Improving Network Performances by Integrating Erasure Coding and Game Theory Techniques in Random Access Protocols

Mohamed Lamine Boucenna, Hadj Batatia and Malek Benslama

Electromagnetism and Telecommunications Laboratory (LET), University Mentouri, 25000 Constantine, Algeria

IRIT/INP-ENSEEIHT, University of Toulouse, 2, rue Charles Camichel, 31071 Toulouse Cedex 7, France

Abstract: This research focuses on network performance and how to solve the problem of low throughput in the Aloha Medium Access Control (MAC) protocol and its derivatives. For this purpose, we propose two complementary solutions. The first consists of the integration of the erasure coding scheme in this protocol to recover collided packets and to reduce the rate of collision between transmitted packets. Here, since each node sends \( N \) coded packets instead of the \( k \) original packets, we have \( (N-k) \) redundant packets. The introduction of redundancy and subsequently structuring it in an exploitable manner, allows serious errors injected by the channel to be corrected. However, if each node attempts to achieve its best output without regard for the other nodes’ actions, this could affect overall system throughput. To analyze such conflicting situations where the action of one node has an impact on the other nodes’ actions, we add a complementary solution, which is based on the game theory technique of acquiring network equilibrium. This makes the network stronger and able to resist many collisions.

Keywords: Erasure coding, game theory, network equilibrium, random access MAC protocols

INTRODUCTION

The ALOHA system is one of the earliest random access systems and in the slotted version; all nodes are synchronized with a global clock and only allowed to start transmission at the beginning of common time slots. The CSMA protocol adds important functionality to the MAC protocols and, as its name suggests, CSMA (and its variants) senses the medium to determine whether any other node is transmitting. Despite a lot of preventative techniques, these protocols still suffer from packet collisions, thus reducing throughput and affecting system performance. In an attempt to improve the efficiency of these protocols, several models and solutions have been proposed. A game theoretic approach for wireless networks was proposed by Akkarajitsakul et al. (2011), a deep analysis of random MAC protocols was presented by Altman et al. (2004, 2005), Boucenna and Benslama (2012) and Mackenzie and Wicker (2001) and considerable improvement in throughput was achieved by King-Sun et al. (2005). In this paper, we proceed in the same context by integrating the erasure coding schema in the Aloha and CSMA protocols to recover collided packets and accordingly increase network throughput in both of protocols with more results and simulations than done by Boucenna and Benslama (2012). However, users wishing to transmit typically want to do as soon as possible by the lower cost and if multiple users try to transmit randomly without following a suitable manner for all users; many accesses fail. Moreover, unsuccessful attempts to transmit may be costly. Thus, users trying to transmit have conflicting objectives in cost and throughput and the appropriate tool for examining interaction between selfish users with conflicting objectives is game theory. Therefore, we formulate a game model and investigate a network equilibrium in which all users are satisfied.

Erasure coding: The purpose of erasure coding is to recover lost packets if their position is known in \((N-k)\) erasure codes. An \((N-k)\) code word consists of \(N\) code packets, with \(k\) original packets and \((N-k)\) redundant packets. The \(k\) original packets can be recovered successfully if we receive \(N'\) packets out of the \(N\) coded packets. If \(N\) is sufficiently large compared to the loss rate, we can achieve high reliability without retransmission. Figure 1 illustrates the mechanism of erasure coding, when \((N-k)\) redundant packets are generated, while Shacham and McKenney (1990) describes how to generate redundant packets.

The classic block codes for erasure correction are called Reed-Solomon codes. An \((N-k)\) Reed-Solomon code (over an alphabet of size \(q = 2^l\), where \(l\) is the
length of a packet) has the ideal property that if any $k$ of the $N$ transmitted symbols are received; the original $k$ source symbols can be recovered. But Reed-Solomon codes have the disadvantage that they are practical only for small $N$, $k$ and $q$; standard implementations of encoding and decoding have a cost of $C$ packet operations (the expected number of arithmetic operations to compute the coded packets, or to recover the original packets), where,

$$C = k \left( N - k \right) \log_2 N$$  \hspace{1cm} (1)

Furthermore, with a Reed-Solomon code, as with any other block code, the erasure probability $f$ must be estimated and the code rate $R = k/N$ selected before the data are transmitted.

