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Evaluation of Variation Coefficient of Slewing Bearing Starting Torque Using Bootstrap Maximum-Entropy Method

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Abstract: This study proposed the bootstrap maximum-entropy method to evaluate the uncertainty of the starting torque of a slewing bearing. Addressing the variation coefficient of the slewing bearing starting torque under load, the probability density function, estimated true value and variation domain are obtained through experimental investigation of the slewing bearing starting torque under various loads. The probability density function is found to be characterized by variational figure, scale and location. In addition, the estimated true value and the variation domain vary from large to small along with increasing load, indicating better evolution of the stability and reliability of the starting friction torque. Finally, a sensitive spot exists where the estimated true value and the variation domain rise abnormally, showing a fluctuation in the immunity and a degenerative disorder in the stability and reliability of the starting friction torque.

Keywords: Small sample, slewing bearing, starting friction torque, uncertainty, variation coefficient

INTRODUCTION

Slewing bearings are widely used in many mechanical systems. The stability and reliability of their performance must be thoroughly evaluated to ensure safe system operation. Many research efforts have been devoted to slewing bearing studies. Mireia *et al.* (2010) developed a procedure for obtaining the load distribution in a four-point-contact slewing bearing, which considered the effect of structure elasticity. Potonik *et al*. (2010) described the maximum contact force on the rolling element and conducted a detailed finite-element analysis of the contact between the ball and the raceway, which took into consideration the geometry parameters, cross-sectional clearance, ratio of curvatures, rolling-ball diameter and initial contact angle. Gao *et al*. (2010) considered the load-carrying capacity and service life of a four-point-contact slewing bearing. Ludwik (2006) proposed a finite-element model for slewing bearing rollers and, based on tribology and contact mechanics, studied the influence of the type and parameters of roller generator correction on the contact stress, as well as the plastic deformation in the contact zone of the roller and the bearing raceway. Chen *et al*. (2010) discussed the fretting wear on a wind turbine blade bearing. Joseph (2008) presented a quality-assurance testing method for a slewing bearing. Liu and Chen (2011) introduced a slewing bearing diagnosis technique based on vibration signal, temperature, friction torque, acoustic emission and stress wave.

These results have significantly improved the service behavior of a running slewing bearing. Many slewing bearings, e.g., blade and yaw bearings for wind turbines demand frequent forward (anticlockwise direction) or backward (clockwise direction) starts and slews; thus, starting torque becomes important indicators of the service behavior of slewing bearings to ensure steady and reliable operation. However, research on starting torque has been inadequate so far and is rarely reported, as enumerated in the next three cases that addressed this issue:

- The first case is concerned with the probability distribution of the starting friction torque, which is unknown and undetermined (Xia *et al*., 2008; Xia, 2012). If loads (a radial load, an axial load and an overturning moment) vary continually, the probability distribution of the starting torque becomes complex.
- **The second case involves unknown trends:** The starting torque of a rolling bearing during operation changes with the operating condition, such as load, ambient temperature and lubrication (Xia *et al*., 2008). If the operating condition changes indeterminately, the starting torque can become uncertain.
- **The third** case **concerns small sample:** Obtaining a large sample for large-size slewing bearings such as blade and yaw bearings for wind turbines is difficult due to high cost.

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Therefore, a large difference exists between the calculated theoretical and the measured practical values, i.e., establishing a purely theoretical model for precise calculation of the starting torque is difficult. In addition, rolling bearing performances are strikingly chaotic and fall under the nonlinear dynamic category. Therefore, the starting torque of slewing bearings is characterized by nonlinear evolvement based on poor-information system. Poor information (Xia *et al*., 2008) in system analysis means incomplete and insufficient information, such as a small sample, an unknown probability distribution and a trend without any prior knowledge.

Traditionally, the evaluation method employed for starting torque relies on the use of statistics and this method inherently requires that the probability distribution and the trend must be known in advance and the sample size must be large. Under poor-information condition, the statistics-based methods may become ineffective. Based on the poor-information theory, the bootstrap maximum-entropy method is proposed to evaluate the variation coefficient of the slewing bearing starting friction torque. Small sample size is permitted by this method and the probability distribution and the trend are not required to be known in advance.

