Research on Matrix Converter Based on Space Vector Modulation

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Abstract: In this study, we study the control strategy for matrix converter. Through absorbing ideas about virtual rectification put forward by P.D. Ziogas, the common control idea of AC-DC-AC converter is introduced into control of matrix converter; SPWM modulation strategy and space vector modulation strategy for matrix converter are studied respectively. In terms of SPWM modulation strategy, it has some advantages: for example, main circuit structure is simple; control program is simple; control switch function does not need complex mathematical derivation and calculation. In terms of space vector modulation strategy, it has advantages as follows: the physical conception is clear; input power factor can be adjusted; output voltage and current sine degree are high. In this chapter, an in-depth analysis will be conducted for this control method.

Keywords: Matrix converter, SPWM modulation, simulation, space vector modulation

INTRODUCTION

The transformation process of matrix converter can be considered to be composed of “rectification” and “inversion” (Brenna et al., 2009). Sinusoidal input current and adjustable input power factor are obtained by using space vector modulation for input phase current in the rectification part and the amplitude value and sinusoidal output voltage with an adjustable frequency are obtained by using space vector modulation for output line voltage in the inversion part (Brooks, 1986); then the middle DC link is eliminated through the principle of equal middle DC voltage and DC current; thus, the method of space vector modulation of matrix converter is obtained (Brooks and Kaupp, 2007).

The classical SPWM modulation mainly focuses on making the output voltage of inverter approaching sinusoidal wave as much as possible, or expecting the fundamental wave component of output PWM voltage to be as large as possible and the harmonic component to be as small as possible (Cao and Dai, 2008). If PWM voltage is controlled by tracking the circular magnetic field and a constant electromagnetic torque is produced, it will have a better effect. As the trajectory of flux linkage is obtained by voltage space vectors versus time integration, it is also called “voltage space vector modulation” (Carmine et al., 2006). It is proved that the maximum amplitude value of output voltage of inverter is DC side voltage, which is 15% higher than general SPWM inverter output voltage and the utilization rate of DC voltage is relatively high while using voltage space vector modulation (Chella et al., 2010).

Voltage space vectors can be obtained by Park transformation of three-phase output voltage in complex plane:

\[ U = \frac{2}{3} \left( U_{a0} + \alpha U_{bc} + \alpha^2 U_{ca} \right) \]  

where, \( \alpha = e^{j\frac{120}{1}} \).

When the power of a motor with symmetric three-phase winding is supplied by three-phase equilibrium sinusoidal voltage, the voltage equation for using blended space vectors to represent stator is as follows:

\[ U = RI + \frac{d\Psi}{dt} \]  

where, \( U \) is a blended space vector of three-phase voltage of stator, \( I \) is a blended space vector of three-phase current of stator and \( \Psi \) is a blended space vector of three-phase flux linkage.

When the revolving speed is not very low, the pressure drop of stator resistance occupies a small proportion which can be ignored. So the approximate relationship between stator voltage and flux linkage is:

\[ U \approx \frac{d\Psi}{dt}, \text{i.e., } d\Psi = UdI \]
The equation above shows that voltage space vector $U$ equals to the change rate of $\Psi$ as a function of time, while its direction is consistent with the direction of motion of $\Psi$. Therefore, the motion trail of flux linkage can be controlled by voltage space vectors.

It can be known from the above that a matrix converter can equal to a virtual AC-DC-AC converter. Therefore, a relatively mature space vector modulation mode with superior performance can be adopted.

**SPACE VECTOR MODULATION OF OUTPUT LINE VOLTAGE OF MATRIX CONVERTER (VSI SVM)**

Now space vector modulation is conducted for the inversion part on the right half of the Fig. 1. The power of this part is supplied by a DC voltage. Enable $U_{mn} = U_{dc}$. There are 8 permissible switch combination modes in VSI, among which 6 will produce non-zero output voltage and 2 will produce zero output voltage. Voltage space vector of output line is defined as:

$$u_{al} = \frac{2}{3}(u_{ab} + u_{bc}e^{i20} + u_{ca}e^{-i20})$$  \hspace{1cm} (4)

Assume that $U_0 \sim U_6$ are seven voltage switch state vectors as shown in Fig. 1b. $U_0 \sim U_6$ six vectors divide a cycle into six sectors, i.e., I, II, III, IV, V and VI. Each sector accounts for 60°. $U_0$ and $U_7$ represent zero voltage vectors. Table 1 shows the relationship between $U_0 \sim U_7$ space vectors and the switch state.

