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A Mathematical Model for the Control of Cholera in Nigeria

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Abstract: In this research, we present and analyze a mathematical model for the control of cholera in Nigeria with modifications as compared to previous cholera models. Our model incorporates treatment, water hygiene and environmental sanitation in curtailing the disease. A system of ordinary differential equations is used. The model studied shows that with proper combination of control measures the spread of cholera could be reduced. Numerical simulation of the full model using maple shows clearly that improvement in treatment, water hygiene and the environmental sanitation offered to about fifty percent is effective to eradicate cholera epidemic.

Keywords: Cholera model, control strategies, simulation

INTRODUCTION

Cholera is a contagious infectious disease that is characterized by extreme vomiting, profuse watery diarrhea and leg pain. It has been found that transmission transpires mostly via absorption of contaminated drinking water or food. Worldwide, almost every year there is an estimated 3-5 million cholera cases and 100, 000-130, 000 deaths due to cholera a year as of 2010 (WHO, 2013a). It has a very short incubation period which starts from a few hours to five days. The health of an infected person disintegrates rapidly and death may occur if treatment is not promptly given. Cholera was first discovered in the Indian subcontinent in 1817. The disease reaches all the way through Asian continent in the 1960s, getting in to Africa in 1970 and Latin America in 1991 (Codecco, 2001; Wearing *et al*., 2005). In many parts of Africa and Asia the disease is still endemic.

Cholera is a disastrous water-borne infectious disease that is caused by the bacterium vibrio cholera. It is a very serious problem in many developing countries due to inadequate access to safe drinking water supply, improper treatment of reservoirs and improper sanitation. In 2012, WHO reported 245, 393 cholera cases and 3034 death cases across 48 countries in which 67% cases occurred in African countries (WHO, 2013b).

In 2005, Nigeria had 4, 477 cases and 174 deaths. There were reported cases of cholera in 2008 in Nigeria in which there were 429 deaths out of 6, 330 cases. Furthermore, 2, 304 cases were reported in Niger State in which 114 were death cases (NBS, 2009). Also in 2009, Nigeria reported 13, 691 cases and 431 deaths (WHO, 2012).

Tian and Wang (2011) evidenced that recent years have seen a strong trend of cholera outbreak in developing countries, such as in India (2007), Iraq (2008), Congo (2008), Zimbabwe (2008-2009), Haiti (2010), Kenya (2010) and Nigeria (2010).

In Nigeria, outbreaks of the disease have been taking place with ever-increasing occurrence ever since the earliest outbreak in recent times in 1970, (Epstein, 1993; Lawoyin *et al*., 2004).

In summary the United Nation (UN) unit, reports: "despite Nigeria's oil wealth, more than 70% of the country's 126 million people live below the poverty line and cholera outbreaks are common in poor urban areas which lack proper sanitation and clean drinking water" (UN Office for the Coordination of Humanitarian Affairs Integrated Regional Information Networks (IFIN) 2005).

In the last few decades, Capasso and Paveri-Fontana (1979), Codecco (2001), Pascual *et al*. (2002), Hartley *et al*. (2006), Jensen *et al*. (2006), Bertuzzo *et al*. (2008), Tien and Earn (2010), Hove-Musekwa *et al*. (2011), Tian and Wang (2011) and Misra and Sigh (2012), have designed mathematical models to explore the transmission dynamics and control of the disease. However, there have not been many studies on cholera in Nigeria using mathematical modeling. Hence we decided to apply mathematical study of disease with control measures which can give support to reduce cholera infection.

MATERIALS AND METHODS

Model formulation: A mathematical model for cholera transmission is developed by making notable

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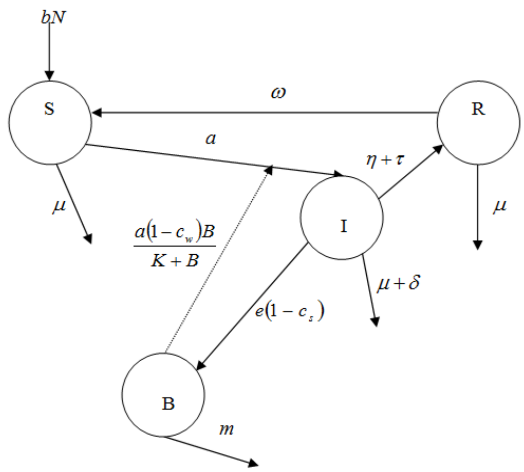

Fig. 1: Schematic diagram of cholera transmission and control model

improvements on the previous models as actual source (Capasso and Paveri-Fontana, 1979; Codecco, 2001). Our model incorporates treatment, water hygiene and environmental sanitation. The model contains 5 variables which are susceptible, infected, recovered, total human population, and the concentration of vibrio cholera in water.

