An Acceptance Sampling Plan under Frechet Distribution Assuring Median Life

S. Balamurali, Muhammad Aslam and Mohammad Saber Fallah Nezhad
1Department of Mathematics, Kalasalingam University, Krishnankoil 626190, TN, India
2Department of Statistics, Forman Christian College University, Lahore 54000, Pakistan
3Department of Industrial Engineering, Yazd University, Iran

Abstract: Reliability sampling plans are used to determine the acceptability of a product at some future point in its effective life. Whenever the life test is performed based on the non-measurement characteristics, attributes sampling plans can be utilized. In this study, we propose an attribute reliability sampling plan for assuring median life time under failure censored life testing when the life time of the product follows a Frechet distribution. The optimal parameters of the proposed plan are determined by using two points on the operating characteristic curve approach. To determine the optimal parameters, an optimization problem is formulated so as to minimize the average number of failures to be observed.

Keywords: Acceptance sampling plans, Frechet distribution, life test, producer’s risk and consumer’s risk

INTRODUCTION

Acceptance sampling is one of the major areas of statistical quality control. Acceptance sampling is the methodology that deals with procedures by which decision to accept or reject the lot based on the results of the inspection of samples. Acceptance sampling prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. In other words, acceptance sampling yields quality assurance. It is also to be pointed out that the acceptance sampling plans are also used to reduce the cost of inspection. The acceptance sampling could be classified in to sampling plans by attributes and sampling plans by variables. In attributes sampling, several sampling plans are available. For example, single sampling plan, double sampling plan, multiple sampling plan, continuous sampling plan, skip-lot sampling plan etc.

If the quality characteristic is regarding the life time of the products, then the acceptance sampling procedure is called a life test. Quality personnel would like to know whether the life time of the products reach the consumer’s standard or not. Generally, when the life test assures that the mean or median life of the products exceeds some specified life time, then the lot of products is accepted otherwise the lot is rejected. In order to save the test time and testing cost, a truncated life test may be conducted to find the smallest sample size to ensure a certain mean or median life time of products when the life test is terminated at the pre-determined time \( t_0 \) and the number of failures observed does not exceed a given acceptance number \( c \). The decision is to accept the lot if a pre-determined mean or median life can be reached with a pre-determined high probability which provides protection to the consumers. Hence the life test is ended when \((c+1)\) st failure is observed or at the pre-assigned time \( t_0 \). Sampling plans based on truncated life tests have been developed and investigated by many authors. Sampling plans based on truncated life tests for exponential distribution was first discussed by Epstein (1954). The results were extended by Goode and Kao (1961) for Weibull distribution, by Gupta and Groll (1961) for gamma distribution, by Kantam and Rosaiah (1998) for half logistic distribution, by Kantam et al. (2001) for log-logistic distribution, by Baklizi and El Masri (2004) for Birnbaum-Saunders distribution, by Rosaiah and Kantam (2005) for inverse Rayleigh distribution, by Tsai and Wu (2006) for generalized Rayleigh distribution and by Balakrishnan et al. (2007) for generalized Birnbaum-Saunders distribution. Rosaiah et al. (2006) developed sampling plans exponentiated log-logistic distribution. Tsai and Wu (2006) developed sampling plans for generalized Rayleigh distribution and by Balakrishnan et al. (2007) for generalized Birnbaum-Saunders distribution. Rosaiah et al. (2006) developed sampling plans exponentiated log-logistic distribution. Tsai and Wu (2006) developed sampling plans for generalized Rayleigh distribution.

In this study, we develop an acceptance sampling plan for sentencing lots when the life time of a product follows Frechet distribution. Tables are also constructed for the selection of optimal parameters for given two...
points on the Operating Characteristic (OC) curve approach along with respective probability of risks. The rest of the study is organized as follows.