The game and its model: We consider $M$ wireless nodes that are willing to transmit data (active nodes) to a designated receiver. Nodes use a slotted Aloha or non-persistent CSMA based protocol to resolve contention at the MAC layer and to reduce the number of collisions among packets by always rescheduling a packet that upon arrival finds the channel to be busy. A slotted version of the Aloha protocol can be considered where the time axis is slotted with a slot size of $\tau$ sec (the propagation delay). All nodes are synchronized and are forced to start transmission only at the beginning of a slot. Slot duration is set to the maximum signal propagation time of $\tau = d_{max}/C$ sec, where $d_{max}$ is the maximum separation distance between nodes and $C$ is the speed of light. This ensures that after transmission stops, all nodes will find the channel to be clear after one slot time. Thus, each transmission must be preceded by an idle slot. We assume also that all packets are of constant length and the number of bits per packet $l$ satisfies the condition for Reed-Solomon coding, $E < q$, ($q = 2^l$). Thus, each transmission must be preceded by an idle slot. We assume also that all packets are of constant length and the number of bits per packet $l$ satisfies the condition for Reed-Solomon coding, $E < q$, ($q = 2^l$).

We assume that the nodes are the players in the game. A player enters the game when he has a packet to transmit (active player) and leaves the game when all his packets have been successfully transmitted. The game in this paper is a repeated non-cooperative game, i.e., there is no coordination between players and they act as free agents and each player attempts to maximize his own payoff according to his strategy. Here the strategy is to select a suitable type of erasure coding and the utility is the player’s throughput. When in the game, the player chooses the type of coding that minimizes the cost of coding, but at the same time maximizes his utility (throughput). After each transmission failure, the player repeats his strategy until transmission succeeds. Figure 2 illustrates the game model where each node selects $\rho_i$ in coding before transmitting via a medium in a wireless network. The strategy of each player $i$ involves setting the number of redundant packets $\rho_i$ to maximize player $i$’s expected utility $u_i$.

A game consists of a principal and a finite set of players $M = \{1, 2, \ldots, M\}$, each of which selects a strategy $S_i \in S$ with the aim of maximizing his utility $u_i$. The utility function $u_i$ ($s$): $S \rightarrow R$ represents each player’s sensitivity to the actions of the other players. Thus, our game can be modeled as a triple, $G = (M, s, u)$ where $M = \{1, 2, \ldots, M\}$ denotes the set of players in the game (action set), $s$ denotes the set of strategies for the players ($s_i$ denotes the strategies for player $i$ and $s_i$ denotes the strategies for all players except player $i$, $s = \{s_1, s_2, \ldots, s_M\} = \{\rho_1, \rho_2, \ldots, \rho_M\}$) and $u$ denotes the payoff assigned to each player (where $U_i$ is the payoff utility assigned to player $i$).

To measure the utility (throughput) function, we follow the probability concept based on that in King-Sun et al. (2005). However in our study, we take into consideration the number of nodes forming the network, as well as the likelihood of choosing a type of encoding (decoding) and also the cost of encoding/decoding, which greatly influences the speed of transmission.
of the system. Next we derive the expressions for throughput for both protocols (slotted Aloha and non-persistent CSMA).

