A Two-Echelon Supply Chain of a Buyback Policy with Fuzzy Demand

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Abstract: In this study, the buyback policy for two-echelon supply chain in fuzzy demand environment is studied. The models of centralized decision and buyback policy are built by the method of fuzzy cut sets theory and their optimal policies are also proposed. Finally, an example is given to illustrate and validate the models and conclusions. It is shown that the optimal order quantity of the retailer fluctuates at the center of the fuzzy demand and decreases with the raise of the customer return rate. The optimal fuzzy expected profits for the manufacturer and retailer in supply chain decrease with increasing of the customer return rate.

Keywords: Buyback policy, fuzzy demand, supply chain, triangular fuzzy number

INTRODUCTION

Buyback policy is an instrument for supply chain coordination, which shifts the demand uncertain from the retailer to the manufacturer, thus encouraging the retailer to increase order quantities.

A large body of literature has explored to coordinate the supply chain with buyback policy during the last two decades. Pasternack (1985) first claimed that an appropriate buyback policy can full coordinate a single-supplier single-retailer supply chain, which was then extended by Mantrala and Raman (1999) to the situation where the retailer has several stores. Lee et al. (2000) and Taylor (2001) studied the return contract with effort depended demand. They showed that in this problem, it attained supply chain coordination combined with feedback policy. Yao et al. (2005a, b) analyzed the profits of both actors when the manufacturer and retailer shared or did not share the forecast information in returns policy. Yue and Raghunathan (2007) discussed the impact of full return policy as well as information sharing on the manufacturer and the retailer under information asymmetry. Bose and Anand (2007) considered the wholesale price as an exogenous price to study returns policies for coordinating the supply chain. They showed that in general, an equilibrium returns policy was not Pareto-efficient with respect to a price only contract, but when the wholesale price was sufficiently high, the equilibrium returns policy was Pareto-efficient. These conclusions were consistent with those of Yao et al. (2007). Ding and Chen (2008) studied the coordination issue of a three level supply in a single period model. Yao et al. (2007) analyzed the impact of price-sensitivity factors on characteristics of buyback policy in a single period product supply chain. Mollenkopf et al. (2007) used an empirical study to explore how internet product returns management systems affect loyalty intentions. Chen and Bell (2009) showed that the customer returns affect the firm’s pricing and inventory decision. Chen and Bell (2011) proposed an agreement between the manufacturer and the retailer that includes two buyback prices. Chen (2011) proposed a returns policy with a wholesale-price-discount scheme that can achieves supply chain coordination. Ai et al. (2012) analyzed the implementation of full returns policies in the chain-to-chain competition.

The conventional studies have focused on the cases that the demands are probabilistic. In other words, the demands follow certain distribution function. However, in practice, especially for new products, the probabilities are not known due to lack of history data. Thus, the uncertain theory, rather than the traditional probability theory is well suited to the supply chain models problem. In this study, the demands are approximately estimated by experts and regarded as fuzzy numbers, which is also used by (Xu and Zhai, 2010a, b; Hu et al., 2011; Sang, 2012). In this study, Buyback policy in fuzzy demand environment will be discussed and the impact of the customer’s return rate on the model will be analyzed.

PRELIMINARIES

Consider a single-period setting for a two-echelon supply, consisting of a manufacturer and a retailer in fuzzy demand. We assume that at the beginning of the selling season, the retailer has no inventory on hand and must decide the order quantity $q$ from the manufacturer. Demand unmet at the end of the selling season is lost. The under age cost is $g$ per unit of unmet demand and there is an overage cost of $h$ per unit of unsold inventory. The retailer is assumed to face customer returns that can not be resold to other customers during the sales period because of the single-period setting.
The customer receives a full refund for returned product from the retailer. Let \( \alpha \) be the customer return rate, \( S \) the cost for the retailer dealing with customer returns, \( w \) be the wholesale price per unit set by the manufacturer and \( c \) the manufacturer’s production cost per unit.

In our models, the buyback policy is composed by two buyback prices \((v, r)\) where buyback price \( v \) applies to unsold inventory resulting from demand uncertainty and buyback price \( r \) applies for returned products from customers.