**FRECHET DISTRIBUTION**

Frechet (1927) had developed one possible limit distribution for the largest order statistic called the Fréchet distribution. Kotz and Nadarajah (2000) discussed the applications of this distribution in over fifty fields ranging from accelerated life testing through to earthquakes, foods, horse racing, rainfall, queues in supermarkets, sea currents, wind speeds and track race records. This distribution is also known as the Extreme Value Type II distribution. Suppose that the product lifetime follows a Frechet distribution which has the following Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) respectively:

\[ f(t) = \left(\frac{\gamma}{s} t^{\gamma-1} e^{-\left(\frac{t}{s}\right)^\gamma}\right), \quad t > 0 \]  
\[ F(t) = e^{-\left(\frac{t}{s}\right)^\gamma}, \quad t > 0 \]  

where,  
\( \gamma \) = The shape parameter  
\( s \) = The scale parameter of the Frechet distribution

The mean of the distribution is given by:

\[ \mu = s \left(1 - \frac{1}{\gamma}\right) \]  

where, \( \sqrt{\Gamma(\gamma)} \) is a gamma function. The probability of the failure of an item before experiment time \( t_0 \) using the mean as the life time is given by:

\[ p = e^{-\left(\frac{\mu}{m/m_0}\right)^\gamma} \]

where, \( \mu/m_0 \) is the ratio of true mean life to the specified median life, \( a \) is the termination ratio and \( w = \sqrt{\left(1 - \frac{1}{\gamma}\right)} \).

The median of the distribution is given by:

\[ m = \frac{s}{\sqrt{\log_e(2)}} \]

The acceptance sampling plans based on truncated life test consists of the minimum number of units on the test \( n \), the acceptance number \( c \) and the ration \( m/m_0 \), where \( m_0 \) is the specified median life time of the Frechet distribution. The proposed sampling plan based on the Fréchet distribution is denoted by the triplet \( (n, c, m/m_0) \).

\[ p = e^{-\left(\frac{a}{b/m/m_0}\right)^\gamma} \}

where,
\( m/m_0 \) = The ratio of true median life to the specified median  
\( a \) = The termination ratio  
\( b = \sqrt{\log_e(2)} \)

**ACCEPTANCE SAMPLING PLANS**

Assume that a truncated life test is conducted for life time of products that have a Frechet distribution defined in (1). The main objective of the test is to set a lower confidence limit on the median life and to test whether the median life of the products is equal to or larger than the specified expected median. Let \( m \) represent the true median life time of a product and \( m_0 \) denote the specified median life time. A product is considered as a good and can be accepted for the consumer’s use if the sample information should satisfy the null hypothesis \( H_0: m \geq m_0 \), otherwise the lot of the products rejected. That is, a lot is said to be a good lot if the true median life of the products in the lot should be at least \( m_0 \) and a lot is classified as a bad one if the true median life of the products is below \( m_0 \). In the concept of acceptance sampling, this hypothesis is tested based on the number of failures observed from the sample in a pre-determined time. We will accept the lot if there is sufficient evidence that \( m > m_0 \), otherwise we reject it. Based on this, the operation of sampling plan is as follows:

- Select a random sample of size \( n \) and put on the tests. An experimenter runs this experiment for a pre-decided experiment time \( t_0 \). The acceptance number (action number) \( c \) is fixed for an experiment.
- The experiment is truncated if more than \( c \) failures are recorded before the end of experiment time or the time of experiment is ended, whichever is earlier. A lot is accepted and released for consumer’s use if no more than \( c \) failures are observed during this time and we accept \( m \geq m_0 \), otherwise we reject it.

As mentioned already, an acceptance sampling plan based on truncated life test consists of the minimum number of units on the test \( n \), the acceptance number \( c \) and the ration \( m/m_0 \), where \( m_0 \) is the specified median life time of the Frechet distribution. The proposed sampling plan based on the Frechet distribution is denoted by the triplet \( (n, c, m/m_0) \).
There are two kinds of risks always involved in any acceptance sampling plan. The probability of rejecting a good lot is called the producer’s risk (α) whereas the probability of accepting a bad quality lot is known as the consumer’s risk (β). The OC function of the proposed plan is given by:

\[ L(p) = \sum_{i=1}^{n} p^i(1-p)^{n-i} \] (7)