**The slotted aloha game:** In our game we have \( M \) nodes, \( \lambda \) (packet/time slot) traffic load with the transmission of packets following a Poisson process with mean \( \lambda \) and throughput \( Th \) defined as:

\[
Th = \lambda P_s
\]

(2)

where,

\[
P_s = P_k + \sum_{n=1}^{k-1} \sum_{m=1}^{n} \frac{m}{k} P_{n,m}
\]

(3)

The probability that a packet is successfully transmitted is given by:

\[
p = (e^{-\lambda N/k})(1 - e^{-\lambda N/k})^{M-1}
\]

(4)

For simplicity, we set \( G = \lambda N/k = \lambda (1 + \rho/k) \), where \( \rho \) is the number of redundant packets, then:

\[
P_k = (e^{-\lambda(1+\rho/k)})(1 - e^{-\lambda(1+\rho/k)})^{M-1}
\]

(5)

Let \( Q_\rho = 1 - P_\rho \). Then, the probability that at least \( k \) encoded packets are received successfully is:

\[
P_k = \sum_{i=0}^N \binom{N}{i} P_\rho^i Q_\rho^{N-i}
\]

(6)

\( P_{n,m} \) is the probability that only \( n \) (\( n \leq k \)) encoded packets are received where \( m \) out of \( n \) are original packets. Then, we have:

\[
P_{n,m} = \binom{k}{m} \binom{N-k}{n-m} P_\rho^m Q_\rho^{N-m}
\]

(7)

An original packet can be successfully received or recovered if at least \( k \) out of \( N \) encoded packets are correctly received or \( n \) (\( n \leq k \)) out of \( N \) encoded packets are received, \( m \) out of \( n \) received packets are original and the packet under consideration is among these packets (with probability \( m/k \)). Thus, from Eq. (3) we conclude:

\[
P_s = \sum_{i=0}^N \binom{N}{i} P_\rho^i Q_\rho^{N-i} + \\
+ \sum_{n=1}^{k-1} \sum_{m=1}^{n} \frac{m}{k} P_{n,m}
\]

(8)

From Eq. (2), the throughput can be expressed as:

\[
Th = \lambda (P_k + \sum_{n=1}^{k-1} \sum_{m=1}^{n} \frac{m}{k} P_{n,m})
\]

(9)

In addition:

\[
Th(\rho) = \lambda \left[ \sum_{i=0}^N \binom{N}{i} P_\rho^i Q_\rho^{N-i} + \\
+ \sum_{n=1}^{k-1} \sum_{m=1}^{n} \frac{m}{k} \binom{k}{m} \binom{N-k}{n-m} P_\rho^m Q_\rho^{N-m} \right]
\]

(10)

Accordingly, the utility function for an active player \( i \) is defined as:

\[
Th(\rho) = \lambda \left[ \sum_{i=0}^N \binom{N}{i} P_\rho^i Q_\rho^{N-i} + \\
+ \sum_{n=1}^{k-1} \sum_{m=1}^{n} \frac{m}{k} \binom{k}{m} \binom{N-k}{n-m} P_\rho^m Q_\rho^{N-m} \right]
\]

(11)

**The non-persistent CSMA game:** In CSMA, when a node attempts to transmit a packet, it senses the channel first. If the channel is idle, the packet is sent immediately; however, if the channel is busy the node delays its transmission until the channel becomes idle. As a result, collision probability can be reduced and performance is improved. Here, by following the same process as in the slotted Aloha game, we can express the utility function as:

\[
Th = \frac{\bar{U}}{B + \bar{I}}
\]

(12)

where, \( B = T + \tau, \bar{I} = \frac{e^{-\alpha G}}{1 - e^{-\alpha G}} \) and \( \bar{U} = U_s P_s T \).

Let:

\[
U_s = \frac{P\{\text{one arrival in interval } T\}}{P\{\text{some arrival occurs}\}}
\]

Then:

\[
U_s = \frac{e^{-\alpha G} (1 - e^{-\alpha G})^{M-1}}{e^{-\alpha GM}}
\]

(13)

Then for \( P_s \), the probability that a packet is successfully transmitted is:

\[
p = (e^{-\alpha G}) (1 - e^{-\alpha G})^{M-1}
\]

(14)

We have \( G = \lambda N/k = \lambda (1 + \rho/k) \), where \( \rho \) is the number of redundant packets, then:

\[
P_\rho = (e^{-\alpha G}) (1 - e^{-\alpha G})^{M-1}
\]

(15)

For simplicity, let \( Q_\rho = 1 - P_\rho \).
Fig. 3: Conventional model throughput in S-aloha with varying \( \lambda \).