Using simulation, the relationship between the starting torque and the load of a slewing bearing is explored to discover the nonlinear evolution characteristics of the variation coefficient of starting friction torque, which establishes experimental foundations for theoretical solution to starting torque problems and proposes a new assessment method for rolling bearing performances.

TEST PLAN AND ANALYSIS STEP

The experiments are conducted on a friction-torque experimental rig. In the experiment, two different large-size slewing bearings, code named B and L, are tested. The outer ring of the slewing bearing is fixed. Under an axial load of 800 kN, the inner ring also bears load M, namely, the overturning moment for bearing B and the radial load for bearing L, as shown in Table 1. For accurate observation and elimination of measurement uncertainty, six equally spaced points are marked on the circumference of the slewing bearing and a number of starting torque values are measured sequentially at these points when the slewing bearing begins to rotate forward and backward.

The following reasons necessitate consideration of six starting torque measurement points. First, the slewing bearing studied in this study is a large-size bearing type that has multiple row raceways and a large number of rolling elements, resulting in complex contact deformation, nonuniform lubrication and different friction conditions at each contact zone of the rolling elements and the raceways during the test. Thus, a number of points must be investigated for complete evaluation of the starting friction torque. Second, the more the number of measuring points are, the larger is the cost of the measuring process, indicating that the number of measuring points should be made as small as possible subject to the condition of ensuring effective assessment. Third, according to the poor-information theory (Xia *et al*., 2008; Xia, 2012), the number of experimental data used for establishing the probability density function of a population using the bootstrap maximum-entropy method should be greater than two. Finally, the recommended number of measuring points is from four to eight, as is customarily done.

Table 1: Plan of load M

Code of bearing	Property of load M	Value of load M
B	Overturning moment, M/kN/m	0, 676, 1352, 2028, 2470
L	Radial load, M/kN	0, 42.5, 85, 127.5, 170

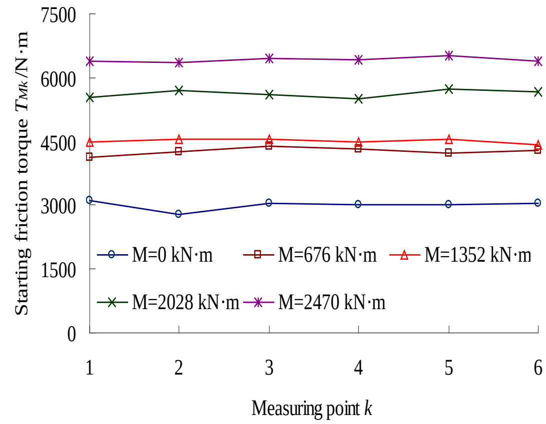


Fig. 1: Starting torque data of bearing B

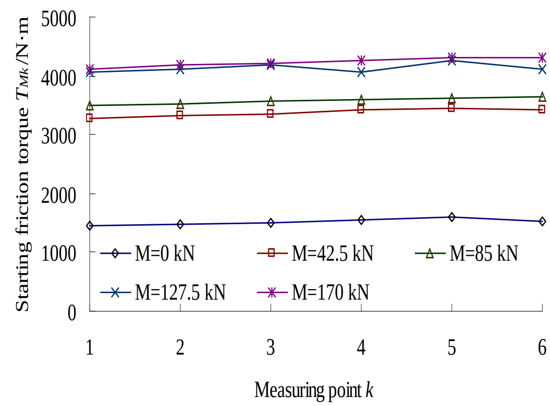


Fig. 2: Starting torque data of bearing L

After the random noise is removed using a filter, the data at each point and under different loads are shown in Fig. 1 and 2.

