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Effects of Friction Damper-brace Design Parameters on Seismic Performance of Multistory Building Structures

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Abstract: In this study, design parameters of Friction Damper-Brace System (FDBS) and their influence on seismic response of low-to-medium-rise building structures are investigated. Numerical analyses are performed on some example building models with different fundamental periods. Improvement of seismic response of the structures with respect to variations of FDBS design parameters including: total slip-load ratio of friction damper devices (FDD), number of FDD installations and arrangement of dampers along height of buildings, is investigated. Results show that for a constant stiffness ratio of the braces and uniform distribution of slip-load ratio amongst FDDs, optimal normalized number of FDD installations, increases or remains invariant in the range of 0.5 to 1.0 when fundamental period of the structure increases.

Keywords: Friction damper, passive control, seismic performance

INTRODUCTION

Past studies have shown that passive structural control using friction dampers is capable to dissipate a large amount of seismic input energy. Several types of friction damper devices exist which have similar energy dissipation mechanisms (Pall and Marsh, 1982; Constantinou *et al*., 1990; Grigorian *et al*., 1992; Dyke *et al*., 1996). Within them, Mualla and Belev (2002) proposed a rotational friction damper with adjustable slip-moment (2002) that is fixated in this study.

There are numerous experimental and analytical studies in the literature on seismic performance of multistory buildings equipped with friction dampers (Aiken *et al*., 1988; Li and Reinhorn, 1995; Cho and Kwon, 2004; Marko *et al*., 2006; Lu *et al*., 2006; Liao *et al*., 2004). When passive devices are considered for seismic structural control, the most important question is how the design parameters should be determined, in order to achieve the desired structural performance under a specified seismic environment. To restate, the topological distribution and mechanical properties of these devices must be designed in accordance with a systematic design method. However, the lack of such design methodology has motivated many researchers to study on optimal design of energy dissipative devices during last decade (Singh and Moreschi, 2001; Xu and Teng, 2002; Park *et al*., 2004; Apostolakis and Dargush, 2009; Levy and Lavan, 2006; Pong *et al*., 2009; Lee *et al*., 2008a, b; Aydin *et al*., 2007; Gluck *et al*., 1996; Inoue and Kuwahara, 1998). Garcia and Soong (2002) proposed a simplified Sequential Search Algorithm (SSSA), which gives optimal floor distribution of viscous dampers by repeated installation of unit viscous damper on the floor with the largest controllability index defined by inter–story drift or relative velocity. That procedure was imposed on a series of example Multi-Degree-of-Freedom (MDOF) structures typically and efficiency of the methodology was evaluated. Lee *et al*. (2008a) investigated design parameters of friction damper-brace system, including allocation and slip load of friction dampers. For this purpose, numerical analyses were performed on a number of example structures (Garcia and Soong, 2002) with short fundamental periods. Results of numerical analyses led to an empirical equation on the optimal number of friction damper installations in building structures with short fundamental periods.

The purpose of this study is to investigate the effects of design parameters of Friction Damper-Brace System (FDBS) on seismic performance of low-to-medium-rise building structures with different fundamental periods. First, seismic responses of Single-Degree-of-Freedom (SDOF) structures with respect to variations of FDBS design parameters, including stiffness and slip-load ratios, are evaluated. Then, results of a large number of numerical analyses on MDOF building structures equipped with FDBS are presented to investigate performance of the structures with respect to variations of generalized FDBS design parameters, including total slip-load ratio, number of Friction Damper Device (FDD) installations and FDD arrangement along height of the structure. At last, the general pattern of appropriate damper allocations along height of the structures is discussed.

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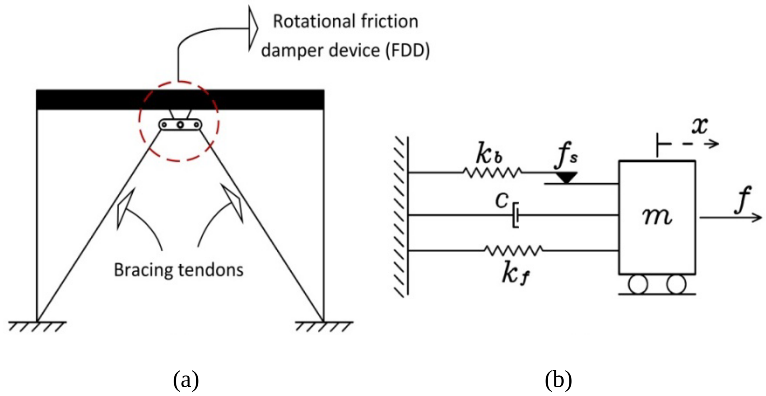