The susceptible population is generated either through birth or through immigration. They acquire infection and move to the infected class at the rate:

$$\frac{a(1-c_w)B}{K+B},$$

where,

- a = The rate of exposure to contaminated water
- c_w = The rate of compliance with water hygiene
- K = The concentration of vibrio cholerae in water that yields 50% chance of catching cholera (Codecco, 2001)
- B = The concentration of vibrio cholerae in contaminated water

The number of infected individuals decreases through recovery from the disease, at the rate η and τ where η is the natural recovery and τ is recovery due treatment. Bacterial populations in the aquatic environment grow in the water at the rate determined by the environmental factors such as temperature. A schematic depiction of the above explanation is shown in Fig. 1 and the variables and parameters of the model are described in Table 1 and 2:

A schematic representation of our model is given below: The model consists of the following system of ordinary differential equations given in (1) to (4):

$$\frac{dS}{dt} = bN - \frac{a(1-c_w)B}{K+B}S + \omega R - \mu S \tag{1}$$

Table 1: Model variables and their interpretations

Symbol	Description
S	Number of susceptible individuals
I	Number of infected individuals
R	Number of recovered individuals
B	Concentration of vibrio cholerae in contaminated water
N	Total population of humans

Table 2: Model parameters and their interpretations

Symbol	Description
b	Per capital birth rate of humans
μ	Per capital natural death rate of humans
a	Rate of exposure to contaminated water
K	Concentration of vibrio cholerae in water
η	Natural recovery
τ	Recovery due to treatment
C_w	Rate of compliance with water hygiene
C_s	Rate of compliance with environmental sanitation
δ	Cholera-induced death rate
m	Growth rate of vibrio cholerae in the aquatic environment
ω	Loss rate (waning) of immunity by recovered individuals
e	Contribution of each infected person to the population of V. cholerae in the aquatic environment

Table 3: Baseline values for variables of the cholera model in Nigeria as at year 2013

Variables	Baseline value	Reference
$N(0)$	170, 123, 740	CIA (2011)
$I(0)$	100	Assumed
$S(0)$	170, 123, 640	Assumed
$R(0)$	0	Assumed
$B(0)$	1000, 000, 000	Assumed

Table 4: Baseline values for parameters of the cholera model

Parameter	Baseline value	Reference
b	0.000107day^{-1}	CIA (2011)
μ	0.0000526 day^{-1}	CIA (2011)
$a = 10000\mu/bN$	0.00000289 day^{-1}	Estimated
K	10^6	Codecco (2001)
η	0.2 day^{-1}	Codecco (2001) and Mwasa and Tchuenche (2011)
τ	(0 – 1) day^{-1}	Assumed
C_w	(0 – 1) day^{-1}	Assumed
C_s	(0 – 1) day^{-1}	Assumed
δ	0.00008 day^{-1}	WHO (2012)
m	0.33 day^{-1}	Codecco (2001)
ω	0.003 day^{-1}	Mwasa and Tchuenche (2011)
e	10 cells/mL/day	Codecco (2001)

$$\frac{dI}{dt} = \frac{a(1-c_w)B}{K+B}S - (\eta+\tau+\mu+\delta)I \tag{2}$$

$$\frac{dR}{dt} = (\eta+\tau)I - (\omega+\mu)R \tag{3}$$

$$\frac{dB}{dt} = e(1-c_s)I - mB \tag{4}$$

where,

$$N = S + I + R. \tag{5}$$

Control measures with various strategies: As shown in Eq. (1) to (4), our model incorporates three new

control measures namely treatment, water hygiene and environmental sanitation. In this study, it is of main interest to see the effect of control measures at different levels of control strategies. Using the baseline values for variables and parameters as in Table 3 and 4 we compute the effect of control measures C_w, C_s, τ at 7 different control strategies as explained below.
The 7 control strategies are:

- **No control:** This is a situation where there is no control ($\tau = C_w = C_s = 0$).
- **Low without treatment:** Low efficiency level of the rate of compliance with water hygiene and rate of compliance with environmental sanitation at 25% ($\tau = 0$, $C_w = C_s = 0.25$).
- **Moderate without treatment:** Moderate effectiveness level of rate of compliance with water hygiene and rate of compliance with environmental sanitation at 50% ($\tau = 0$, $C_w = C_s = 0.50$).
- **High without treatment:** High effectiveness level of rate of compliance with water hygiene and rate of compliance with environmental sanitation at 75% ($\tau = 0$, $C_w = C_s = 0.75$).
- **Low universal:** Low level of all the three control strategy at 25% ($\tau = C_w = C_s = 0.25$).
- **Moderate universal:** Moderate level of all the three control strategy at 50% ($\tau = C_w = C_s = 0.50$).
- **High universal:** High level of all the three control strategy at 75% ($\tau = C_w = C_s = 0.75$).

RESULTS AND DISCUSSION

The model is simulated using the parameter values in Table 3 to assess the effect of the three main control measures considered in this study, namely, treatment, rate of compliance with water hygiene and rate of compliance with environmental sanitation (τ, C_w and C_s). The results obtained from various strategies are shown in Table 5 and Fig. 2 to 5.

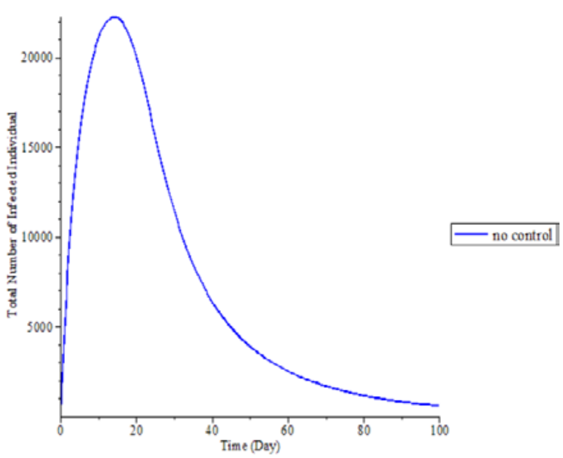

Fig. 2: Total number of infected individuals without any control measures (Control parameters used are _ = cw = cs = 0)

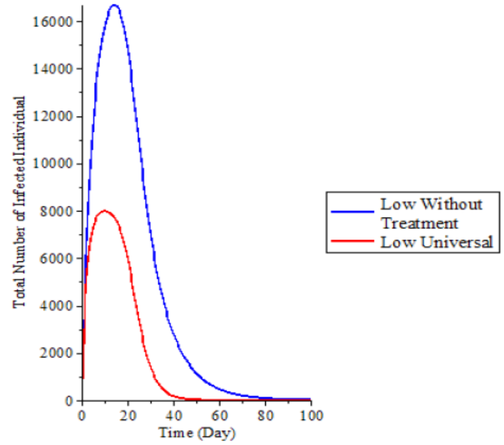

Fig. 3: Comparison between low universal strategy and low without treatment (Control parameters used are cw = cs = _= 0.25)

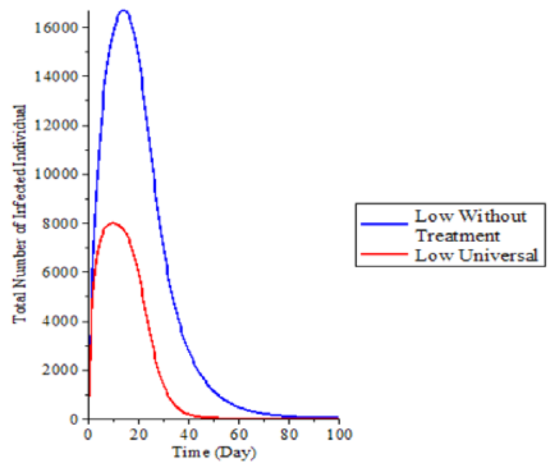

Fig. 4: Comparison between moderate universal strategy and moderate without treatment (Control parameters used are _ = cw = cs = 0.50)