Figure 1 and 2 shows that the six measuring points are combined under a given load for each slewing bearing, obtaining only one starting torque filtering datum at each point. Thus, six filtering data are obtained for each test. The problem studied in this study considered that the load changes five times, each time processing the six data to explore the degree of variation of the starting torque under the load. Using the six data from each test, the probability density function of the starting torque variation coefficient under load can be structured to determine the evolution of the nonlinear characteristics of the slewing bearing starting friction torque. Obviously, this is a poor-information problem. This can be eliminated using the bootstrap maximum-entropy method, with the steps enumerated as follows:

- The six data of the slewing bearing starting torque under a given load are re-sampled by the bootstrap, generating large quantities of simulated data.
- The simulated data are processed using the maximum-entropy method, which establishes the probability density function of the variation coefficient of the starting torque under the load.
- Under a given confidence level, the probability density function is processed via the statistical theory, obtaining the estimated true value and the variation domain of the variation coefficient of the starting torque under the load.
- The evolution characteristics of the probability density function, the estimated true value and the variation domain are diagramed and analyzed, ultimately obtaining the variation rule and the expression for the uncertainty of the slewing bearing starting friction torque.

EVALUATION METHOD

Under load M, the data series T_M of the starting torque at each point on the circumference of the inner ring is assumed to be obtained as:

$$T_M = (T_{M1}, T_{M2}, T_{M3}, ..., T_{Mk}, ..., T_{Mn}); k = 1,2,...,n \tag{1}$$

where,

T_M = The data series of the starting friction torque

T_{Mk} = The datum at the kth point under load M and n (n = 6) is the number of data in T_M.

According to the bootstrap method (Xia *et al*., 2008), the bootstrap resampling samples, namely, B simulation samples of size $m_D = n$, can be obtained by an equiprobable sampling with replacement of T_M, as follows:

$$T_b = (t_b(1), t_b(2), ..., t_b(k), ..., t_b(m_D)); \\ k = 1,2,..., m_D; b = 1,2,..., B \tag{2}$$

where T_b is the bth bootstrap sample, m_D is a bootstrap evaluation factor and B is the number of bootstrap-resampling samples (generally, B = 10,000-20,000, according to the poor-information theory).

From Eq. (2), two large size B samples can be obtained. The first sample is called the simulated mean sample, given by:

$$T = (t_1, t_2, ..., t_b, ..., t_B) \tag{3}$$

and the second sample is called the simulated standard deviation sample given by:

$$S = (s_1, s_2, ..., s_b, ..., s_B) \tag{4}$$

with

$$t_b = \frac{1}{m_D} \sum_{k=1}^{m_D} t_b(k) \tag{5}$$

and

$$s_b = \sqrt{\frac{1}{m_D - 1} \sum_{k=1}^{m_D} (t_b(k) - t_b)^2} \tag{6}$$

where,

T = The simulated mean sample

t_b = The bth datum in

T, S = The simulated standard deviation sample

s_b = The bth datum in S

From Eq. (3-6), one large size B sample can be obtained as:

$$\Theta = (\theta_1, \theta_2, ..., \theta_b, ..., \theta_B) \tag{7}$$

with:

$$\theta_b = \frac{s_b}{t_b} \tag{8}$$

where,

Θ = The simulated variation coefficient sample

θ_b = The simulated value of the variation coefficient, namely, the bth datum in Θ.

Assuming that the variation coefficient is a continuous random variable θ and its probability density function is $f(\theta)$, based on the information entropy theory, the information entropy H of $f(\theta)$ can be defined as:

$$H = -\int_{-\infty}^{+\infty} f(\theta) \ln f(\theta) \mathrm{d}\theta \tag{9}$$

The basic idea of the maximum-entropy method is that in all the feasible solutions to a problem, the solution that maximizes the information entropy is the most unbiased solution. Accordingly, let:

$$H \to \max \tag{10}$$

The constraint condition is:

$$\int_{\Theta} \theta^i f(\theta)\mathrm{d}\theta = m_{\mathrm{M}i}\,;\ i = 0,1,2,...,m_{\mathrm{M}} \tag{11}$$

where,

- Θ = Stands for the integral domain and $\Theta \in [\Theta_{\min}, \Theta_{\max}]$
- m_M = The origin moment order and
- m_{Mi} = The ith origin moment