![Conventional model throughput in S-aloha with varying \( \lambda \)](image)

Fig. 4: Conventional model Th in S-aloha with varying values of \( \lambda \) and \( \alpha \). When \( \lambda \in [0.7, 1.5] \), Th is maximized.

The probability that at least \( k \) encoded packets are received successfully is:

\[
P_k = \sum_{i=1}^{N} \binom{N}{i} P_i^i Q_{1-i}^{N-i}
\]

(16)

Let \( P_{n,m} \) be the probability that only \( n \) (\( n \leq k \)) encoded packets are received, where \( m \) out of \( n \) are original packets; then we have:

\[
P_{n,m} = \binom{k}{m} \binom{N-K}{n-m} P_i^i Q_{1-i}^{N-m}
\]

(17)

We also have:

\[
P_s = P_k + \sum_{n=1}^{k} \sum_{m=1}^{n} (P_k + m/k) P_{n,m}
\]

(18)

Moreover, the utility function for an active player \( i \) is defined as:

\[
Th(\rho_i) = \frac{(1 - e^{-\alpha(1+\rho_i)/k})^M P_s e^{-\alpha(1+\rho_i)(M-1)}}{1 + \alpha - e^{-\alpha(1+\rho_i)/k}}
\]

(21)

Existence of Nash equilibrium:

Theorem 1: (Fudenberg and Tirole, 1991): Debreu and Glicksberg (1952) have considered a strategic-form game whose strategy spaces \( S_i \) are nonempty compact convex subsets of a Euclidean space. If the payoff functions \( U_i \) are continuous and quasi-concave in \( S_i \), there exists a pure strategy Nash Equilibrium (NE).

Theorem 2: (Fudenberg and Tirole, 1991): An Equilibrium exists for every concave \( n \)-person game.

For such a case, Rosen proved the existence of equilibrium for a concave utility function. It is thus clear that since our utility (11) satisfies all preceding conditions \( \left( \frac{\partial^2 U_i}{\partial \rho_i^2} = 0 \right) \), both theorems are applicable to this game. Thus, a pure NE exists. To find the NE of the game we analyze the player’s best response function (Osborne, 2004). The best response of player \( i \) is the number of redundant packets that maximizes the utility function.

Definition 1: (Jerzy, 2007): An action \( S \) is the best reply to \( S_i \), if \( u_i(s, s_i) \geq u_i(s, s_i') \) for all \( s' \in S \). Let \( BR(S_i) \) denote the set of best replies to \( S_i \). An NE is an action profile \( s = (s, s_i) \) in which \( s_i \in BR(S_i) \) for all \( i = 1... M \).

First, we investigate whether there exists a value of \( \rho_i \) such that a better throughput can be achieved. We consider the scenario where all nodes use the same value \( \rho_i = \rho \) and modify this in synchronization with the other nodes in the network. Note that all our simulations were carried out using the MATLAB simulator.

In slotted aloha: We investigated the behavior of the conventional system throughput (before integrating erasure coding) by varying \( \lambda \). According to Fig. 3 and 4, throughput is maximized when \( \lambda \in [0.7, 1.5] \). Figure 4 plots in 3D the average throughput obtained by the network for different values of \( \lambda \) and \( \alpha \). For better and more precise results, we use \( \alpha = 0.05 \) in the reminder of this work (like the conventional model).