Fig. 1: a) A SDOF structure equipped with FDBS; b) Dynamic model of the original system

MATERIALS AND METHODS

Dynamic Model of Friction Damper-Brace System (FDBS):

FDBS in SDOF structures: A SDOF structure equipped with FDBS is presented in Fig. 1a. As shown, it consists of a shear single-story frame, a chevron brace and a rotational FDD (Mualla and Belev, 2002) on mid-span of the girder. Figure 1b represents a basic dynamic model of the original system, where k_f, m and c denote the lateral stiffness, mass and damping of the frame, respectively and k_b, f_s, x and f represent the lateral stiffness of the brace, the slip-load of the friction damper, the displacement of the frame and the external load, in the same order.

Dynamic behavior of the model during seismic excitations can be described in the two following stages:

- **Stage 1: Stick stage:** Internal force of FDD is less than slip-load (f_s) and consequently no rotation is observed in FDD. In this stage, FDBS is the same as an ordinary chevron brace.
- **Stage 2: Slip stage:** Internal force of the FDD is equal to f_s and consequently the FDD yields. Lateral stiffness of the FDD is negligible in this condition. The sliding stage endures while the internal force remains equal to f_s.

Accordingly, the FDD is assumed to have rigid-perfectly-plastic behavior. On the other hand, the primary structure and the brace are assumed to behave elastically. Therefore, the force-displacement relationship of a system equipped with the FDBS can be modeled as a bilinear behavior as displayed in Fig. 2. As shown, (k_f+k_b) is primary linear-elastic stiffness; k_f is secondary strain hardening stiffness of the system; and f_y denotes equivalent yield strength which can be defined as follows (Moreschi and Singh, 2003):

$$f_y = f_s\left(\frac{k_f + k_b}{k_b}\right) \tag{1}$$

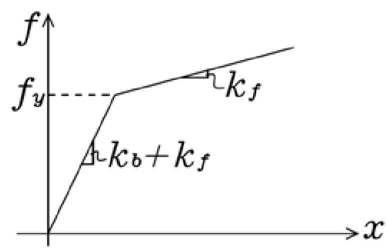


Fig. 2: Bilinear force-displacement relationship of a SDOF system equipped with FDBS

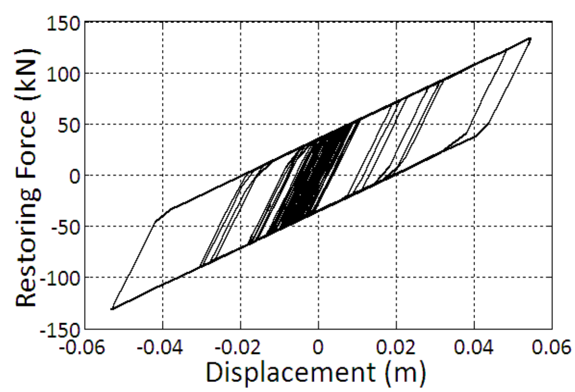
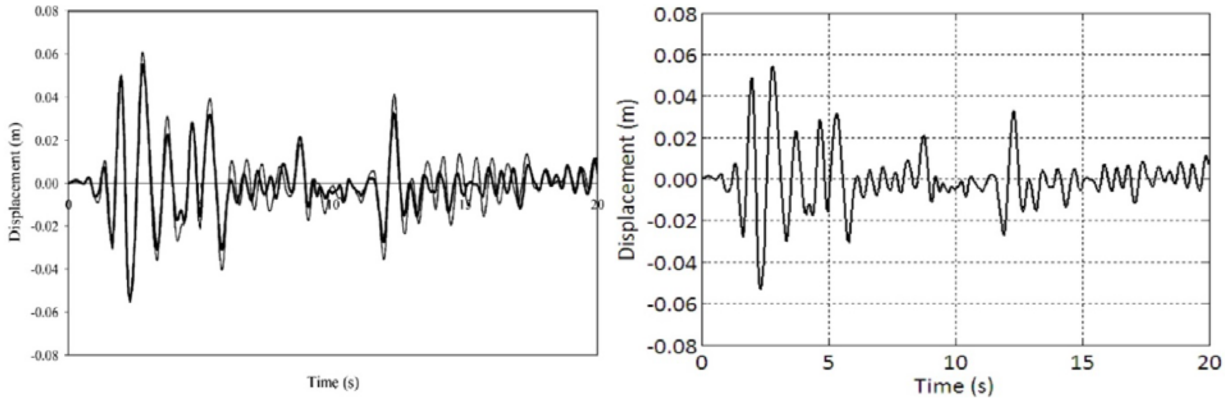