The variation coefficient θ_b is Rearranged from a small to a large order by employing the histogram principle and is divided into Q groups. Hence, the ith origin moment $m_{\mathrm{M}i}$ is given by:

$$m_{\mathrm{M}i} = \sum_{q=1}^{Q} \xi_q^i F_q; i = 0,1,2,...,m_{\mathrm{M}}; q = 1,2,...,Q \tag{12}$$

where,

- ξ_q = The median of the qth group
- F_q = The frequency at ξ_q

According to the Lagrange method of multipliers, the probability density function that satisfies Eq. (10) and (11) is:

$$f(\theta) = \exp\left(\lambda_0 + \sum_{i=1}^{m_{\mathrm{M}}} \lambda_i \theta^i\right) \tag{13}$$

where λ_i is the ith Lagrange multiplier and should satisfy Eq. (10), i.e:

$$m_{\mathrm{M}i} = \frac{\int_{\Theta} \theta^i \exp(\sum_{i=1}^{m_{\mathrm{M}}} \lambda_i \theta^i)\mathrm{d}\theta}{\int_{\Theta} \exp(\sum_{i=1}^{m_{\mathrm{M}}} \lambda_i \theta^i)\mathrm{d}\theta}; i = 1,2,...,m_{\mathrm{M}} \tag{14}$$

The Lagrange multiplier λ_0 is given by:

$$\lambda_0 = -\ln\left(\int_{\Theta} \exp(\sum_{i=1}^{m_{\mathrm{M}}} \lambda_i \theta^i)\mathrm{d}\theta\right) \tag{15}$$

The estimated true value is defined as:

$$C_0 = \int_{\Theta} \theta f(\theta)\mathrm{d}\theta \tag{16}$$

where, C_0 is the estimated true value of the variation coefficient of the starting friction torque.

Assuming that the significance level $\alpha \in [0,1]$; then, confidence level P is defined as:

$$P = (1-\alpha) \times 100\% \tag{17}$$

The estimated interval of the variation coefficient of the starting torque is defined as:

$$[C_{\mathrm{L}}, C_{\mathrm{U}}] = [\theta_{\alpha/2}, \theta_{1-\alpha/2}] \tag{18}$$

where,

- $\theta_{\alpha/2}$ = The value of the variable θ corresponding to the probability $\alpha/2$
- $\theta_{1-\alpha/2}$ = The value of the variable θ corresponding to the probability $1-\alpha/2$
- C_{L} = The lower boundary of the estimated interval
- C_{U} = The upper boundary of the estimated interval

Based on statistics, we have:

$$\frac{\alpha}{2} = \int_{\Theta_{\min}}^{\theta_{\alpha/2}} f(\theta)\mathrm{d}\theta \tag{19}$$

$$1 - \frac{\alpha}{2} = \int_{\Theta_{\min}}^{\theta_{1-\alpha/2}} f(\theta)\mathrm{d}\theta \tag{20}$$

The variation domain of the variation coefficient of the starting torque is defined as:

$$U = C_{\mathrm{U}} - C_{\mathrm{L}} \tag{21}$$

where, U is the variation domain of the variation coefficient of the starting friction torque.

Two statistics, viz., the estimated true value C_0 and the variation domain U and one function, viz., the probability density function f (θ), are proposed to determine the evolution characteristics of the slewing bearing starting torque with the load.

For the starting friction torque, the estimated true value can be employed to indicate the degree of variation, the variation domain can be adopted to describe the uncertainty of variation and the probability density function can be applied to project the internal mechanism of variation.

Based on statistics, the performances of a population vary with working conditions. This change is characterized by the mutation of some statistics of a sample output by the population. A variation coefficient is a common statistics for evaluating the variation between samples with obviously different truth values.

Equations (5), (6) and (8) show that the variation coefficient is equal to the standard deviation of the mean ratio. Therefore, its estimated true value is a relative uncertainty of the starting friction torque. With the increase in load, the smaller the estimated true value