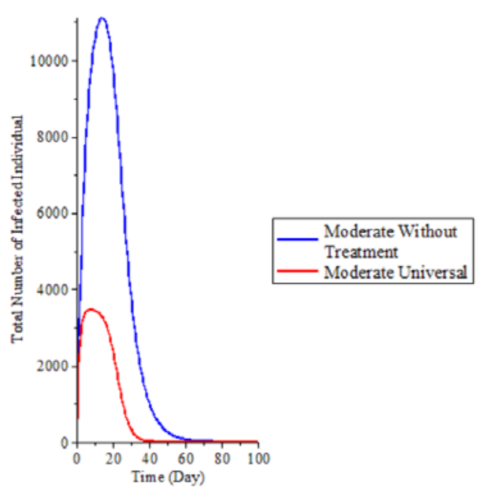

Fig. 5: Comparison between high universal strategy and high without treatment (Control parameters used are cw = cs=_= 0.75

Table 5: Effect of 7 different control strategies on the infected population

Time (day)	a	b	c	d	e	f	g
0	100	100	100	100	100	100	100
5	15528	11656	7783	3910	7316	3396	1279
10	21019	15768	10517	5265	7979	3446	1269
15	22207	16640	11087	5544	7587	3210	1173
20	20082	14847	9782	4854	5990	2394	847
25	15642	10867	6743	3199	3344	1113	359
30	11463	7044	3829	1633	1396	334	93
35	8466	4428	2002	731	512	82	20
40	6405	2791	1020	310	180	19	4
45	4956	1775	516	128	62	4	1
50	3902	1136	261	53	22	1	0
55	3115	731	132	21	7	0	0
60	2514	472	67	9	3	0	0

Table 5 shows the effect of the three control measures at various levels of control strategies for sixty days. At day 0, the number of infected individuals is 100 for all seven control strategies. As the days increase, the number of infected individuals also tends to increase until it reaches a certain level at approximately day 15. The number of infectives is highest when there is no control measures at all as shown in Table 5 column (a). For high without treatment (d), low Universal (e), moderate Universal (f) and high Universal (g), the number of infectives are low compared to (a), (b) and (c).

Figure 2 illustrates the situation in which there is no control. The number of infected individuals increases until it reaches a maximum of approximately 23000 in less than 20 days time.

Figure 3 shows that for low without treatment the number of infectives has risen up at the high rate with the peak value of about 17000 individuals. The low universal shows a lower peak value of about 7000 individuals being infected. Therefore, low universal have highly positive effect on morbidity of the infected individuals than low without treatment. It can also be seen that the disease reaches extinction faster (approximately day 40) for low universal compared to low without treatment.

Figure 4 depicts the total number of the infected individuals as a function of time, for the various level of control strategy. The figure shows an increase in the moderate without treatment which resulted in over 11, 000 and a decrease in the moderate universal with only 3500 individuals being infected. Endemicity also occurs but with a much of lower infection level. Holding the rate of $\tau = C_w = C_s$ at 50% the numbers of infected individuals drop from 11000 to 3500. It can also be seen that increasing the rates of control parameters decreases the total number of infected individual.

Clearly, we observed in Fig. 5, there is a drastic reduction in total number of infected individual. High universal gave results that are marked lower than high without treatment. Increasing the rates of control parameters to 75% yield a rapid decay of the infection curve. This means that the disease is not endemic and it dies out.

CONCLUSION

In this study, we presented an improved mathematical model for the transmission and control of cholera dynamics in Nigeria. The model incorporates treatment, water hygiene and environmental sanitation. Different control strategies are used to see the effect of control measures on eradication of cholera.

Our study shows that 50% level of control measures is sufficient to effectively control the spread of cholera in Nigeria. The total number of the infected individuals decreases with the increase in level of the control measures stopping the disease from reaching an alarming level. Among the control strategies used, universal is more effective for controlling cholera.

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