Fig. 3: Bilinear hysteretic behavior of the proposed model subjected to El Centro NS ground motion

In this study, the numerical analyses are performed using well-known Newmark-β method (assuming: β = 1/6 and γ = 1/2) (Clough and Penzien, 1993) for simulating the dynamic response of bilinear systems.

Bilinear hysteretic behavior of the model is shown in Fig. 3, in which summation of confined areas of hysteresis loops represents total dissipated energy due to frictional behavior of the FDBS. The proposed numerical method of a single-story steel frame equipped with FDBS is verified by experiment (Mualla and Belev, 2002). Figure 4a (thick line) illustrates displacement time history of the structure subjected to El Centro NS, 1940 ground motion reported by Mualla



(a) (b)

Fig. 4: Displacement time history of a single-story steel frame subjected to El Centro NS ground motion studied by Mualla and Belev (2002), a) experimental (thick line), b) proposed model

and Belev (2002). On the other hand, Fig. 4b depicts the same response in the proposed model. The results manifest reasonable accuracy of the proposed model compared to experimental data.

Generalization to MDOF building structures: When FDBS is installed on MDOF building structures, each FDD has an independent bilinear hysteretic behavior under seismic loading. Therefore, the entire system has a more complicated nonlinear dynamic response that can be modeled with generalizing the previous proposed numerical method. For this purpose, stick or slip phase of each FDD is an independent potential source of nonlinearity during each time step of dynamic analysis.

KEY PARAMETERS FOR SDOF STRUCTURES

FDBS design parameters formulation: According to Fig. 2, the bilinear force-displacement relationship of the SDOF system depends on two design parameters of the FDBS.

The first design parameter is stiffness of the brace (k_b) that can be normalized by stiffness of the frame (k_f). The obtained stiffness ratio (*SR*) is expressed in the following:

$$SR = \frac{k_b}{k_f} \tag{2}$$

The second design parameter is slip-load of the FDD (f_s) that can be normalized by story weight (W) of the SDOF system, as follows:

$$\rho = \frac{f_s}{W} \tag{3}$$

where, ρ is normalized slip–load of the FDD.

Performance index: For linear structures, where the structure does not suffer structural damage, the peak inter-story drift becomes an important response parameter, since it is a measure of nonstructural damage (Levy and Lavan, 2006). Therefore, peak inter-story drift ($\left|x_i(t)\right|_{\max}$) is considered as performance index of the equipped structure in this study and is assumed to be normalized by peak inter-story drift of the bare frame ($\left|x_{0,i}(t)\right|_{\max}$) (Lee *et al*., 2008b), as follows:

$$R_d = \frac{\left|x_i(t)\right|_{\max}}{\left|x_{0,i}(t)\right|_{\max}} \tag{4}$$

where, R_d denotes relative peak inter-story drift as nondimensional performance index of the equipped SDOF structure.

EFFECTS OF FDBS DESIGN PARAMETERS ON RESPONSE OF SDOF STRUCTURES

Results of numerical analyses are presented in this section to investigate the effects of design parameters of the FDBS on performance of SDOF structures. Figure 5 and 6 typically illustrate variations of performance index, R_d, versus design parameters of *SR* and ρ, respectively. A SDOF system with different fundamental periods of the primary structure equal to 0.1 s through 0.5 s and damping ratio equal to 0.02, subjected to El Centro ground motion is considered. In Fig. 5, R_d decreases when the first design parameter, *SR*, increases while ρ = 0.25. As shown, R_d decreases rapidly as *SR* increases within the range of $0<SR<5.0$, so that response reduction is equal to or greater than almost 80% when *SR* is set equal to 5. On the other hand, in Fig. 6 in which *SR* is assumed equal to 5, R_d exhibits a steep descent when ρ varies from zero to