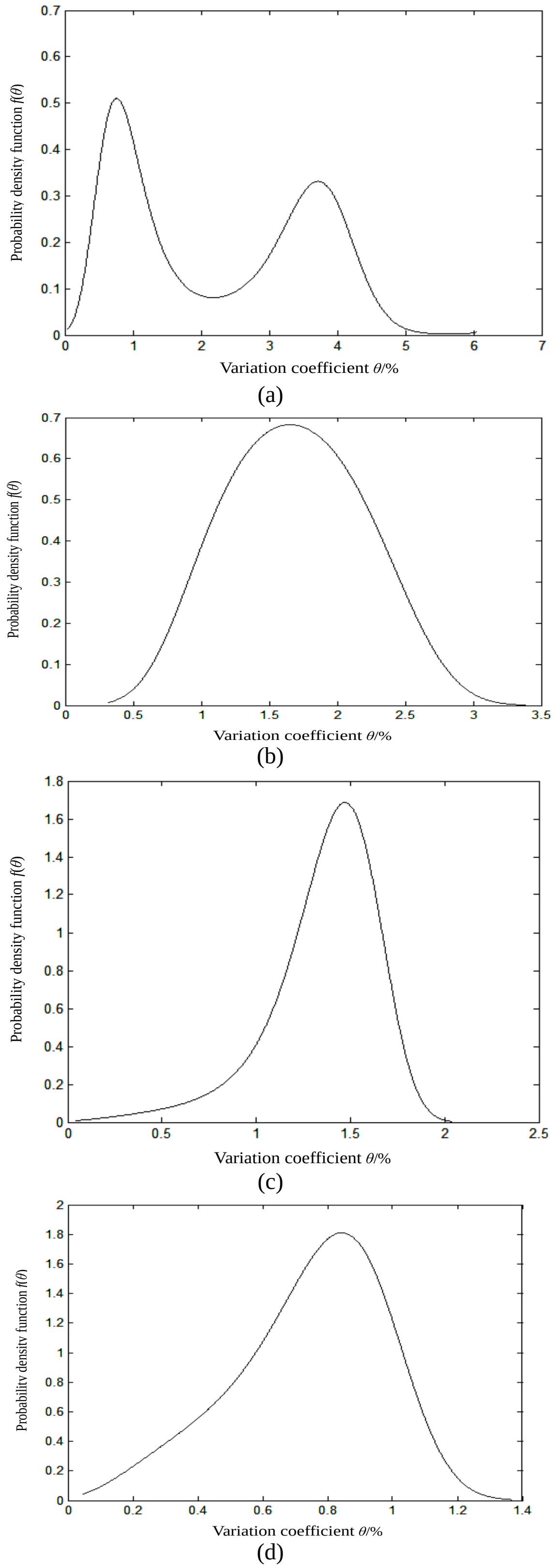


Fig. 3: Probability density function of starting torque variation coefficient (bearing B); (a) M = 0 kN/m; (b) M = 676 kN/m; (c) M =2,028 kN/m; (d) M = 2,470 kN/m

is, the more stable is the starting friction torque; otherwise, the starting torque would be more unstable.

MUTATION ANALYSIS OF STARTING FRICTION TORQUE

Evolvement of probability density function of variation coefficient: The probability density functions have been well known to be capable of simulating the essence of population. In this section, the evolvement of the probability density function of the variation coefficient is studied to reveal the inherent mechanism of the uncertainty of the slewing bearing starting torque under load.

For the two slewing bearings studied, the probability density functions of the starting torque variation coefficient are shown in Fig. 3 and 4 (*B*=10,000), which show that, as a curve, the probability density function with load is characterized by the following three features:

- The first feature is the variational figure, which presents three conditions, viz., bimodal, unimodal and symmetrical and unimodal and asymmetrical distributions, indicating that the estimated true value and the variation domain vary with the load, resulting in mutative reliability, stability and uncertainty of the slewing bearing starting state
- The second feature is the variational scale, expressed as the height-to-width ratio of a curve, viz. and tall, short, fat and thin, meaning that the variation domain changes with the load. The larger the value the scale takes, the smaller is the variation domain, causing an increasing reliability of maintaining an initial starting state of the slewing bearing and vice versa
- The third feature is the variational location, which represents the different values the abscissa of a curve summit assumes, signifying that the estimated true value, viz., the variation coefficient that takes the maximum probability, changes with the load. That is to say, different loads lead to different slewing bearing starting states (stationary or nonstationary). The smaller the value the location takes, the more stable is the starting state, showing better evolution in the stability of the starting torque and better performance of the slewing bearing and vice versa

In this study, the variational figure, scale and location are called the polytrope of a function, which can be used to illustrate the evolvement of the probability density function of the variation coefficient.

