Chemical Reaction and Slip Condition Effects on MHD Oscillatory Flow Through a Porous Medium

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Abstract: We investigate the combined effects of chemical reaction and slip condition on unsteady heat and mass transfer to MHD oscillatory flow through a channel filled with saturated porous medium. Exact solution of the governing equations for fully developed flow is obtained in closed form. The effects of various material parameters like Hartmann number, radiation parameter, Reynolds number, magnetic Reynolds number, Grashof number, modified Grashof number, porosity parameter, and frequency of the oscillation are discussed on flow variables and presented by graphs.

Key words: Chemical reaction, MHD, oscillatory flow, porous medium, slips condition

INTRODUCTION

Applications of combined heat and mass transfer flow with chemical reaction play important role in the design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees damage of crops due to freezing, food processing and cooling of towers. Investigation of periodic flow through a porous channel is important from practical point of view because fluid oscillations may be expected in many magnetohydrodynamics devices and natural phenomena, where fluid flow is generated due to oscillating pressure gradient or due to vibrating walls. Abdussattar and Alam (1995) investigated MHD free convective heat and mass transfer flow with hall current and constant heat flux through a porous medium. Alam et al. (2009) analyzed transient MHD free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeated plate in the presence of variable chemical reaction and temperature dependent viscosity. Mansour and Aly (2009) presented the effects of chemical reactions and radiation on MHD free convective heat and mass transfer from a horizontal cylinder of elliptic cross section saturated porous media with considering suction or injection. Muhamin and Hashim (2009) presented variable viscosity and thermophoresis effects on Darcy mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. Sedeeek and Almushige (2010) investigated effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Muthucumaraswamy (2010) examined chemical reaction effects on vertical oscillating plate with variable temperature. In all these investigations “no-slip” boundary condition is considered for the velocity field. However in some application e.g. micro fluidic and nano fluidic devices where the surface to volume ratio is large, the slip behavior is more typical and slip boundary condition is usually used for the velocity field (Darhuber and Troian, 2005).


The aim of the present study is to investigate the effects of chemical reaction and slip condition on unsteady heat and mass transfer flow in a channel filled with porous medium, and other parameters on the flow
like Hartmann number, porosity parameter, radiation parameter, thermal Grashof number, modified Grashof number, Reynolds number, magnetic Reynolds number and frequency of the oscillation.

Mathematical formulation: Consider a two-dimensional free and force convection effects on unsteady incompressible laminar MHD radiative heat and mass transfer flow with temperature oscillating in a channel filled with saturated porous medium. It is assumed that the fluid has a small electrical conductivity, the electromagnetic force produced is very small and a uniform magnetic field. The x-axis is taken along the plate in the vertical upward direction and the y-axis is taken normal to the plate. All the fluid properties are considered constant except the influence of the density variation in the buoyancy term, according to the classical Boussinesq approximation. Under these assumptions along with Boussinesq approximations for incompressible fluid model, the equations governing the motion are given as:

\[
\begin{align*}
\frac{\partial u'}{\partial t} & = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{v}{k'} - \frac{\sigma_B B_0^2 u'}{\rho} g \beta^* (T - T_0) + g \beta^* (C - C_0) \\
\frac{\partial T'}{\partial t} & = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q'}{\partial y'} \\
\frac{\partial C}{\partial t} & = D \frac{\partial^2 C}{\partial y'^2} - K_s^* (C' - C_0) \\
\end{align*}
\]

\[
\begin{align*}
u' - \gamma * \frac{\partial u'}{\partial y'} = 0, \quad T' - T'_{\omega} \quad \text{only} & = 0 \\
u' = 0, T' = T_0 + (T'_0 - T_0) \cos \omega t' \quad \text{only} & = 1 \\
\end{align*}
\]

where \( u' \) is the axial velocity, \( t' \) is the time, \( T' \) the fluid temperature, \( T'_{\omega} \) and \( T_0 \) are walls temperatures \( P \) the pressure, \( g \) the gravitational force, \( q' \) the radiative heat flux, \( C' \) the fluid concentration, \( C'_{\omega} \) and \( C_0 \) are walls concentrations, \( D \) is mass diffusivity, \( K_s \) is the permeability of the porous medium \( \omega \) is the frequency of the oscillation, \( \beta \) is the coefficient of thermal expansion, \( \beta^* \) is the coefficient of concentration expansion, \( B_0 \) the electromagnetic induction, \( s \) the conductivity of the fluid, \( \rho \) the density of the fluid, \( v \) is the kinematics viscosity coefficient. It is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by:

\[
\frac{\partial q'}{\partial y'} = 4a^2(T'_0 - T')
\]

where \( a \) is the mean radiation absorption coefficient. On introducing the following dimensionless quantities:

\[
\begin{align*}
\text{Re} & = \frac{Ua}{v}, \quad x = \frac{x}{a}, \quad y = \frac{y'}{a}, \quad u' = \frac{U}{a}, \quad \theta = \frac{T - T_0}{T_0 - T_0}, \\
H^2 & = \frac{a^2 \sigma_B B_0^2}{\rho \nu}, \quad K_s = \frac{K_s^* a^2}{D}, \quad \omega = \frac{t'}{a}, \\
p & = \frac{Ua \rho C_p}{k}, \quad N^2 = \frac{4a^2 \gamma^2}{k}, \quad C = \frac{C'_{\omega}}{C_0}, \quad C = \frac{C_0 - C_0}{C_0} \\
Gc & = \frac{g \beta^* (C'_{\omega} - C_0) a^2}{vU}, \quad Rm = \frac{Ua^2}{D^2 v}
\end{align*}
\]

In Eq. (1) to (4), leads to:

\[
\begin{align*}
\text{Re} \frac{\partial u}{\partial t} & = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2)u + Gr\theta + Gc C \\
\text{Pe} \frac{\partial \theta}{\partial t} & = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \\
\text{Rm} \frac{\partial C}{\partial t} & = \frac{\partial^2 C}{\partial y^2} - K_s C
\end{align*}
\]

With the boundary conditions:

\[
\begin{align*}
u - \gamma \frac{du}{dy} & = 0, \quad \theta = 0, \quad C = 0, \quad \text{on} \quad y = 0 \\
u = 0, \quad \theta = \cos \omega t, \quad C = \cos \omega t, \quad \text{on} \quad y = 1
\end{align*}
\]

where, Gr is the thermal Grashof number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, D is the Darcy number, Pr is the Prandtl number, \( \gamma \) is the slip parameter and \( s \) is the porous medium shape factor.

Method of solution: To solve Eq. (7) to (9), we assumed the pressure gradient, fluid velocity, fluid temperature, and fluid concentration in non-dimensional form, as:

\[
\begin{align*}
u & = \frac{\partial u}{\partial t} + (s^2 + H^2)u + Gr\theta + Gc C \\
\text{Pe} \frac{\partial \theta}{\partial t} & = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \\
\text{Rm} \frac{\partial C}{\partial t} & = \frac{\partial^2 C}{\partial y^2} - K_s C
\end{align*}
\]
\[ -\frac{\partial p}{\partial x} = Re(e^{i\omega t} + e^{-i\omega t}) \]  

\[ u(y, t) = u_0(y)e^{i\omega t} + u_1(y)e^{-i\omega t} \]  

\[ \theta(y, t) = \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t} \]  

\[ C(y, t) = C_0(y)e^{i\omega t} + C_1(y)e^{-i\omega t} \]  

where \( R < 0 \) for favorable pressure.

Equation (7) to (10) with the use of (11) and (12) reduce to

\[ \frac{d^2 u_0}{dy^2} - m_1^2 u_0 = -R - Gr \theta_0 - GcC_0 \]  

\[ \frac{d^2 u_1}{dy^2} - m_2^2 u_1 = -R - Gr \theta_1 - GcC_1 \]  

\[ \frac{d^2 \theta_0}{dy^2} + N_1^2 \theta_0 = 0 \]  

\[ \frac{d^2 \theta_1}{dy^2} + N_2^2 \theta_1 = 0 \]  

\[ \frac{d^2 C_0}{dy^2} - S_1^2 C_0 = 0 \]  

\[ \frac{d^2 C_1}{dy^2} - S_2^2 C_1 = 0 \]  

With boundary conditions:

\[ u_0(y) = u_0, \quad u_1(y) = u_1, \quad \theta_0 = \theta_0, \quad \theta_1 = \theta_1, \quad C_0 = C_0, \quad C_1 = C_1 \]  

where,

\[ m_1 = \sqrt{S_1^2 + H^2 + Rei\omega}, \quad m_2 = \sqrt{S_2^2 + H^2 - Rei\omega} \]  

\[ N_1 = \sqrt{N_1^2 - Pei\omega}, \quad N_2 = \sqrt{N_1^2 - Pei\omega} \]  

\[ S_1 = \sqrt{K_1 + Pei\omega}, \quad S_2 = \sqrt{K_1 - Pei\omega} \]  