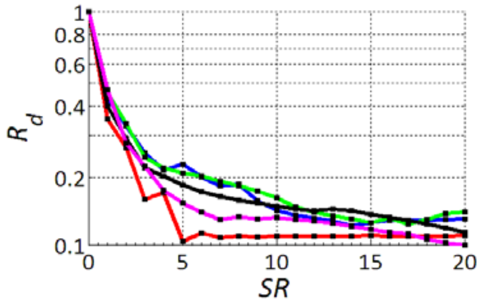


Fig. 5: Variations of R_d versus *SR* in a SDOF system subjected to El Centro ground motion (ρ = 0.25)

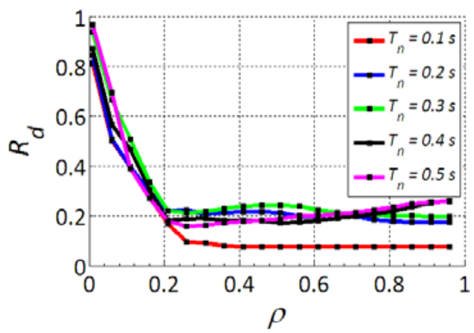


Fig. 6: Variations of R_d versus ρ in a SDOF system subjected to El Centro ground motion (*SR* = 5.0)

almost 0.2 and then remains constant. This fact shows that if slip–load ratio of the FDD is greater than a threshold value, seismic loading cannot activate the FDD and then no slip stage occurs during earthquake. In such condition, the FDBS behaves the same as an ordinary chevron brace.

KEY PARAMETERS FOR MDOF STRUCTURES

FDBS design parameters formulation: In generalization to MDOF building structures, four design parameters can be defined. The first set of design parameters is defined in the following:

$$SR_i = \frac{k_{bi}}{k_{fi}} \tag{5}$$

where, SR_i is stiffness ratio of FDBS which is installed on *i*-th floor of the MDOF building structure. In addition, k_{bi} and k_{fi} denote stiffness of the brace and lateral story stiffness of the primary structure at *i*-th elevation, respectively.

The second design parameter is number of FDDs which are installed at different levels of the MDOF structure. Number of FDD installations is denoted by N_f in this study.

The third design parameter is total slip-load ratio (ρ) of the entire FDDs installed at the MDOF building structure that can be expressed as follows:

$$\rho = \frac{\sum_{i=1}^{N_f} f_{si}}{\sum_{i=1}^{N} W_i} \tag{6}$$

where, ρ is total slip-load ratio of the entire FDDs and f_{si}, W_i, N_f and N denote slip-load of *i*-th FDD, weight of *i*-th story, number of FDD installations and number of stories, respectively. Once the total slip-load ratio is distributed amongst entire FDDs, the portion of each one indicates normalized slip–load of the given FDD. In this study, total slip-load ratio is assumed to be distributed amongst FDDs identically.

The last FDBS design parameter is arrangement of FDDs along height of the MDOF building structure. This parameter indicates which stories are chosen and equipped for any arbitrary N_f.

Performance index: Performance index in MDOF building structures is a generalized form of Eq. (5) as expressed by the following equation:

$$R_d = \frac{\max\limits_{i=1,\dots,N} \left\{ \left| x_i(t) \right|_{\max} \right\}}{\max\limits_{i=1,\dots,N} \left\{ \left| x_{0,i}(t) \right|_{\max} \right\}} \tag{7}$$

where, R_d, $x_i(t)$ and $x_{0,i}(t)$ are relative peak inter-story drift and time history of *i*-th inter-story drift before and after damper installation, respectively and N is total number of stories.

EFFECTS OF FDBS DESIGN PARAMETERS ON RESPONSE OF MDOF BUILDING STRUCTURES

Numerical analyses show that performance of MDOF building structures, equipped with FDBS, correlates with:

- Structural properties
- Seismic loading
- FDBS design parameters, with strong nonlinearity

Therefore, in order to investigate effects of the FDBS design parameters on improvement of structural performance, numerous example building structures are investigated in this study, considering variations in dynamic characteristics, input ground motions and FDBS design parameters

Description of example building structures and ground motions: In order to study the seismic performance of low-to-medium-rise buildings equipped with FDBS, 4-, 6-, 8-, 10- and 12-story structures are taken into consideration. The distribution of lateral

Table 1: Lateral story stiffness of the building models before FDBS installations (i.e., bare frames)

Story stiffness (kN/cm)