Moreover, Fig. 5 confirms that operating with \( \rho = 0 \) leads to network collapse. However, there exists an optimal point of operation (\( \rho = 2 \) packets) at which throughput is maximized \( Th(\rho = 2) = \arg \max Th \) for every node in the network. We refer to this optimal point of operation as \( \rho^* \), which corresponds to the NE for our scheme. In other words, when \( \rho \neq \rho^* \), congestion dominates resulting in lower throughput, whereas when \( \rho = \rho^* \) throughput decreases. Thus, \( Th(\rho, \rho^*) > Th(\rho, \rho) \) for all \( \rho' \in \rho \) and \( \rho = BR(\rho_i) \).
In **CSMA**: We investigate whether there exists a value of $\rho_i$ such that better throughput can be achieved. We consider the scenario where all the nodes use the same value $\rho_i = \rho$ and modify it in synchronization with the other nodes in the network. Figure 6 and 7 plot the average throughput obtained by a node for different values of $\rho$. These figures confirm that operating with $\rho = 0$ leads to network collapse. However, there exists an optimal point of operation ($\rho = 5$ packets) at which throughput is maximized, $Th(\rho = 5) = \text{argmax}Th$, for every node in the network. We refer to this optimal point of operation as $\rho^*$, which corresponds to the NE for our scheme. It shows that when $0 < \rho \leq \rho^*$, congestion dominates, resulting in lower throughput. However, when $\rho \geq \rho^*$ throughput decreases. Thus, $Th(\rho, \rho - \lambda) \geq Th(\rho, \rho - \lambda')$, for all $\rho' \in \rho$ and $\rho = \text{BR}(\rho_i)$. In the remainder of this section, we use $\alpha = 0.2$.

**Performance improvement:**

In **slotted aloha**: Now, we investigate the importance of the obtained results. We begin this assessment with the cost of coding employed in our model. The cost of coding denotes the number of arithmetic operations performed during encoding, or decoding operations to recover encoded packets. In other words, it represents the speed of the system. The fastest system is the one with the lowest cost. Figure 5 shows that cost decreases at equilibrium.

In Fig. 8 we evaluate the cost while varying the number of redundant packets as well as the number of original packets used during erasure coding. At equilibrium, we have the lowest cost [0.9, 2.2%]; in this case, the encoder (decoder) carried out 90 arithmetic operations to generate two redundant packets (or recover eight original packets). Furthermore, it performed 950 arithmetic operations for ten redundant packets. This difference greatly affects the speed and also system performance. Figure 9 shows a comparison of throughput at equilibrium and without equilibrium (conventional system). In Fig. 9 and 10, we can see the improvement due to equilibrium, which becomes even clearer when we mimic reality by using a greater number of active nodes ($M = 20$). In Fig. 9, there is a small difference in throughput, which is possible only
Figure 10 shows a remarkable improvement in throughput when using the NE model, where the difference can reach 0.02 packets/sec, with $\lambda = 2$. Note that when we increase the number of active nodes (users), the advantage of equilibrium becomes more important.

At Equilibrium (NE), throughput increases quickly to reach its maximum, after which it begins to deteriorate slowly until it reaches its lowest level at $\lambda = 10$ packets/sec. It should be noted that, by increasing the number of active nodes, network throughput decreases; this is due to the number of collisions, which increases when increasing the number of active nodes (users) (Fig. 11).

The use of the erasure coding scheme and game theory analysis in the slotted Aloha protocol results in a stable and more efficient network. However, to confirm this advantage in general, in the following section we apply the same process to the non-persistent CSMA protocol.

In CSMA: Here, we investigate the importance of the obtained results. Game theory has allowed us to discover the equilibrium in the MAC layer after integrating erasure coding in the non-persistent CSMA protocol. However, in order to evaluate these results, we need to examine the behavior of certain particular characteristics and then compare these to the results obtained using other values not in equilibrium.

As before, we begin the assessment considering the cost of coding used in our model. The cost of coding gives the number of arithmetic operations carried out during the encoding and also during the decoding operations to recover encoded packets. In other words, it represents the speed of the system. The fastest system is the one with the lowest cost. Figure 12 shows the lower cost at equilibrium. From Fig. 12 we can evaluate the cost of coding when varying the number of redundant packets as well as the number of original packets used during erasure coding. At equilibrium, we have the lowest cost [1-3.5%]; in this example, the encoder (decoder) carried out 350 arithmetic operations to generate five redundant packets (or recover 14 original packets). Furthermore, it performed 1050 arithmetic operations for 12 redundant packets. This difference greatly affects the system speed and also its performance. Figure 13 presents a comparison of throughput at Equilibrium (NE) and without equilibrium (conventional). Figure 14 describes in more detail, the behavior of throughput when $\lambda \in [0.04, 0.14]$. For values beyond this interval, use of the NE model becomes ineffective.