Any voltage space vector of output line $\overline{U}_{al}$ can be expressed by two adjacent switch state vectors $\overline{U}_a$ and $\overline{U}_b$ and zero vector $\overline{U}_{ov}$. It can be obtained through sine theorem within the switching period $T_s$ that:

$$\overline{U}_{al} = d_a \overline{U}_a + d_b \overline{U}_b + d_{ov} \overline{U}_{ov}$$  \hspace{1cm} (5)

where, $d_a$, $d_b$ and $d_{ov}$ are respectively the duty cycle of $\overline{U}_a$, $\overline{U}_b$ and zero vector $\overline{U}_{ov}$. It can be obtained through sine theorem within the switching period $T_s$ that:

$$d_a = \frac{T_a}{T_s} = m_s \sin(60° - \theta_{sv})$$

$$d_b = \frac{T_b}{T_s} = m_s \sin(\theta_{sv})$$

$$d_{ov} = \frac{T_{ov}}{T_s} = 1 - d_a - d_b$$  \hspace{1cm} (6)

where: $T_s$ refers to a PWM switching period; $m_s$ is voltage modulation factor. Modulation factor $0 \leq m_s = \sqrt{3U_{om}/U_{dc}} \geq 1$; $\theta_{sv}$ is the included angle between

![Diagram of sectors of voltage space vectors of output line](image)

![Vector synthesis schematic diagram](image)

![Diagram of voltage space vectors of output line](image)

**Table 1: Relationship between VSI space vectors and switch state**

<table>
<thead>
<tr>
<th>Switch state</th>
<th>$S_{AP}$</th>
<th>$S_{BP}$</th>
<th>$S_{CP}$</th>
<th>$S_{AN}$</th>
<th>$S_{BN}$</th>
<th>$S_{CN}$</th>
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<tr>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(1: represents switch on, 0: represents switch off)
output voltage vector and the initial position of sectors and \( U_{PN} \) is the voltage between buses.

When the voltage space vector of output line is located in sector I, the local average of output line voltage is:

\[
\begin{bmatrix}
\bar{u}_{ab} \\
\bar{u}_{bc} \\
\bar{u}_{ca}
\end{bmatrix} = \begin{bmatrix}
U_{dc} \\
-U_{dc} \\
0
\end{bmatrix} d_u + \begin{bmatrix}
U_{dc} \\
0 \\
-U_{dc}
\end{bmatrix} d_v = \begin{bmatrix}
d_u + d_v \\
-d_u \\
-d_v
\end{bmatrix} U_{dc}
\]

\[= m_l \begin{bmatrix}
\cos(\theta_{SP} - 30^\circ) \\
-\sin(60^\circ - \theta_{SP}) \\
-\sin(\theta_{SP})
\end{bmatrix} U_{dc}
\]

\[\text{Eq. (7)}\]

\[-30^\circ \leq \omega_0 t - \phi_o \leq 30^\circ \text{ in the first sector and } \theta_{SV} = (\omega_0 t - \phi_o + 30^\circ) + 30^\circ. \text{ Therefore, Eq. (3-7) changes into:}\]

\[
\begin{bmatrix}
\bar{u}_{ab} \\
\bar{u}_{bc} \\
\bar{u}_{ca}
\end{bmatrix} = m_v \begin{bmatrix}
\cos(\omega_0 t - \phi_o + 30^\circ) \\
\cos(\omega_0 t - \phi_o + 30^\circ - 120^\circ) \\
\cos(\omega_0 t - \phi_o + 30^\circ + 120^\circ)
\end{bmatrix} U_{dc} = \bar{T}_{VSI} U_{dc} \quad \text{Eq. (8)}
\]

where, \( \bar{T}_{VSI} \) represents the transfer matrix of voltage source inverter at a low frequency.