	Number of stories				
Story	4	6	8	10	12
1-2	1000	1000	1000	1000	1000
3-4	1000	1000	1000	1000	1000
5-6		850	850	850	850
7-8			850	850	850
9-10				725	725
11-12					725

Table 2: Mass properties of the building models before FDBS installations (i.e., bare frames)

MDOF structure	Fundamental period (sec)	Story mass (kg)
4-story	0.8	195 530
	1.0	305 500
	1.2	439 950
	1.4	598 800
6-story	1.0	133 620
	1.2	192 420
	1.4	261 900
	1.6	342 080
8-story	1.2	108 990
	1.4	148 330
	1.6	193 750
	1.8	245 200
10-story	1.4	90 000
	1.6	117 550
	1.8	148 780
	2.0	183 680
12-story	1.6	79 600
	1.8	100 740
	2.0	124 380
	2.2	150 500

story stiffness along height of the buildings before FDBS installations (i.e., bare frames) is presented in Table 1. According to Table 1, it is noteworthy that conclusions of this study are valid only for buildings regular in height. For each number of stories, five fundamental periods of bare frames (i.e., buildings before FDBS installations) are considered. The total number of building models constructed is then (5×5). For a given model, all the story masses are assumed to be identical. Mass properties corresponding to different fundamental periods are given in Table 2. The inherent damping ratio ξ_0 of the structures is set equal to 2% for all modes and Rayleigh damping model is considered (Clough and Penzien, 1993).

Three input records including: 1940 El Centro NS, 1994 Northridge, Pacoima Dam (upper left) station and 1989 Loma Prieta, Gilroy Array #2 station ground motions are used to perform the numerical analyses.

FDBS design assumptions in numerical analyses: According to section 5.1, there are four design parameters that must be determined to capture the best efficiency of the FDBS. The first design parameter SR_i, as defined in Eq. (6), is set equal to 5.0 for all FDDs during all numerical analyses, with regard to Fig. 5. It is shown that application of braces with greater *SR* than almost 5.0 no longer decrease response of the SDOF structure significantly. This fact is also observed at MDOF buildings.

Results of a study presented by Lee *et al*. (2008b) on slip-load and allocation of FDDs at MDOF buildings, postulate that optimal number of FDD installations (N_f) is almost half of total number of stories (N) at short-period buildings. On the other hand, fundamental periods of example structures in this study (Table 2), are generally greater than those of Lee *et al*. (2008b) (i.e., the example structures presented in this study have less stiffness). Therefore, for a given N-story building structure with periods shown in Table 2, the expected optimal value of normalized number of FDD installations (N_f/N) must be in the range of 0.5 through 1.0. Consequently, variations of the second FDBS design parameter are expected to range within $0.5 \leq (N_f/N) \leq 1.0$.

Total slip-load ratio of FDDs (ρ), as defined in Eq. (7), is assumed to vary in the range of 0.1 through 2. Results of numerical analyses show that optimal value of ρ, as the third FDBS design parameter, occurs within this range. In addition, all possible states of FDDs arrangement along height of the building structure, as the forth FDBS design parameter, are considered in this study and the best configurations are explored.

RESULTS AND DISCUSSION

According to results of Table 3, given values of N_f show that optimal number of FDD installations increases or remains constant when fundamental period of the structure increases. As stated before, Lee *et al*. (2008b) had shown that the optimal values of normalized number of FDD installations (N_f/N) are generally equal to 0.5 in short-period structures. Accordingly, it is concluded that $(N_f/N)_{opt}$ increases or remains constant in the range of 0.5 through 1.0 when fundamental period of the structure increases, as shown in Fig. 7.

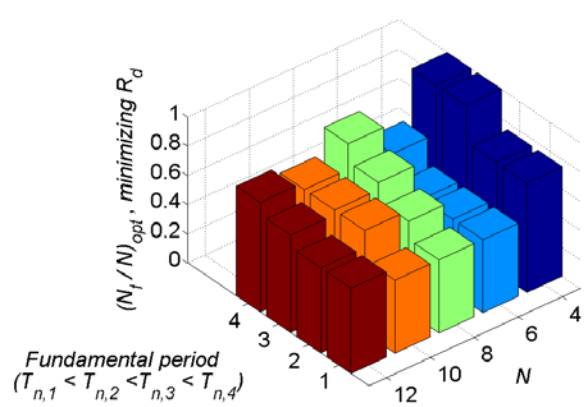


Fig. 7: Variations of $(N_f/N)_{opt}$ minimizing R_d versus fundamental period of the structures of different heights subjected to El Centro NS ground motion