Equation (13) to (18) subject to boundary conditions (19) are solved and the solution for fluid temperature, fluid concentration and fluid velocity are presented in the following form:

\[ \theta(y, t) = \frac{1}{2} \left[ \frac{\sin N_1 y}{\sin \lambda_1} e^{i\omega t} + \frac{\sin N_2 y}{\sin \lambda_2} e^{-i\omega t} \right] \]  

\[ C(y, t) = \frac{1}{2} \left[ \frac{\sinh S_1 y}{\sinh \lambda_1} e^{i\omega t} + \frac{\sinh S_2 y}{\sinh \lambda_2} e^{-i\omega t} \right] \]  

\[ u(y, t) = [A \cosh m_1 y + B \sinh m_1 y + C \cosh m_2 y + D \sinh m_2 y] e^{i\omega t} \]  

The rate of heat transfer across the channel’s wall is given as:

\[ \frac{\partial \theta}{\partial y} \bigg|_{y=0} = \frac{1}{2} \left[ \frac{N_1}{\sin \lambda_1} e^{i\omega t} + \frac{N_2}{\sin \lambda_2} e^{-i\omega t} \right] \]  

The rate of mass transfer across the channel’s wall is given as:

\[ \frac{\partial C}{\partial y} \bigg|_{y=0} = \frac{1}{2} \left[ \frac{S_1}{\sinh \lambda_1} e^{i\omega t} + \frac{S_2}{\sinh \lambda_2} e^{-i\omega t} \right] \]  

The shear stress at the lower wall of the channel is given:

\[ \frac{\partial u}{\partial y} \bigg|_{y=0} = [Bm_1 + \eta_1 - \eta_2] e^{i\omega t} \]  

\[ + [Dm_2 + \eta_1 - \eta_2] e^{-i\omega t} \]  

where, \( A = a_1 - a_2, \quad B = b_1 - b_2 - b_3 - b_4 + b_5 + b_6, \quad D5 = \frac{R}{m_5^2 d_2}, \quad D = D1 - D2 - D2 + D4 + D5 + D6, \quad C = c_1 - c_2, \quad \lambda_1 = \frac{R}{m_1}, \quad \lambda_2 = \frac{\sin N_1 y}{2(N_1^2 + m_1^2) \sin \lambda_1}, \quad \lambda_3 = \frac{\sin S_1 y}{2(S_1^2 + m_2^2) \sinh \lambda_1}, \quad \xi_1 = \frac{R}{m_2}, \quad \xi_2 = \frac{\sin N_2 y}{2 \cosh m_2 (N_2^2 + m_2^2)} \sinh \lambda_2, \quad \xi_3 = \frac{\sin S_2 y}{2(S_2^2 - m_1^2) \sin \lambda_1}, \quad a_1 = \frac{B \sinh m_1}{\cos m_1} - \frac{R}{m_7 \cosh m_1}. \]
RESULTS AND DISCUSSION

In order to illustrate the influence of various parameters on the velocity, temperature and concentration field, numerical calculations of the solutions, obtained in the preceding section, have been carried out for different values of the chemical reaction parameter, Reynolds number, radiation parameter, slip parameter, magnetic parameter, porosity parameter, Grashof number, frequency of the oscillation, Peclet number, modified Grashof number, and magnetic Reynolds number. We