We consider the uncertain demand faced by the retailer as a positive triangular fuzzy variable \( \tilde{D} = (d_1, d_2, d_3) \) with the most possible value \( d_2 \), where \( 0 < d_1 < d_2 < d_3 \). The fuzzy demand \( \tilde{D} \) means the demand is about \( d_2 \). \( d_1 \) and \( d_3 \) are the lower limit and upper limit respectively of the fuzzy demand \( \tilde{D} \) and described by a general membership function \( \mu_{\tilde{D}} (x) \):

\[
\mu_{\tilde{D}}(x) = \begin{cases} 
\tilde{D}_2(x), & x \in [d_1, d_2], \\
\tilde{D}_3(x), & x \in (d_2, d_3], \\
0, & x \notin (d_1, d_3).
\end{cases}
\tag{1}
\]

For \( x \in [d_1, d_3] \), the left membership function \( \tilde{D}_L (x) = (x-d_1)/(d_2-d_1) \) is an increase function of \( x \). For \( x \in (d_2, d_3] \), the right membership function \( \tilde{D}_R (x) = (d_3-x)/(d_3-d_2) \) is a decrease function of \( x \).

In order to measure the expected value of a fuzzy number \( \tilde{D} \), we use the ranking method of fuzzy numbers proposed by Dubois and Prade (1987):

\[
E[\tilde{D}] = \frac{1}{2} \int \left[ \tilde{D}_L^1(\lambda) + \tilde{D}_R^1(\lambda) \right] d\lambda
\tag{2}
\]

Where \( [\tilde{D}_L^1(\lambda), \tilde{D}_R^1(\lambda)] \) is the \( \lambda \) cut set of \( \tilde{D} \).

**BUYBACK POLICY WITH FUZZY DEMAND**

Consider a supply chain occupied by an integrated-actor, which can also be regarded as the retailer and the manufacturer making cooperation. The fuzzy profit of two-stage supply chain can be expressed as follows:

\[
\Pi_{sc} = p(1-\alpha)\min\{\tilde{D}, q\} - h\max\{q-\tilde{D}, 0\} - q\max\{\tilde{D} - q, 0\} - cq - S
\tag{3}
\]

The integrated-actor tries to maximize its fuzzy expected profit \( E[\Pi_{sc}] \) by choosing the optimal order quantity \( q \), which solves the following model:

\[
\text{Max. } E[\Pi_{sc}] = E[p(1-\alpha)\min\{\tilde{D}, q\} - h\max\{q-\tilde{D}, 0\} - q\max\{\tilde{D} - q, 0\} - cq - S] \\
\text{s.t. } d_1 < q \leq d_3
\tag{4}
\]

Since the fuzzy demand \( \tilde{D} = (d_1, d_2, d_3) \) in Eq. (4), is a positive triangular fuzzy number, we know that the order quantity \( q \) has two cases, i.e., \( q \in [d_1, d_2] \) or \( q \in [d_2, d_3] \).

**Lemma 1:** When \( q \in [d_1, d_2] \), the fuzzy expected value of \( E[\min\{\tilde{D}, q\}], E[\max\{q - \tilde{D}, 0\}], \) and \( E[\max\{\tilde{D} - q, 0\}] \), are respectively given by:

\[
E[\min\{\tilde{D}, q\}] = q - \frac{1}{2}qL(q) + \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) d\lambda
\]

\[
E[\max\{q - \tilde{D}, 0\}] = \frac{1}{2}qL(q) - \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) d\lambda
\]

\[
E[\max\{\tilde{D} - q, 0\}] = \frac{1}{2}qL(q) - \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) + R^{-1}(\lambda) d\lambda
\]

**Proof:** We can get the \( \lambda \) cut set of min\{\( \tilde{D}, 0 \), , max\{q - \( \tilde{D}, 0 \), and max\{\( \tilde{D} - q, 0 \)\}, under this case:

\[
\begin{align*}
\min\{\tilde{D}, q\} &= \left\{ \begin{array}{c}
L^{-1}(\lambda), \lambda \in [0, L(q)] \\
q, \lambda \in (L(q), 1]
\end{array} \right. \\
\max\{q - \tilde{D}, 0\} &= \left\{ \begin{array}{c}
0, \lambda \in [0, L(q)] \\
q - L^{-1}(\lambda), \lambda \in (L(q), 1]
\end{array} \right. \\
\max\{\tilde{D} - q, 0\} &= \left\{ \begin{array}{c}
0, \lambda \in [0, L(q)] \\
L^{-1}(\lambda) - q, \lambda \in (L(q), 1]
\end{array} \right.
\end{align*}
\]