The variational figure has an effect on the uncertainty of the estimated true value and the variation domain, the variational scale has an effect on the variation domain and the variational location has an effect on the estimated true value.

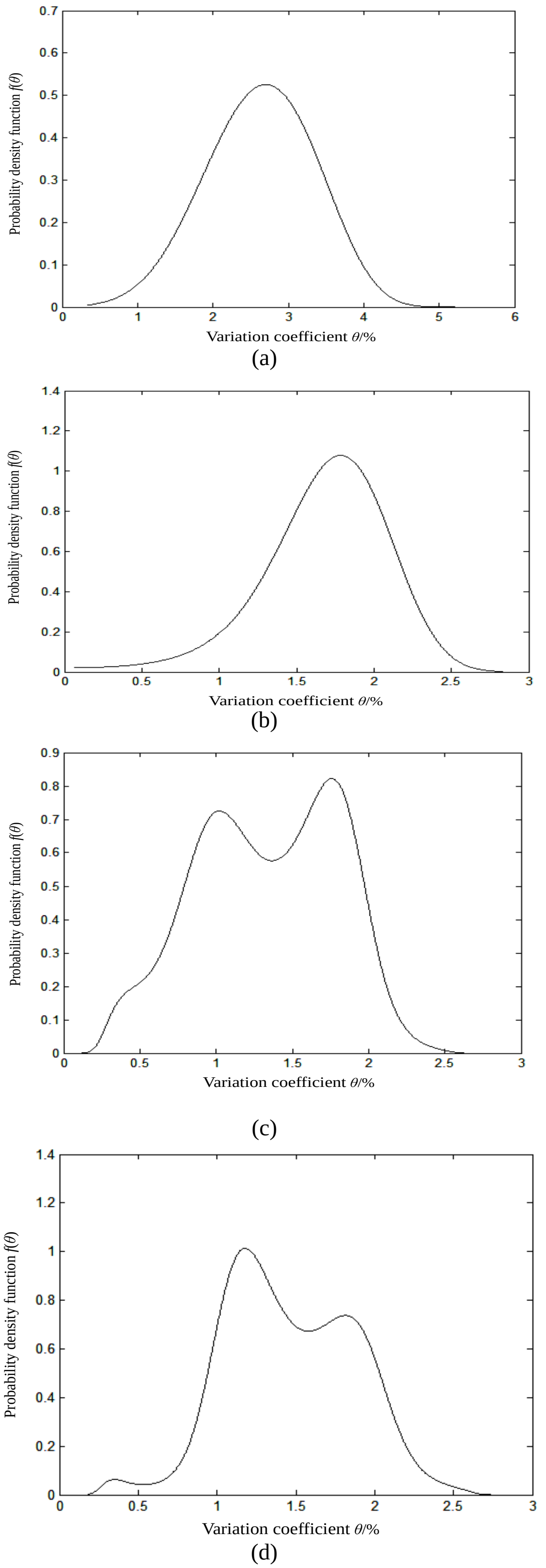


Fig. 4: Probability density function of starting torque variation coefficient (bearing L); (a) M = 0 kN; (b) M = 42.5 kN; (c) M = 127.5 kN; (d) M = 170 kN