Table 3: Maximum efficiency of the FDBS in enhancing seismic response of multi-story buildings

Event	Structure	Fundamental period (sec)	Total slip-load ratio (ρ)	Number of FDD installations (N_f)	Response reduction (%)
El Centro	N = 4	0.8	1.5	3	58.3
		1.0	1.1	3	55.4
		1.2	0.9	4	51.7
		1.4	0.3	4	67.3
	N = 6	1.0	0.3	3	42.7
		1.2	1.7	3	65.6
		1.4	1.5	3	69.0
		1.6	1.1	4	52.6
	N = 8	1.2	1.1	4	45.7
		1.4	1.1	5	53.9
		1.6	1.7	6	57.4
		1.8	1.7	7	55.8
	N = 10	1.4	0.7	5	40.4
		1.6	0.5	7	48.6
		1.8	1.1	7	61.6
		2.0	0.3	7	62.4
	N = 12	1.6	0.5	7	55.7
		1.8	0.5	7	61.3
		2.0	0.5	8	65.0
		2.2	0.5	9	70.8
Northridge	N = 4	0.8	0.5	3	63.8
		1.0	0.7	3	72.0
		1.2	1.1	4	70.5
		1.4	0.9	4	53.5
	N = 6	1.0	0.9	4	71.8
		1.2	1.1	4	66.1
		1.4	0.9	4	57.9
		1.6	1.9	4	65.7
	N = 8	1.2	1.9	5	63.9
		1.4	1.1	5	53.9
		1.6	1.7	5	84.4
		1.8	1.3	5	84.3
	N = 10	1.4	0.3	5	54.2
		1.6	1.9	5	76.0
		1.8	1.9	7	81.2
		2.0	1.7	7	78.1
	N = 12	1.6	0.5	7	73.1
		1.8	1.3	8	71. 6
		2.0	1.7	9	75.9
		2.2	0.5	9	74.6
Loma Prieta	N = 4	0.8	0.7	2	63.7
		1.0	0.7	3	76.7
		1.2	1.1	4	77.3
		1.4	1.1	4	69.5
	N = 6	1.0	0.7	4	65.9
		1.2	1.7	4	65.7
		1.4	1.1	5	59.8
		1.6	1.5	5	39.1
	N = 8	1.2	0.7	5	49.1
		1.4	1.9	6	53.4
		1.6	0.3	7	59.2
		1.8	1.3	4	74.8
	N = 10	1.4	0.5	6	44.9
		1.6	0.5	6	68.6
		1.8	1.5	6	68.9
		2.0	1.9	6	77.6
	N = 12	1.6	0.9	7	66.9
		1.8	0.9	7	67.4
		2.0	1.7	7	75.9
		2.2	1.7	7	71.5

Regarding to results of numerical analyses, optimal configurations of FDD installation along height of structure do not follow a particular pattern. However, a partial conformity is observed in results of numerical analyses when different FDBS design parameters are assumed. Figure 8 shows probability distributions of FDD installations along height of the structures for different values of N_f. These probability distributions are obtained from comparison of various optimal states of FDD arrangement when other design parameters