By Eq. (2), we can get the fuzzy expected value of

\[
E[\min\{\tilde{D}, q\}] + E[\max\{q - \tilde{D}, 0\}] + E[\max\{\tilde{D} - q, 0\}]
\]

as:

\[
E[\min\{\tilde{D}, q\}] = \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) + q) d\lambda + \frac{1}{2}\int_{L^{-1}(\lambda)}^{1} (q + q) d\lambda
\]

\[
E[\max\{q - \tilde{D}, 0\}] = \frac{1}{2}qL(q) - \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) d\lambda
\]

\[
E[\max\{\tilde{D} - q, 0\}] = \frac{1}{2}qL(q) - \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) + R^{-1}(\lambda) d\lambda
\]

**Lemma 2:** When \( q \in (d_2, d_3) \), the fuzzy expected value of \( E[\min\{\tilde{D}, q\}], E[\max\{q - \tilde{D}, 0\}], \) and \( E[\max\{\tilde{D} - q, 0\}] \), are:

\[
E[\min\{\tilde{D}, q\}] = \frac{1}{2}qR(q) + \frac{1}{2}\int_{R^{-1}(\lambda)}^{1} R^{-1}(\lambda) d\lambda + \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) d\lambda
\]

\[
E[\max\{q - \tilde{D}, 0\}] = q - \frac{1}{2}qR(q) - \frac{1}{2}\int_{R^{-1}(\lambda)}^{1} R^{-1}(\lambda) d\lambda - \frac{1}{2}\int_{0}^{L^{-1}(\lambda)} L^{-1}(\lambda) d\lambda
\]

\[
E[\max\{\tilde{D} - q, 0\}] = \frac{1}{2}\int_{R^{-1}(\lambda)}^{1} R^{-1}(\lambda) d\lambda - \frac{1}{2}qR(q)
\]
Proof: We can get the λ cut set of min{ĥ, q}, max{ĥ - D, 0}, and max{D - q, 0} under this case:

\[
\begin{align*}
\min\{\hat{D}, q\} &= \left\{ \frac{L'(\lambda), q}{L'(\lambda), R'(\lambda)}, \lambda \in [0, R(q)] \right\}, \\
\max\{q - \hat{D}, 0\} &= \left\{ \frac{\hat{D} - \lambda}{\hat{D} - \lambda, q - \lambda}, \lambda \in [0, R(q)] \right\}, \\
\max\{\hat{D}, q - \lambda\} &= \left\{ \frac{\hat{D} - \lambda}{\hat{D} - \lambda, q - \lambda}, \lambda \in [R(q), 1] \right\}.
\end{align*}
\]

Similar to the proof of Lemma 1, we can obtain Lemma 2.

Theorem 1: When \( c < p(1 - \alpha) + g \leq h + 2c \), the optimal order quantity \( q^* \) is:

\[
q^* = E^\alpha \left( \frac{2(p(1 - \alpha) + g - c)}{p(1 - \alpha) + h + g} \right)
\]

Proof: When \( q \in [d_1, d_2] \), By Eq. (4) and Lemma 1, we have the fuzzy expected profit \( E[\pi_{SC}] \) as:

\[
E[\pi_{SC}] = E[p(1 - \alpha)\min\{\hat{D}, q\} - h\max\{q - \hat{D}, 0\} - g\max\{\hat{D}, q - \lambda\} - cq - S]
\]

\[
= p(1 - \alpha)E[\min\{\hat{D}, q\}] - hE[\max\{\hat{D} - \lambda, q - \lambda\}] - gE[\max\{\hat{D}, q - \lambda\}] - cq - S
\]

\[
= p(1 - \alpha) + g - c + \frac{1}{2}p(1 - \alpha) + h + g \int_0^1 L'(\lambda) d\lambda - q - L(q) - gE[\hat{D}] - S
\]

The first and second derivative of \( E[\pi_{SC}] \) in Eq. (5), can be obtained:

\[
dE[\pi_{SC}] = p(1 - \alpha) + g - c + \frac{1}{2}p(1 - \alpha) + h + g L(q)
\]

\[
d^2E[\pi_{SC}] = \frac{1}{2}p(1 - \alpha) + h + g L(q)
\]

Since \( L(q) \) is an increasing function with \( L(q) > 0 \) and, \( (p(1 - \alpha) + h + g) > 0 \) therefore \( d^2E[\pi_{SC}] \) is negative and \( E[\pi_{SC}] \) is concave in \( q \).