where, \( p \) is the function of cdf of underlying lifetime distribution and is obtained from Eq. (2). We need to find the values of plan parameters mentioned above. It is only possible, if both producer’s risk and consumer’s risk are used in simulation process. This approach to simulate the optimal plan parameters is known as the two points on the OC curve approach. In acceptance sampling schemes, producer and consumer are more conscious about the rejection of a good lot and acceptance of a bad lot respectively. So, the producer wants that the probability of acceptance should be larger than his confidence level \((1 - \alpha)\) and consumer desires that the lot acceptance probability should be less than his risk \(\beta\). The main advantage of the simulation process using this approach is that we get the plan parameters that are according to the producer’s and consumer’s risk desire. Other than two conditions stated above, we need to impose some other constraints to reduce the space to reach on appropriate combinations of the plan parameters. Let \( p_1 \) (Acceptable Quality Level (AQL)) be the probability of failure corresponding to producer’s risk and \( p_2 \) (Limiting Quality Level (LQL)) is the probability of failure of the product corresponding to consumer’s risk. We are interested to find four values of the plan parameters such that the following two inequalities should be satisfied:

\[
L(p_i) = \sum_{i=1}^{n} p_i^i (1-p_i)^{n-i} \geq 1 - \alpha
\]

\[
L(p_2) = \sum_{i=1}^{n} p_2^i (1-p_2)^{n-i} \leq \beta
\] (8)

Under the Frechet distribution \( p_1 \) and \( p_2 \) obtained from producer’s risk at \( m/m_0 \) by considering median ratios larger than 1 and consumer’s risk at \( m/m_0 = 1 \). These probabilities under the Frechet distribution can be obtained from Eq. (6). We obtained the plan parameters of the proposed plan by minimizing the sample size at consumer’s risk. These optimal parameters of the proposed plan under the Frechet distribution are placed in Table 1 to 3. We considered three values of shape parameters of the Frechet distribution \((\gamma = 1, 2, 3)\), experiment termination ratios \((a = 0.5, 1.0)\), producer risk \((\alpha = 0.05)\), consumer’s risk \((\beta = 0.25,0.10,0.05,0.01)\) and various values of percentile ratios \(m/m_0 \) = 1.2, 1.4, 1.6, 1.8, 2.2, 2.2. Table 1 is constructed for shape parameter 1, Table 2 is made for shape parameter 2, Table 3 is for shape parameter 3.

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(m/m_0)</th>
<th>(n)</th>
<th>(c)</th>
<th>(\text{ASN at } p_2 )</th>
<th>(\text{ASN at } p_1 )</th>
<th>(\text{ASN at } p_2 )</th>
<th>(\text{ASN at } p_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.2</td>
<td>253, 58</td>
<td>253</td>
<td>0.9523</td>
<td>129, 65</td>
<td>129</td>
<td>0.9511</td>
</tr>
<tr>
<td>1.4</td>
<td>77, 16</td>
<td>77</td>
<td>0.9558</td>
<td>94, 43</td>
<td>94</td>
<td>0.9517</td>
<td>73, 52</td>
</tr>
<tr>
<td>1.6</td>
<td>38, 7</td>
<td>38</td>
<td>0.9511</td>
<td>46, 20</td>
<td>46</td>
<td>0.9501</td>
<td>73, 49</td>
</tr>
<tr>
<td>1.8</td>
<td>39, 22</td>
<td>39</td>
<td>0.9622</td>
<td>29, 12</td>
<td>29</td>
<td>0.9526</td>
<td>48, 31</td>
</tr>
<tr>
<td>2.0</td>
<td>26, 14</td>
<td>26</td>
<td>0.9517</td>
<td>23, 9</td>
<td>23</td>
<td>0.9591</td>
<td>30, 19</td>
</tr>
<tr>
<td>2.2</td>
<td>15, 2</td>
<td>15</td>
<td>0.9685</td>
<td>16, 6</td>
<td>16</td>
<td>0.9591</td>
<td>24, 15</td>
</tr>
<tr>
<td>0.10</td>
<td>1.2</td>
<td>309, 70</td>
<td>309</td>
<td>0.9562</td>
<td>129, 65</td>
<td>129</td>
<td>0.9511</td>
</tr>
<tr>
<td>1.4</td>
<td>118, 23</td>
<td>118</td>
<td>0.9526</td>
<td>129, 58</td>
<td>129</td>
<td>0.9585</td>
<td>73, 52</td>
</tr>
<tr>
<td>1.6</td>
<td>64, 11</td>
<td>64</td>
<td>0.9582</td>
<td>72, 30</td>
<td>72</td>
<td>0.9526</td>
<td>73, 49</td>
</tr>
<tr>
<td>1.8</td>
<td>40, 6</td>
<td>40</td>
<td>0.9567</td>
<td>46, 18</td>
<td>46</td>
<td>0.9543</td>
<td>70, 44</td>
</tr>
<tr>
<td>2.0</td>
<td>30, 4</td>
<td>30</td>
<td>0.9633</td>
<td>33, 12</td>
<td>33</td>
<td>0.9516</td>
<td>49, 30</td>
</tr>
<tr>
<td>2.2</td>
<td>25, 3</td>
<td>25</td>
<td>0.9713</td>
<td>26, 9</td>
<td>26</td>
<td>0.9600</td>
<td>37, 22</td>
</tr>
</tbody>
</table>

