profiles for different values of prandtl number Pr

Table 1: Values of shear stress with different values of material parameters

Re	H	S	Gr	ω	λ	τ
1	1	1	1	0.2	1	2.3046
2	1	1	1	0.2	1	2.3026
1	1	1	2	0.2	1	3.9699
1	2	1	1	0.2	1	2.3071
1	1	1	1	1	1	2.2375
1	1	2	1	0.2	1	2.3071
1	1	1	1	0.2	2	1.0675
1	1	1	1	2	1	2.1281

Table 2: Values of nusset1 number with different values of material parameter

Pr	N	λ	ω	Nu
0.71	1	1	0.2	1.9900
0.71	2	1	0.2	2.0428
1	1	1	0.2	2.2501
0.71	1	2	0.2	2.7237
3	1	1	0.2	6.6523

Grashof number, Peclet number, Hartmann number, epsilon, radiation parameter, Prandtl number, and time. The purpose of the calculations is to assess the effects of the parameters Gr, Pe, H, ε, N, Pr, and t upon the nature of the flow and transport. We made use of the following parameter values except otherwise indicated, Pr = 0.71, H = 1, Gr = 1, Re = 1, S = 1, λ = 1, A = 0.2, t = 0.2, ω = 0.2, Pe = 1, B = 1, ε = 0.2. The velocity profiles have been studied and presented in Fig. 1 to 6. The velocity profiles for different values of the Grashof number (Gr = 1.0, 3.0, 5.0, 7.0) and Reynolds number (Re = 1, 10, 20, 50) are shown in Fig. 1 and 2. It is observed that the velocity increase with increasing the Grashof number or Reynolds number. The velocity profiles for different values of suction parameter (λ = 1.0, 2.0, 3.0, 4.0) and frequency of the oscillation (ω = 1, 5, 10, 15) are presented in Fig. 3 and 4. It is observed that the velocity decreases with increasing suction parameter and the frequency of the oscillation. The velocity profiles for different values of the Hartmann number (H = 1.0, 5.0, 10, 50) and porosity parameter (S = 1, 5, 20, 50) are studied in Fig. 5 and 6. It is observed that the velocity does not change with increasing Hartmann number and porosity parameter. The temperature profiles have been studied and presented in

Fig. 7 to 9. The temperature profiles for different values of the Prandtl number (Pr = 0.22, 0.71, 1.0, 3.0), suction parameter (λ = 1.0, 3.0, 5.0, 7.0), and radiation parameter (N = 1.0, 5.0, 10, 50) are shown in Fig. 7, 8 and 9 respectively. It is observed that the temperature increases with increasing Prandtl number, suction and radiation parameter respectively. The variation of the skin friction and Nussetl number on the porous plate with material parameters are presented in Table 1 and 2. It is observed that an increasing Grashof number lead to the increase in shear stress and increasing Prandtl number and suction causes an increase in Nussetl number.

CONCLUSION

This paper investigates the transient heat transfer flow in a channel with porous medium and periodic suction. The velocity and temperature profiles are obtained analytically using perturbation technique. The effect of different parameters namely, the radiation parameter, Grashof number, Hartmann number, porosity parameter, suction parameter, Reynolds number and Prandtl number are studied. The conclusions of the study are as follows.

- It is observed that the velocity increases with increasing Grashof number and Reynolds number, increasing suction and frequency of the oscillation weaken the flow and magnetic field intensity and porosity parameter have a very small significant effect on the flow.
- It is observed that the temperature increases with increasing the Prandtl number, suction and radiation parameter.
- An increase in Grashof number causes an increase in the shear stress, and increasing suction or frequency of the oscillation decreases shear stress, while increasing Prandtl number or suction lead to an increase in the temperature gradient at the plate.

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