\begin{equation}
\begin{aligned}
a_2 &= \frac{Gr}{2 \cosh m_1 \left(N_1^2 + m_1^2\right)} - \frac{Ge}{2 \cosh m_1 \left(N_1^2 + m_1^2\right)} \\
b_1 &= \frac{Ge}{2 \cosh m_1 \left(S_1^2 - m_1^2\right) d_1}, \quad b_2 = \frac{Ge}{2 \cosh m_1 \left(N_1^2 - m_1^2\right)} \\
b_3 &= \frac{\gamma GrN_1}{2 \sinh N_1 \left(N_1^2 - m_1^2\right) d_1}, \\
b_4 &= \frac{R}{m_1^2 \cosh m_1 d_1}, \quad b_5 = \frac{R}{m_1^2 d_1} \\
b_6 &= \frac{\gamma GrS_2}{2 \sinh S_2 \left(S_2^2 - m_2^2\right) d_2}, \quad b_7 = \frac{\gamma GrS_1}{2 \sinh S_1 \left(S_1^2 - m_1^2\right) d_1}. \\
d_1 &= \frac{\sin m_1}{\cosh m_1} + \gamma m_1, \quad d_2 = \frac{\sinh m_2}{\cosh m_2} + \gamma m_2, \\
c_1 &= -D \sinh m_2 \frac{\cosh m_1}{\cosh m_2} - \frac{R}{m_2^2 \cosh m_2}, \\
a_2 &= \frac{Gr}{2 \cos m_2 \left(N_2^2 + m_2^2\right)} - \frac{Ge}{2 \cosh m_2 \left(N_2^2 + m_2^2\right)}, \\
D_1 &= \frac{Ge}{2 \cosh m_2 \left(S_2^2 - m_2^2\right) d_2}, \\
D_2 &= \frac{Gr}{2 \cosh m_2 \left(N_2^2 - m_2^2\right) d_2}, \quad D_3 = \frac{\gamma GrN_2}{2 \sin N_2 \left(N_2^2 - m_2^2\right) d_2}, \\
D_4 &= \frac{R}{m_2^2 \cosh m_2 d_2}, \quad D_5 = \frac{R}{m_2^2 d_2}, \\
\eta_1 &= \frac{GrN_1}{2 \sin N_1 \left(N_1^2 + m_1^2\right)}, \quad \eta_2 = \frac{GrS_1}{2 \sin S_1 \left(S_2^2 - m_2^2\right)}, \\
l_1 &= \frac{GrN_2}{2 \sin N_2 \left(N_2^2 + m_2^2\right)}, \quad l_2 = \frac{GrS_2}{2 \sin S_2 \left(S_2^2 - m_2^2\right)}.
\end{aligned}
\end{equation}
Fig. 5: Temperature profiles for different values of the Peclet number

Fig. 6: Temperature profiles for different values of the frequency of the oscillation

Fig. 7: Velocity profiles for different values of the slip parameter

Fig. 8: Velocity profiles for different values of the Hartmann number

Fig. 9: Velocity profiles for different values of the porosity parameter

Fig. 10: Velocity profiles for different values of the Reynolds number

Fig. 11: Velocity profiles for different values of the Grashof number

Fig. 12: Velocity profiles for different values of the modified Grashof number
made use of the following parameter values except otherwise indicated.  
\[ R_e = 1, N = 1, Kr = 1, Pe = 1, Rm = 1, R = -1, S = 1, \\
H = 1, Gr = 0.2, Gc = 0.2, \alpha t = \pi/2, \omega = 0.1, \gamma = 1 \]

The concentration profiles for different values of the chemical reaction parameter (K = 5, 10, 15, 20), magnetic Reynolds number (Rm = 5, 10, 15, 20) and frequency of the oscillation are shown in Figs. 1, 2 and 3, respectively. It is observed that the concentration decreases with increasing chemical reaction parameter or magnetic Reynolds number and increase with increasing frequency of the oscillation. The temperature profiles for different values of the radiation parameter (N = 0.1, 0.5, 1.0, 1.5), Peclet number (Pe = 1, 2, 3, 4) and frequency of the oscillation are presented in Figs. 4, 5 and 6 respectively. It is observed that the temperature profiles increases with increasing radiation parameter, Peclet number and frequency of the oscillation. The velocity profiles for different values of the slip parameter (\(46 = 0, 1, 2, 3\)), magnetic parameter (H = 1, 3, 5, 7), porosity parameter (S = 0.5, 1.0, 1.5, 2.0), Reynolds number (Re=0.02, 0.04, 0.06, 0.08), Grashof number (Gr = 0.2, 0.4, 0.6, 0.8) and modified Grashof number (Gc = 1, 3, 5, 7) are shown in Fig. 7, 8, 9, 10, 11 and 12, respectively. It is observed that the velocity increases with increasing slip parameter, magnetic parameter, porosity and Grashof number, and decreases with increasing Reynolds number or modified Grashof number.

**CONCLUSION**

This study investigated the effects of chemical reaction and slip condition on transient MHD heat and mass transfer flow through a channel filled with porous medium. The results reveal among other things that the velocity increases with increasing slip parameter, magnetic parameter or Grashof number, and decreases with increasing Reynolds number or modified Grashof number. The concentration profiles increases with increasing frequency of the oscillation, and decreases with increasing chemical reaction parameter and magnetic Reynolds number.

**REFERENCES**