Table 1: Optimal parameters for the SSP under the Frechet distribution with shape parameter \(\gamma = 1\)
constructed for the shape parameter 2 and at the same time, Table 3 is constructed for the shape parameter 3.

**ILLUSTRATIVE EXAMPLES**

Suppose that manufacturer wants to adopt a single sampling plan for assuring that the median life of the products under inspection is at least 1000 h when \( \beta = 0.10 \) and \( \alpha = 0.05 \) at the median ratio = 2. He wants to run this experiment 1000 h. From the past data, it is observed that the lifetime of the product follows a Frechet distribution with shape parameter 1. Table 1 can be used to find the optimal parameters of the single sampling plan. The optimal plan from Table 1 for
specified requirements such as \( \alpha = 0.05, \beta = 0.10, \gamma = 1, \frac{m}{m_0} = 2 \) and \( a = 1.0 \) is obtained as: \( n = 23 \) and \( c = 9 \). It is to be pointed out that most of the life testing sampling plans for various life time distributions like exponential, Weibull etc available in the literature is based on one point on the OC curve approach for assuring mean or percentile life time. But in this study, we have designed sampling plans based on two-points on the OC curve approach for assuring median life time of the products under Frechet distribution. Hence we cannot make any comparative study of the results of the proposed plan with the results of the existing sampling plans.

**Real world example:** Now we will explain the proposed plan using the failure times of air conditioning system of an air plane (Bain and Engelhardt, 1991). The lifetimes of 11 air conditioning system of an air plane are as: 33, 47, 55, 56, 104, 176, 182, 220, 239, 246 and 320. We found that the failure time data is well fitted to Frechet distribution with \( \gamma = 1.19 \) (close to 1). Suppose that the specified median life of air conditioning system is 40 and we want to run the test for 80 h (this leads to termination ratio \( a = 2.0 \)) with \( \alpha = 0.05, \beta = 0.25, \gamma = 1, \frac{m}{m_0} = 2 \) and \( a = 2.0 \). From Table 1, we have plan parameters \( n = 30 \) and \( c = 19 \). According to the plan, we will accept the products if no more than 19 failures are noted in air conditioning systems before 80 h. From the ordered lifetime air conditioning data, we note that four failures 33, 47, 55 and 56 are recorded before the termination time 80 h. So, according to the plan, the air conditioning system should be accepted.

**CONCLUSION**

In this study, we have proposed a single sampling plan for the truncated life test when the life time of the product follows a Frechet distribution since Frechet distribution has been shown to be a useful model in the areas of quality and control and reliability studies. To ensure that the life quality of products exceeds a specific standard, acceptance sampling plans based on median can be used. In this study, a single sampling plan has been developed based on the median of Frechet distribution when the life test is truncated at a pre-determined time. Some useful tables have also been provided selection of optimal parameters. The optimal parameters are determined in such a way that it should satisfy two points on the OC curve.

**REFERENCES**