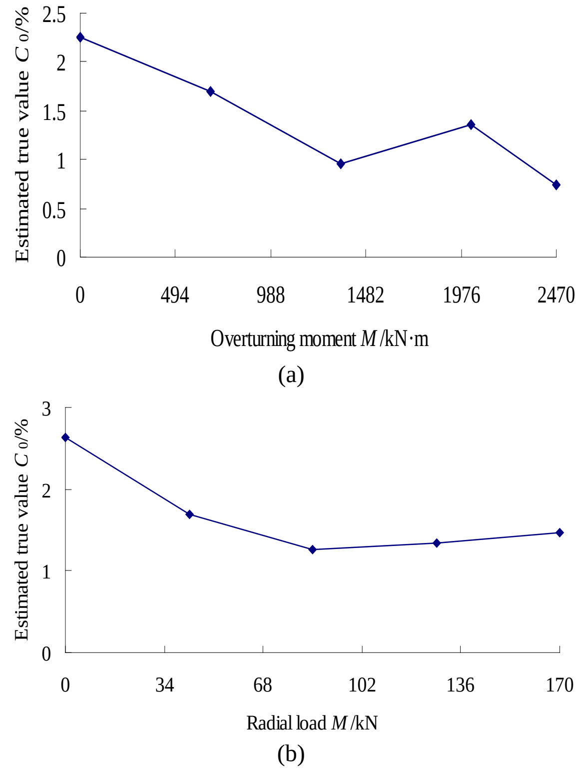


Fig. 5: Estimated true and sample values of the starting torque variation coefficient; (a) Bearing B; (b) Bearing L

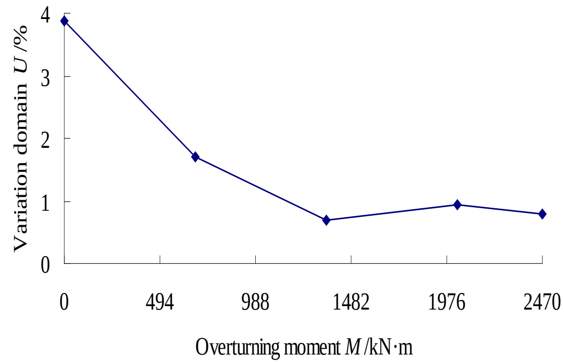
For the variation coefficient of the slewing bearing starting friction torque, the probability density function determines the characteristics of the estimated true value and the variation domain. Therefore, the stability and reliability of the slewing bearing starting torque depend on the probability density function of the variation coefficient.

Evolvement of estimated true value of variation coefficient: The results of the estimated true value C_0 are shown in Fig. 5 (B = 10,000).

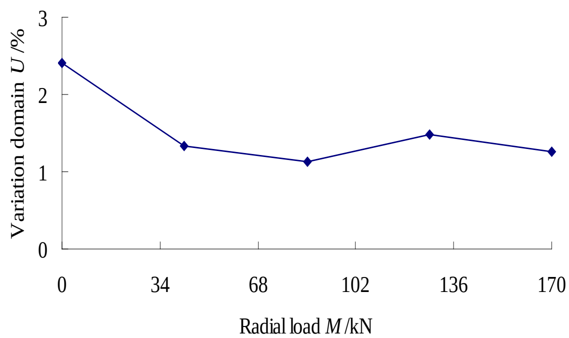
Figure 5a shows that the estimated true value C_0 for bearing B obeys an obvious law, i.e., the C_0 values for all M, except for M = 2,028 kN/m, reduce monotonously as the overturning moment M increases. This condition shows that with the increase in the overturning moment, the variation in the starting torque decreases nonlinearly in view of the overall situation where a local maximum of M = 2,028 kN/m exist.

Figure 5b shows that the estimated true value C_0 for bearing L shows an obviously nonlinear trend, like a parabola, as the radial load M increases. Figure 5b differs from Fig. 5a, but similarity is also present in some aspects. For example, local maxima exist at M = 170 kN in Fig. 5b and at M = 2,028 kN/m in Fig. 5a.

The estimated true value C_0 can intuitively reveal how the load affects the variation of the uncertainty in the starting friction torque, indicating that a problem arises.



(a)



(b)

Fig. 6: Variation domain of starting torque variation coefficient; (a) Bearing B; (b) Bearing L

Based on starting performance requirements, when the load is large, the smaller the variation coefficient is, the more stable is the starting performance. Figure 5 show that the studied slewing bearing meets the requirement in relation to the overall situation. From the nonlinear evolvement of the rolling bearing performance, the condition that the variation coefficient changes from large to small denotes a better evolution in the stability of the starting torque and better performance of the slewing bearing. Thus, within allowable load, the slewing bearing is immune to larger loads. The condition where the variation coefficient rises abnormally at M = 2,028 kN/m for bearing B and at M = 170 kN for bearing L predicates a fluctuation of the immunity and a degenerative disorder of the stability of the starting friction torque, i.e., a gene mutation occurs.