The local average of input current of VSI is:

\[
\bar{i}_p = \bar{T}_{VSI} \cdot \bar{i}_{ia} = \frac{\sqrt{3}}{2} I_{im} \cdot m_v \cdot \cos(\theta_L) = \text{CONST}
\]

\[\text{Eq. (9)}\]

**SPACE VECTOR MODULATION OF INPUT PHASE CURRENT OF MATRIX CONVERTER (VSR SVM)**

The left half of Fig. 2 to 4 is a virtual rectifier (VSR). Enable it to produce direct current \( i_p = I_{dc} \), similar to VSR input current space vector and VSI output voltage space vector as shown in Fig. 3.

If \( \bar{I}_\mu \) and \( \bar{I}_\nu \) are used to represent two adjacent phase current space vectors and \( \bar{I}_p \) is used to represent zero current space vectors, in each VSR switching period:

\[
\bar{I}_p = d_\mu \bar{I}_\mu + d_\nu \bar{I}_\nu + d_{0c} \bar{I}_{0c}
\]

\[\text{Eq. (10)}\]

where, \( d_\mu, d_\nu \) and \( d_{0c} \) are respectively the duty cycle of \( \bar{I}_\mu, \bar{I}_\nu \) and \( \bar{I}_{0c} \) and:

\[
\begin{bmatrix}
\bar{i}_x \\
\bar{i}_y \\
\bar{i}_z
\end{bmatrix} = \begin{bmatrix}
I_{dc} \\
-I_{dc} \\
0
\end{bmatrix} d_u + \begin{bmatrix}
I_{dc} \\
0 \\
-I_{dc}
\end{bmatrix} d_v = \begin{bmatrix}
d_u + d_v \\
-d_u \\
-d_v
\end{bmatrix} I_{dc}
\]

\[\text{Eq. (12)}\]
Substitute Eq. (3-12) with \(-30^\circ \leq \omega_t - \varphi_i \leq +30^\circ\) and \(\theta_{SC} = (\omega_t - \varphi_i)+30^\circ\). It can be obtained that:

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = m_c \begin{bmatrix}
\cos(\omega t - \varphi) \\
\cos(\omega t - \varphi - 120\^\circ) \\
\cos(\omega t - \varphi + 120\^\circ)
\end{bmatrix} I_{dc} = T_{vSR} I_{dc}
\]

where, \(T_{vSR}\) represents the transfer matrix of voltage source rectifier at a low frequency. While the local average output voltage of VSR is:

\[
\bar{u}_{pn} = T_{vSR}^T u_{iph} = \frac{3}{2} m_c U_{lm} \cos(\varphi_i) = \text{CONST}
\]

EQUIVALENT DIRECT SPACE VECTOR MODULATION

It is inferred from the above that VSR space vector modulation and VSI space vector modulation can be directly linked from the perspective of local average since the local average output voltage in VSR space vector modulation and the local average input current of VSI space vector modulation are constants.

An equivalent (i.e., actual) direct space vector modulation method can be obtained by combining VSR and VSI space vector modulation and eliminating the middle DC link. The synthesis method of switch control law is as shown in Fig. 3 to 5. Since input current and output voltage respectively have six effective static vectors, 36 possible switch combination states can be obtained according to the law shown in Fig. 3 to 5 and Table 3 each state corresponds to a combination mode of 9 switches, as shown in Table 3. In the table, acc represents that input phase a and output phase A, input phase c and output phase B, input phase c and output phase C respectively connect through break-over switch. Three zero vectors not included in the table are aaa, bbb and ccc.