For a small $\lambda$: $\lambda \in [0.04, 0.14]$. For values beyond this interval, use of the NE model becomes ineffective.

Figure 10 shows a remarkable improvement in throughput when using the NE model, where the difference can reach 0.02 packets/sec, with $\lambda = 2$. Note that when we increase the number of active nodes (users), the advantage of equilibrium becomes more important.

At Equilibrium (NE), throughput increases quickly to reach its maximum, after which it begins to deteriorate slowly until it reaches its lowest level at $\lambda = 10$ packets/sec. It should be noted that, by increasing the number of active nodes, network throughput decreases; this is due to the number of collisions, which increases when increasing the number of active nodes (users) (Fig. 11).

The use of the erasure coding scheme and game theory analysis in the slotted Aloha protocol results in a stable and more efficient network. However, to confirm this advantage in general, in the following section we apply the same process to the non-persistent CSMA protocol.

In CSMA: Here, we investigate the importance of the obtained results. Game theory has allowed us to discover the equilibrium in the MAC layer after integrating erasure coding in the non-persistent CSMA protocol. However, in order to evaluate these results, we need to examine the behavior of certain particular characteristics and then compare these to the results obtained using other values not in equilibrium.

As before, we begin the assessment considering the cost of coding used in our model. The cost of coding gives the number of arithmetic operations carried out during the encoding and also during the decoding operations to recover encoded packets. In other words, it represents the speed of the system. The fastest system is the one with the lowest cost. Figure 12 shows the lower cost at equilibrium. From Fig. 12 we can evaluate the cost of coding when varying the number of redundant packets as well as the number of original packets used during erasure coding. At equilibrium, we have the lowest cost [1-3.5%]; in this example, the encoder (decoder) carried out 350 arithmetic operations to generate five redundant packets (or recover 14 original packets). Furthermore, it performed 1050 arithmetic operations for 12 redundant packets. This difference greatly affects the system speed and also its performance. Figure 13 presents a comparison of throughput at Equilibrium (NE) and without equilibrium (conventional). Figure 14 describes in more detail, the behavior of throughput when $\lambda \in [0.1]$. Figure 14 illustrates the behavior of network throughput versus the offered traffic load and the number of redundant packets. Throughput is maximized when $\rho = 5$ and $\lambda \in [0.5, 5]$.

At Equilibrium (NE) throughput increases quickly to reach its maximum, $T_{\text{th max}} = 0.5$, which is better than that found in the conventional model, $T_{\text{con max}} = 0.05$.\n
Fig. 10: S-aloha throughput versus $\lambda$ with $M = 20$

Fig. 11: Behavior of throughput versus the active nodes in S-aloha

Fig. 12: Cost of coding versus $\rho$ in CSMA

Fig. 13: CSMA throughput versus $\lambda$.
This confirms the advantage of the Nash equilibrium discovered by applying game theory analysis. Thereafter, throughput begins to deteriorate slowly until it reaches its lowest level ($T_{th_{\min}} = 0.05$) at $\lambda = 10$ packets/sec.

As in the slotted Aloha protocol, there is a remarkable improvement in the performance of slotted non-persistent CSMA with the application of an erasure coding scheme and game theory analysis.

**CONCLUSION**

In this paper, we created non-cooperative and repeated game models for the slotted Aloha and slotted non-persistent CSMA protocols to improve network performance. At equilibrium, network throughput is maximized and all nodes are satisfied, without the need to change their strategies, which makes the network stable and more efficient. Use of our proposed solutions based on erasure coding and game theory makes the Aloha and CSMA protocols stronger and able to resist many collisions. Our results were verified by appropriate simulations, where we compared both the conventional and game models. The simulations show acceptable progress in terms of throughput and speed (coding cost). Thus, the use of game theory is beneficial for contention based medium access protocols.

**REFERENCES**