Hence, the optimal order quantity of the retailer can be obtained by solving \( dE[\pi_{SC}] = 0 \), which gives\n
\[
q^* = E^\alpha \left( \frac{2(p(1 - \alpha) + g - c)}{p(1 - \alpha) + h + g} \right).
\]

Since \( 0 < L(q^*) \leq 1 \), thus \( p(1 - \alpha) + g \leq h + 2c \).

Theorem 2: When \( p(1 - \alpha) + g > h + 2c \), the optimal order quantity \( q^* \) is:

\[
q^* = R^\alpha \left( \frac{2(h + c)}{p(1 - \alpha) + h + g} \right)
\]

Proof: When \( q \in [d_2, d_3] \), By Eq. (4) and Lemma 2, we have the fuzzy expected profit \( E[\pi_{SC}] \) as:

\[
E[\pi_{SC}] = E[p(1 - \alpha) + h + g]q + \frac{1}{2}p(1 - \alpha) + h + g \int_0^1 L'(\lambda) d\lambda - h + c - gE[\hat{D}] - S
\]

The first and second derivative of \( E[\pi_{SC}] \) in Eq. (6), can be obtained:

\[
dE[\pi_{SC}] = \frac{1}{2}p(1 - \alpha) + h + g R(q)
\]

\[
d^2E[\pi_{SC}] = \frac{1}{2}p(1 - \alpha) + h + g R'(q)
\]

Since \( R(q) \) is a decreasing function with \( R(q) < 0 \) and, \( (p(1 - \alpha) + h + g) > 0 \), therefore \( d^2E[\pi_{SC}] \) is negative and \( E[\pi_{SC}] \) is concave in \( q \).

Hence, the optimal order quantity of the retailer can be obtained by solving \( dE[\pi_{SC}] = 0 \), which gives\n
\[
q^* = R^\alpha \left( \frac{2(h + c)}{p(1 - \alpha) + h + g} \right) \quad \text{Since } 0 < R(q) \leq 1, \quad \text{thus we get } p(1 - \alpha) + g > h + 2c.
\]

From Eq. (5, 6) and Theorem 1, 2, we can easily obtain the optimal fuzzy expected value of the profit for the integrated supply chain, which is given by:

\[
E[\pi_{SC}] = \left\{ \begin{array}{ll}
p(1 - \alpha) + h + g \int_0^1 L'(\lambda) d\lambda - h + c - g & \text{if } p(1 - \alpha) + g \leq h + 2c \\
& \text{and } \frac{1}{2}p(1 - \alpha) + h + g \int_0^1 L'(\lambda) d\lambda - gE[\hat{D}] - S & \text{if } p(1 - \alpha) + g \geq h + 2c
\end{array} \right.
\]

We can also express the fuzzy profit of the retailer and manufacturer under buyback policy as follows:

\[
\pi_r = p(1 - \alpha)\min\{\hat{D}, q\} + ra \min\{\hat{D}, q\} + v \max\{q - \hat{D}, 0\} - h \max\{q - \hat{D}, 0\} - cq - S
\]

\[
\pi_m = (w - c) q - ra \min\{\hat{D}, q\} - v \max\{q - \hat{D}, 0\}
\]

The retailer tries to maximize its fuzzy expected profit \( E[\pi_{SC}] \) with the buyback policy \( (w, v, r) \) from the manufacturer.

Theorem 3: When \( v = ra \) and \( w - c = ra \), the buyback strategy \( (w, v, r) \) can achieve supply chain coordination.

Proof: When \( q \in [d_1, d_3] \), the fuzzy expected profit of the retailer is:

\[
E[\pi_r] = \left\{ \begin{array}{ll}
p(1 - \alpha) + ra + c - g & \text{if } p(1 - \alpha) + ra + c - g \leq h + 2c \\
& \text{and } \frac{1}{2}p(1 - \alpha) + ra + c - g \int_0^1 L'(\lambda) d\lambda - gE[\hat{D}] - S & \text{if } p(1 - \alpha) + ra + c - g \geq h + 2c
\end{array} \right.
\]

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From Eq. (9), we obtain the optimal order quantity $q^*$ as:

$$q^* = L\left(\frac{2 \left( p(1-\alpha) + ra + h - v \right)}{p(1-\alpha) + ra + h - v + g}\right)$$

Since in order to fully coordinate the whole channel, thus we require $q^* = q$ in fuzzy buyback policy contract. From Eq. (10) and Theorem 1, we can obtain $v = ra$ and $w - c = ra$.