It is required to first find the switch combination of matrix converter corresponding to the equivalent AC-DC-AC structure in order to realize space vector modulation in real matrix converter and accomplish
and current and each comprehensive duty cycle PWM period corresponds to 5 duty cycles of voltage input phase current space vector modulation and one output line voltage space vector modulation but also matrix converter, the same switch conducts not only AC-DC-AC equivalent structure. However, in the real matrix converter, the same switch conducts not only output line voltage space vector modulation but also input phase current space vector modulation and one PWM period corresponds to 5 duty cycles of voltage and current and each comprehensive duty cycle corresponds to one switch combination, respectively:

\[
\begin{align*}
T_1 &= T_{ap} = d_a \cdot d_p \cdot T_i \\
T_2 &= T_{bp} = d_p \cdot d_p \cdot T_i \\
T_3 &= T_{ac} = d_a \cdot d_c \cdot T_i \\
T_4 &= T_{bc} = d_p \cdot d_c \cdot T_i \\
T_5 &= T_{ap} - T_{bc} - T_{ac} - T_{bp} \\
T_6 &= T_{bc} - T_{ac} - T_{bp} - T_{ap} \\
\end{align*}
\]

(14)

Take the following example to explain the high-frequency synthesis steps of double space vector modulation of matrix converter, in which the output line voltage space vector is located in the first sector and the input phase current space vector is located in the first sector. It can be obtained through Eq. (3-8) and Eq. (3-13) that:

\[
\begin{align*}
m_1 m_1 \begin{bmatrix}
\cos(\theta_{SV} - 30\degree) \\
-\sin(60\degree - \theta_{sv}) \\
-\sin(\theta_{sv})
\end{bmatrix} + m_2 m_2 \begin{bmatrix}
\cos(\theta_{SC} - 30\degree) \\
-\sin(60\degree - \theta_{sc}) \\
-\sin(\theta_{sc})
\end{bmatrix} &= T_{VSI} T_{VSR}^T
\end{align*}
\]

(15)

Enable \( m = m_1 = m_2 \) and \( T_{PML} = T_{VSI} T_{VSR}^T \), thus

\[
T_{PML} = m \begin{bmatrix}
\cos(\theta_{SV} - 30\degree) \\
-\sin(60\degree - \theta_{sv}) \\
-\sin(\theta_{sv})
\end{bmatrix} + m \begin{bmatrix}
\cos(\theta_{SC} - 30\degree) \\
-\sin(60\degree - \theta_{sc}) \\
-\sin(\theta_{sc})
\end{bmatrix}
\]

(16)

To simplify work, enable \( m_1 = 1 \) and \( m = m_2 \) and substitute the equation above with Eq. (3-5 and 3-10). Thus, the local average of output line voltage is:

\[
\begin{align*}
\begin{bmatrix}
\bar{u}_{ab} \\
\bar{u}_{bc} \\
\bar{u}_{ca}
\end{bmatrix} = \begin{bmatrix}
-d_a + d_p & d_p + d_c & u_{av} \\
-d_p & d_p + d_c & u_{av} \\
-d_c & -d_p & u_{av}
\end{bmatrix}
\end{align*}
\]

(17)

Because: \( u_{ab} = u_{ao} - u_{bo} \) and \( u_{ca} = u_{ao} - u_{co} \), therefore,
It can be seen from the analysis above that each output line voltage can be made up of input line voltage combination within each switching period. It can be obtained from equation 3-18 that:

\[
\begin{align*}
\begin{bmatrix} u_{AB} \\ u_{BC} \\ u_{CA} \end{bmatrix} &= \begin{bmatrix} d_{aB} + d_{B\mu} \\ -d_{aB} \\ -d_{B\mu} \end{bmatrix} u_{ab} + \begin{bmatrix} d_{av} + d_{fv} \\ -d_{av} \\ -d_{fv} \end{bmatrix} u_{ac} \\
\end{align*}
\]

where,

\[
\begin{align*}
d_{aB} &= d_a d_{ab} = m \sin(60^\circ - \theta_{ab}) \sin(60^\circ - \theta_{ac}) = T_{aB} / T_s \\
d_{B\mu} &= d_B d_{b\mu} = m \sin(\theta_{b\mu}) \sin(60^\circ - \theta_{ac}) = T_{b\mu} / T_s \\
d_{av} &= d_a d_{av} = d_a \sin(60^\circ - \theta_{ab}) \sin(\theta_{ac}) = T_{av} / T_s \\
d_{fv} &= d_f d_{fv} = m \sin(\theta_{fv}) \sin(\theta_{ac}) = T_{fv} / T_s \\
d_0 &= 1 - d_{av} - d_{fv} - d_{aB} - d_{B\mu} = T_0 / T_s
\end{align*}
\]