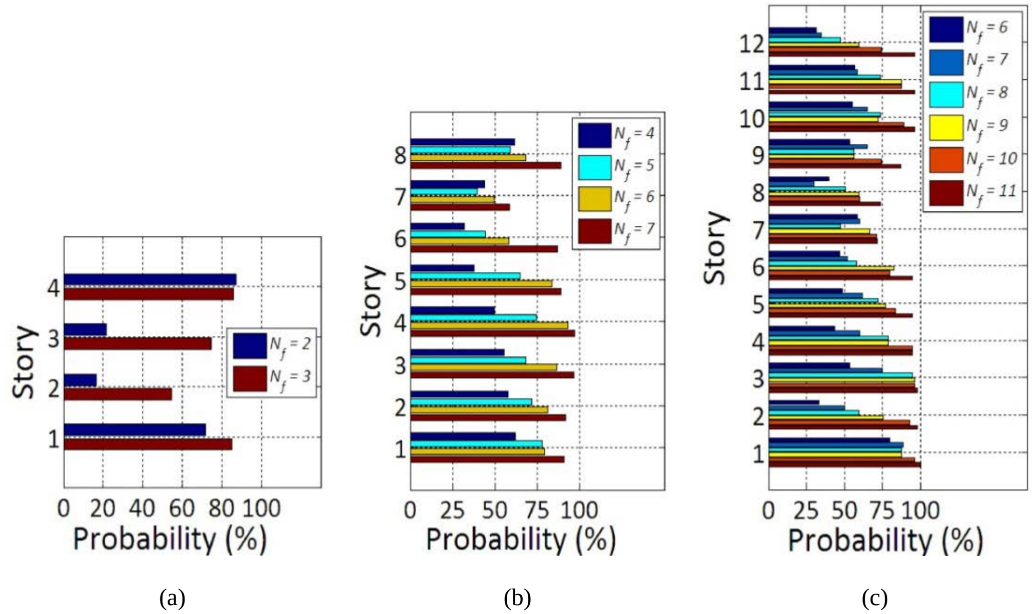


Fig. 8: Probability distribution of FDD installation along height of the structures for different values of N_f, a) 4-story, b) 8-story, c) 12-story structure

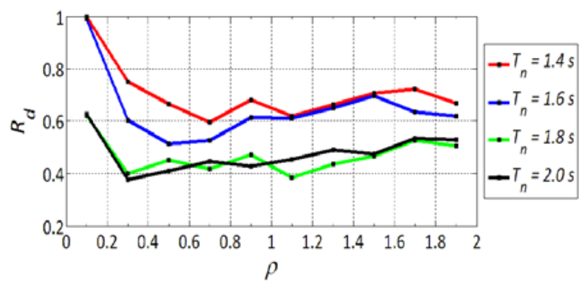


Fig. 9: Variations of R_d versus ρ for the 10-story structure subjected to El Centro ground motion

vary. In general, as shown in Fig. 8 middle stories along height have less priority for FDD installation when smaller values of N_f are considered.

In accordance with results of numerical analyses, as briefly shown in Table 3, total slip-load ratio (ρ) of the FDDs has a strongly nonlinear impact on dynamic response of the building structures. Consequently, no particular optimality exists for this design parameter of the FDBS and different values of ρ must be checked. However, an optimal range can be considered for ρ minimizing performance index of the structure if another FDBS design parameter, N_f, is well assumed. The results show that variations of the performance index (R_d) are limited to about 10% within the optimal range of ρ. Figure 9 typically displays variations of R_d versus ρ for the 10-story building subjected to El Centro ground motion for different fundamental periods when N_f is optimized earlier. The optimal range of ρ in the example structures of Fig. 9 is between 0.5 through 1.5.

CONCLUSION

In this study, effects of FDBS design parameters on seismic performance of low-to-medium-story building structures are investigated. For this purpose, design parameters of FDBS in SDOF structures are introduced and their influence on dynamic response of the system is examined. Results show that there is a threshold for stiffness ratio of braces beyond which no further response mitigation is achieved. Then, design parameters of FDBS are generalized to MDOF building structures. In this stage, improvement of seismic response of the structures with respect to variations of design parameters of FDBS including: total slip-load ratio of FDDs, number of FDD installations and arrangement of dampers along height of building structures, is investigated. In order to examine effects of fundamental period of the structure on design procedure of the FDBS, different periods are considered. It is concluded that that for a constant stiffness ratio of the braces and uniform distribution of slip-load ratio amongst FDDs, optimal normalized number of FDD installations, $(N_f /N)_{opt}$, increases or remains invariant in the range of 0.5 through 1.0 when fundamental period of the structure increases. To examine arrangements of FDD installations along height of the structures, entire possible states of damper placement are compared. The obtained results show that configuration of FDD placement is quite case-sensitive and no particular optimal pattern can be prescribed in general. However, a partial conformity observed in results of numerical analyses demonstrate that middle stories have less priority for FDD installation when smaller values of N_f are considered. To study influence of total slip-load ratio of FDDs, it is concluded that an optimal range can be found for ρ minimizing performance index of the structure if another FDBS design parameter, N_f, is well assumed.

REFERENCES

Aiken, I.D., J.M. Kelly and A.S. Pall, 1988. Seismic response of a nine-story steel frame with friction damped cross-bracing. Proceeding of the 9th World Conference on Earthquake Engineering. Tokyo and Kyoto, Japan.