When $q \in (d_2, d_1]$, the fuzzy expected profit of the retailer is:

$$E[\bar{\Pi}_R] = E[\bar{P}(1-\alpha) + ra + h - v + g] \Phi(q) + \int_{\alpha}^{1} R'(\lambda) d\lambda \int_{\lambda(\lambda)}^{1} (h - v + w) q - gE[\bar{D}] - S $$

From Eq. (11), we obtain the optimal order quantity $q^*$ as:

$$q^* = R\left(\frac{2(h - v + w)}{p(1-\alpha) + ra + h - v + g}\right)$$

In this case, we can also require $q^* = q^*$, which leads to $v = ra$ and $w - c = ra$.

**Remark 1**: It should be noted that the result in Theorem 3, this buyback is unsatisfactory since it provides the retailer with all of the supply chain’s fuzzy expected profit unless there is a complementary profit sharing agreement between the manufacturer and retailer.

The manufacturer just needs to negotiate a profit share ($\beta$) with the retailer such that the manufacturer obtains $\beta E[\bar{\Pi}_{SC}]$ and the retailer earns $(1-\beta)E[\bar{\Pi}_{SC}]$ of the total fuzzy expected profit of the integrated supply chain. The manufacturer can negotiate a profit share ($\beta$) to ensure that the retailer’s fuzzy expected profit is at least the profit with no buyback policy $(1-\beta)E[\bar{\Pi}_{SC}] \geq E[\bar{\Pi}_R| r,v = 0]$ in order to motivate the retailer to accept the $(w, v, r)$ buyback policy. For the manufacturer to earn at least the fuzzy expected profit when it does not offer a buyback policy $(1-\beta)E[\bar{\Pi}_{SC}] \geq E[\bar{\Pi}_R| r,v = 0]$. The above discussion leads to the proof of Theorem 4.

**Theorem 4**: When $\beta \in \left[ E[\bar{\Pi}_R| r,v = 0], 1 - E[\bar{\Pi}_R| r,v = 0] \right]$, the $(w, v, r)$ buyback policy is win-win for both the manufacturer and retailer.

**NUMERICAL EXAMPLE**

In this section, we tend to further elucidate the proposed fuzzy models with a numerical example. We will analyze the effectiveness of the parameter $\alpha$ on the optimal equilibrium values as below:

- It is obviously from the Table 1 that the optimal order quantity $q^*$ will decrease along with the customer return rate $\alpha$ when the other parameters fixed. Especially, in this numerical example, the optimal order quantity equals to the most possible value of fuzzy demand when $\alpha = 1/3$. When $1/3 < \alpha < 1$ and $\alpha < 1/3$, the optimal order quantity of retailer locate at the left and right of the most possible value of fuzzy demand $\bar{D}$, respectively.

- From Table 1, it can be noted that when the other parameters fixed in buyback contract, the fuzzy expected profit of the manufacturer, retailer and two-echelon supply chain will all decrease along with the raise of parameter $\alpha$.

In addition, the manufacturer can choose any set of prices $(w, v, r)$ that satisfy the Theorem 3.3 with $w < p$ and will not affect the results in Table 1. For example, when $\alpha = 0.5$, the buyback policies $(w = 7.5, r = 7.0, v = 3.5)$ and $(w = 7.0, r = 6.0, v = 3.0)$ lead to identical results in Table 1.

**CONCLUSION**

This study formulates fuzzy supply chain models based on fuzzy set theory, where the manufacturer and the retailer adopt buyback policy. In order to examine models performance in fuzzy demand, we use fuzzy cut sets method to solve this problem. The manufacturer and the retailer can gain win-win condition by choosing the set of prices $(w, v, r)$. The method proposed in this study is easier to implement and requires less data. It is appropriate when the environment is complex, ambiguous, or there is lack of statistical data.
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REFERENCES