It can be seen from the analysis above that each output line voltage can be made up of input line voltage combination within each switching period. It can be obtained from equation 3-18 that:

\[
\begin{align*}
\begin{bmatrix} u_{AB} \\ u_{BC} \\ u_{CA} \end{bmatrix} &= \begin{bmatrix} d_{aB} \\ -d_{aB} \\ 0 \end{bmatrix} u_{ab} + \begin{bmatrix} d_{B\mu} \\ 0 \\ -d_{B\mu} \end{bmatrix} u_{ac} + \begin{bmatrix} d_{av} \\ -d_{av} \\ 0 \end{bmatrix} u_{ac} + \begin{bmatrix} d_{fv} \\ 0 \\ -d_{fv} \end{bmatrix} u_{ac}
\end{align*}
\]

There are five different kinds of combination corresponding to five duty cycles in each PWM period. The control of matrix converter can be concluded as the determination of 5 duty cycles and their corresponding 5 kinds of switch combination state. Mutually nested double space vector PWM modulation strategies can not only ensure good sinusoidal property of output line voltage but also ensure that of input phase current, which realize the purpose of applying space vector modulation in control strategies of matrix converter and provide matrix converter with the effect of double PWM converter. The analysis above actually reflects the process of AC-AC direct transformation control of matrix converter (Table 4).

**OPTIMIZED SPACE VECTOR MODULATION OF MATRIX CONVERTER**

The section above introduces space vector synthesis methods of matrix converter. Assume that the adjacent basic vectors in the section where output voltage vectors of matrix converter locate are \(V_M\) (vector M for short) and \(V_N\) (vector N for short) and the adjacent basic vectors in the section where input current vectors of matrix converter locate are \(I_a\) (vector \(a\) for short) and \(I_\beta\) (vector \(\beta\) for short). So the comprehensive modulation of both space vectors is realized by using mutual nesting method. The overall vector synthesis process of input phase current and output line voltage has 5 kinds of combination in total, i.e., \(\alpha-M, \alpha-N, \beta-N, \beta-M\) and zero vector \(I_0 - U_o\).
The action time of each vector combination is expressed by duty cycle.

Since matrix converter is working under high-frequency PWM modulation mode and the switching loss in the switching process cannot be neglected, how to reduce the switching loss of matrix converter has become a significant problem. We adopt a new nine-section optimized modulation strategy on the basis of common space vector modulation methods introduced above.

Using the common space vector modulation method, the vector transformation order in one switching period is: \( \alpha M - \alpha N - \beta N - \beta M - 0 - \beta M - \beta N - \beta M - 0 \), while there are three switch states of zero vector: aaa, bbb and ccc, as shown in Fig. 3 to 6.

In the process of realizing the transformation of a vector sequence, the selection of zero vectors is related to the required switching times. Up to now, the selection principle of zero vectors is still focusing on reducing the switching times of MC.

As shown in Fig. 3 to 5a, assuming that voltage vectors and current vectors are both in their respective first section, first consider output voltage vector modulation and the order of voltage vectors is \( M - N - N - M - 0 \) in the first half of switching period based on the general (standard) vector transformation order; and then consider input current vector modulation and its order of vectors (in the first half of switching period) is \( \alpha - \beta - 0 \). As the last effective vector (in the first half of switching period) \( \beta M \) is aca, zero vector aaa is often selected because only one switching is required from vector \( \beta M \) to zero vector aaa. It can be known from Fig. 3 to 5b that the switching times necessary for realizing the order of vectors is 10. Assuming that the order of voltage vectors in the first half of switching period is changed into \( N - M - N - 0 \), the order of input current vectors remains unchanged. As the last effective vector \( \beta N \) is acc, select ccc as zero vector similarly, as shown in Fig. 3 to 5c. The switching times necessary for realizing the order of vectors is 8. For a vector sequence with nine-section and eight-step transformation, this is the least times that can be achieved. However, it can be known from the analysis above that such times cannot always be ensured in any situation.