In this study, the load that corresponds to the gene mutation is called the sensitive spot; if the slewing bearing starts running under such a load, its service life will be affected because the sensitive spot is related to the structure, design parameter, lubrication, manufacturing quality and load characteristics of the measurement and the bearing systems. Therefore, design, manufacturing and measurement of the bearing system should consider the sensitive spot of the load. This finding is a novel discovery and a new perspective and it is not reported in existing literature. Hopefully, it can lay a new foundation for analysis of the starting torque measurement system and for designing a slewing bearing system.

Evolvement of variation domain of variation coefficient: The results of the variation domain of the variation coefficient are shown in Fig. 6 (B = 10,000 and p = 90%).

The trends shown in Fig. 6a and b are almost the same. More specifically, the variation domain assumes the maximum value when $M = 0$ for the two bearings and becomes very small when $M \geq 676$ kN/m for bearing B and $M \geq 42.5$ kN for bearing L.

From the perspective of statistics, the estimated true value has an uncertainty for randomness of an event. This uncertainty can be expressed as the variation domain under a given confidence level at which the smaller the variation domain is, the higher is the reliability, meaning that the reliability of maintaining an initial starting state of the slewing bearing under load is higher than that under no-load, with a 90% confidence level. In addition, the variation domain in Fig. 6 fluctuates when $M \geq 2{,}028$ kN/m for bearing B and $M \geq 127.5$ kN for bearing L, corresponding to the situation of sensitive spots in Fig. 5. Therefore, the variation domain has uncertainty under load.

CONCLUSION

For the variation coefficient of the slewing bearing starting torque under load, the probability density function is characterized by variational figure, scale and location. The variational figure has an effect on the uncertainty of the estimated true value and the variation domain, the variational scale has an effect on the variation domain and the variational location has an effect on the estimated true value.

The estimated true value and the variation domain of the variation coefficient vary from large to small with increasing load, predicating better evolution in the stability and reliability of the starting friction torque, viz. and better performance of the slewing bearing. Thus, within the allowable load, the slewing bearing is immune to larger load.

A sensitive spot exists where the estimated true value and the variation domain rise abnormally, which indicates a fluctuation of immunity and a degenerative disorder of the stability and reliability of the starting friction torque. If the slewing bearing starts running at the sensitive spot, its service life will be affected.

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REFERENCES

Chen, L., X.T. Xia and M. Qiu, 2010. Analysis and control of fretting wear for blade bearing in wind turbine. Appl. Mech. Mater., 26-28: 167-170.

Gao, X.H., X.D. Huang, H. Wang, R.J. Hong and J. Chen, 2010. Effect of raceway geometry parameters on the carrying capability and the service life of a four-point-contact slewing bearing. J. Mech. Sci. Technol., 24(10): 2083-2089.

Joseph, M., 2008. Quality assurance testing of a slewing bearing. J. Failure Anal. Prevent., 8(5): 431-433.

Liu, Z.J. and J. Chen, 2011. Monitoring and diagnosis technique on slewing bearing. Adv. Mater. Res., 156-157: 1478-1483.

Ludwik, K., 2006. Modelling of rollers in calculation of slewing bearing with the use of finite elements. Mech. Mach. Theory, 41(11): 1359-1376.

Mireia, O., S. Xabier, D. Jorge and S. Alberto, 2010. Design of four contact-point slewing bearing with a new load distribution procedure to account for structural stiffness. J. Mech. Des. Trans. ASME, 132(2): 0210061-02100610.

Potonik, R., P. Gönczb, J. Flašker and S. Glodež, 2010. Fatigue life of double row slewing ball bearing with irregular geometry. Proc. Eng., 2(1): 1877-1886.

Xia, X.T., 2012. Forecasting method for product reliability along with performance data. J. Fail. Anal. Prevent., 12(5): 532-540.

Xia, X.T., X.Y. Chen, Y.Z. Zhang and Z.Y. Wang, 2008. Grey bootstrap method of evaluation of uncertainty in dynamic measurement. Measurement, 41(6): 687-696.