Apostolakis, G. and G.F. Dargush, 2009. Optimal seismic design of moment-resisting steel frames with hysteretic passive devices. Earthq. Eng. Struct. D., 39(4): 355-376.

Aydin, E., M.H. Boduroglu and D. Guney, 2007. Optimal damper distribution for seismic rehabilitation of planar building structures. Eng. Struct., 29: 176-185.

Cho, C.G. and M. Kwon, 2004. Development and modeling of a frictional wall damper and its applications in reinforced concrete frame structures. Earthq. Eng. Struct. D., 33(7): 821-838.

Clough, R. and J. Penzien, 1993. Dynamics of Structures. 2nd Edn., McGraw-Hill, New York.

Constantinou, M.C., A. Mokha, A.M. Reinhorn and Teflon, 1990. Bearings in base isolation II Modeling. J. Struct. Eng., 116(2): 455-474.

Dyke, S.J., B.F. Jr. Spencer, M.K. Sain and J.D. Carlson, 1996. Modeling and control of magnetorheological dampers for seismic response reduction. Smart Mater. Struct., 5(5): 565-575.

Garcia, D.L. and T.T. Soong, 2002. Efficiency of a simple approach to damper allocation in MDOF structures. J. Struct. Control, 9: 19-30.

Gluck, N., A.M. Reinhorn, J. Gluck and R. Levy, 1996. Design of supplemental dampers for control of structures. J. Struct. Eng-ASCE, 122(12): 1394-1399.

Grigorian, C.E., T.S. Yang and E.P. Popov, 1992. Slotted bolted connection energy dissipaters. Report of National Science Foundation, University of California Berkeley.

Inoue, K. and S. Kuwahara, 1998. Optimum strength ratio of hysteretic damper. Earthq. Eng. Struct. D., 27: 577-588.

Lee, S.K., J.H. Park, B.W. Moon, K.W. Min, S.H. Lee and J. Kim, 2008a. Design of a bracing-friction damper system for seismic retrofitting. Smart Struct. Syst., 4(5): 685-696.

Lee, S.H., J.H. Park, S.K. Lee and K.W. Min, 2008b. Allocation and slip load of friction dampers for a seismically excited building structure based on storey shear force distribution. Eng. Struct., 30: 930-940.

Levy, R. and O. Lavan, 2006. Fully stressed design of passive controllers in framed structures for seismic loadings. Struct. Multidiscip. O., 32: 485-498.

Li, C. and A.M. Reinhorn, 1995. Experimental and analytical investigation of seismic retrofit of structures with supplemental damping, Part II-friction devices. Technical Report NCEER-95-0009, Buffalo (NY), State University of New York, Buffalo.

Liao, W.I., I. Mualla and C.H. Loh, 2004. Shaking-table test of a friction-damped frame structure. Struct. Des. Tall Spec., 13: 45-54.

Lu, L.Y., L.L. Chung, L.Y. Wu and G.L. Lin, 2006. Dynamic analysis of structures with friction devices using discrete-time state-space formulation. Comput. Struct., 84: 1049-1071.

Marko, J., D. Thambiratnam and N. Perera, 2006. Study of viscoelastic and friction damper configurations in the seismic mitigation of medium-rise structures. J. Mech. Mater. Struct., 1(6): 1001-1039.

Moreschi, L.M. and M.P. Singh, 2003. Design of yielding metallic and friction dampers for optimal seismic performance. Earthq. Eng. Struct. D., 32: 1291-1311.

Mualla, I.H. and B. Belev, 2002. Performance of steel frames with a new friction damper device under earthquake excitation. Eng. Struct., 24(3): 365-371.

Pall, A.S. and C. Marsh, 1982. Response of friction damped braced frames. J. Struct. Div-ASCE, 108(9): 1313-1323.

Park, J.H., J. Kim and K.W. Min, 2004. Optimal design of added viscoelastic dampers and supporting braces. Earthq. Eng. Struct. D., 33: 465-484.

Pong, W., Z.H. Lee, C.S. Tsai and B.J. Chen, 2009. Heuristic design procedure for structures with displacement-dependent damping devices. Eng. Computation., 26(4): 347-359.

Singh, M.P. and L.M. Moreschi, 2001. Optimal seismic response control with dampers. Earthq. Eng. Struct. D., 30: 553-572.

Xu, Y.L. and J. Teng, 2002. Optimum design of active/passive control devices for tall buildings under earthquake excitation. Struct. Des. Tall Buil., 11: 109-127.