As shown in Fig. 3 to 6a, the current vector enters into the second sector and the voltage vector is still in the first sector. Only 8 times of switching is required based on the general (standard) vector transformation order, that is, the minimum switching times.

The reason that the switching times are not the minimum one is that the transformation from the second vector to the third vector requires two times of switching instead of one. Similar situation occurs in the second half of switching period, making the total switching times twice more than the minimum times. It can be known from the analysis above that it is only necessary to change the order of voltage vectors, change the general (standard) transformation order \( M - N - N - M - 0 \) into \( N - M - M - N - 0 \) and meanwhile change the selection of zero vector switch correspondingly when the switching times is not the minimum one. Thus, the switching times necessary for realizing vector transformation can be decreased to the minimum value-8 times. Figure 3 to 6b shows the transformation sequence diagram after the order of voltage vectors is changed.

In short, as long as an appropriate vector sequence and zero vector switches is selected, the switching times can be the minimum one. Through careful observation and analysis of all switching combinations using the method above, it is known that the minimum switching times can be ensured in any situation if the vector sequence and zero vector switches are selected according to the following rules.

- The order of input current vectors is maintained as \( \alpha - \beta - 0 \)
- If the sum of input sector number and output sector number is odd, the sequence of output vectors is \( M - N - N - M - 0 \)
- If the sum of input sector number and output sector number is even, the sequence of output vectors is \( N - M - M - N - 0 \)
- When the current sector number is 4 and 1, the output zero vector is aaa; When the current sector is 5 and 2, the output zero vector is bbb; otherwise, it is ccc
It is thus clear that the modulation of voltage vectors and current vectors are no longer mutually independent and that the selection of zero vectors is determined by current vectors. If the traditional (general) order of vectors is adopted for modulation, the switching period in which 10 transformation times occur will account for 50% among all switching periods, making the average switching times in a switching period as 9. If the optimized order of vectors is adopted, the average switching times is 8, with exceptions. When the sector where a reference vector locates in the switching period changes, the switching state of the first vector no longer equals to that of the last vector; thus, the switching times increases. The switching times corresponding to both modulation strategies above can be expressed as:

General method : \[ N = t \times 9 \times f_s + 6(fo + fi) \]
Optimized method : \[ N_{opt} = t \times 8 \times f_s + 6(fo + fi) \]

where, \( f_i \) is input frequency, \( f_o \) is output frequency. As \( f_i \) and \( f_i \) are much less than \( f_s \) (modulation frequency), the switching times increased due to sector change can be neglected. Then, the switching times corresponding to both modulation strategies above can be expressed as:

General method : \[ N = t \times 9 \times f_s \]
Optimized method : \[ N_{opt} = t \times 8 \times f_s \]

It is thus clear that the total switching times decreases by 11% (1/9) compared to the traditional strategy after the optimized modulation strategy is adopted and that it’s switching loss will also decrease by the same proportion correspondingly.

CONCLUSION

This study studies the control strategies of matrix converter, introduces the control idea of general AC-DC-AC converters into the control of matrix converter through absorbing the virtual rectification idea put forward by P.D.Ziogas et al. and respectively studies SPWM modulation strategy and space vector modulation strategy of matrix converter. It also proposes the optimized method of space vector modulation strategy, that is, selecting an applicable vector sequence and zero vector based on a certain law can make the switching times reaching the minimum, thus achieving the purpose of reducing switching loss. The analysis in this study has significant theoretical meanings.

REFERENCES