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The Relationship between the Friction Coefficient and the Asperities Original Inclination Angle

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Abstract: Because of the contact deformation, the inclination angle of the contact face is decreased gradually when contact and deformation. Base on the change of inclination angle of the contact surface, the concept “friction repose angle” set out. The tangent of the initial inclination angle of two asperities is three time of the tangent of the “friction repose angle”. The relationship set up a bridge between the initial surface geometric configuration (can be detect) and the configuration which after the deformation (can not be detect). Static Friction Coefficient is the max value of Kinetic Friction Coefficient before the deformation process of instantaneous contact surface. The Ratio of Kinetic and Static Friction Coefficient distribute from 0.771 to 0.9117 and were inversely proportional to the inclination angle of the contact face .Kinetic Friction Coefficient is the average friction coefficient of the deformation process of instantaneous contact surface. In the sandstone, the value of Kinetic Friction Coefficient which more than 0.5546 is because of the coupling of different classes zigzag-shape surface asperities mostly. The study puts forward the new ideas “dynamic deformation tribology” which will promote the development of the Tribology.

Keywords: Dynamic friction, friction repose angle, inclination angle of the asperities, instantaneous contact surface, kinetic friction coefficient, static friction coefficient

INTRODUCTION

In the past, the Friction Coefficient was defined as tan ψ which was determined by presumptive angle of an inclined planar face but not related to the surface micro-friction coefficient of the inclined planar face. The macro-friction coefficient should and always get by experiment and the inclination angle of the asperities is difficult to grasp in practice. The fractal dimension in the (He-Ping and Zhong-Hui, 2004) is a parameter which is related to the inclination angle of the asperities. And this is studying the geometric configuration of rock macro-crack surface from different perspectives. Cheng-Yao *et al*. (2012) point out that is a coupling relationship between friction coefficient and stress ratio. The average inclination angle of surface asperities and fractal dimension of macro-crack have some memory function on stress condition and may be a potential method to remember the paleo-stress field. Above study and some classics of tribology (Valentin, 2010; Shi-Zhu and Huang-Ping, 2002) are all from the static geometric configuration of friction surface and hidden hypothesis that no tangential deformation of friction surface asperities in the friction process. In fact the tangential deformation must being and the tangential deformation process must affect the macro-friction coefficient of a friction surface (Mahmoud *et al*., 2009; Timpe and Komvopoulos, 2006; Kim *et al*., 2011). The studies of geological and engineering domain on rocks friction coefficient and the geometric configuration of rock surface is just start. The academic friction coefficient theory of rocks will helpful to someone to study the causes and values rocks friction coefficient, the faults causes and influencing factors, the values of lithosphere friction and dynamic friction of other materials. The “dynamic friction” should a new development direction. There seldom study on surface asperities and its deformation at present. Exact surface asperities (contact) and its deformation model need to be set up. The author only induces the dynamic geometric configuration and tangential deformation laws. The author want the study

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could bring much better ideas into the development of “dynamic contact” and “dynamic deformation friction” theory which maybe birth in the future.

THE FRICTION MODEL OF ASPERITIES

Dynamic deformation of friction: The study supposes that the cross section of surface asperities of a friction face is all triangular, as Fig. 1 show. Firstly, we study two asperities who opposites to each other with the extrusion of compressive stress. We supposes that the cutting depth of the two asperities who opposites to each other is δ. We look the deformation of footwall in the Fig. 1 as the research object. The initial inclination angle of two asperities is β_0 and after deformation transform to β, the β should be named “instantaneous inclination angle”. The β and the β_0 like the “gradient of the slope” concept (He-Ping and Zhong-Hui, 2004) in some tribology literature. The inclination angle of asperities concept not only the “gradient of the slope” of a particle but also the meso-asperities. We may find that the inclination angle of the Meso-asperities be not determined by the geometric configuration of asperities (Valentin, 2010).

In the friction process, in order to assure the cutting depth δ not change all along. The instantaneous inclination angle βof two contact face should become smaller and reach the minimum angle θ in Fig. 1 shows. The study call the θ as the “friction repose angle” and express the minimum angle in the friction process. When at the minimum angle θ, the vertical maximum deformation of is δ/2, the contact face AB change to A\`B\` after the deformation. AA\`>CC\`, In order to simplify the model, the study supposes that AA\`≈ DB\`, this assumption will induct some errors, But not a bad assumption, the errors will be a possible little than the “triangular cross section of surface asperities” assumption. So, the study consider that A'B'//AD and A'B' = AD. The “friction repose angle” θ should meet as the formula 1 show:

$$\angle B^{'} A^{'} C^{'} = \angle DAC = \theta \tag{1}$$

The relationship between θ and β_0 should meet as the formula 2 shows:

$$\tan\theta = \tan(\beta_0)/3 \tag{2}$$

That is to say, the tangent of the initial inclination angle of two asperities is three time of the tangent of the “friction repose angle”. The simple “three time relationship” will be a important application value, because the relationship set up a bridge between the initial surface geometric configuration (can be detect) and the configuration which after the deformation (cannot be detect). The initial value is initial point and the 1/3 is end point, the average of initial point and end point make up of the macro-value.

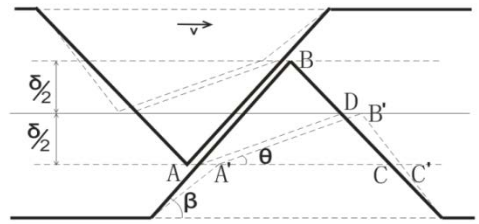

Fig. 1: Original inclination angle and deformation model of the asperities in a friction surface

Kinetic friction coefficient model: The friction coefficient should be divided in to three levels (Cheng-Yao *et al*., 2013) for the sandstone, that is respectively of sand particles surfaces μ_{fg}, of asperities inclined plane (equated with the contact face AB as Fig. 1) μ_{fs}, of rocks surfaces (or macroscopic crack) μ_{fc}, those three mechanisms and their coupling caused the friction coefficients differentiation (An-Ou, 1992). The μ_{fg} and μ_{fs} is often a constant quantity, but μ_{fc} and the coupling of different scale asperities (An-Ou, 1992). The instantaneous inclination angle β become smaller, the stress become larger, the friction coefficient becomes smaller in the contact and deformation process. So, the kinetic coefficient friction is a instantaneous changing process in the slipping friction process. To average the slipping friction process which from initial angle β_0 to angle θ. If let the friction coefficients asperities inclined plane (contact face AB as Fig. 1) μ_{fg} = tan ω, the minimum value μ_{fcmin} of instantaneous friction coefficient μ_{fc} as formula 3 shows:

$$\mu_{fc\min} = \frac{\mu_{fs} + \tan\theta}{1 - \tan\theta \cdot \mu_{fs}} = \tan(\theta + \omega) \tag{3}$$

The maximum value $\mu_{fc\max}$ of instantaneous friction coefficient μ_{fc} as formula 4 shows:

$$\mu_{fc\max} = \frac{\mu_{fs} + \tan\gamma}{1 - \tan\gamma \cdot \mu_{fs}} = \tan(\beta_0 + \omega) \tag{4}$$

The theoretical kinetic friction coefficient $\overline{\mu_{fc}}$ of the asperities cell deformation as formula 5 shows:

$$\overline{\mu_{fc}} = \int_{\theta}^{\beta_0} \tan(\chi + \omega) d\chi = \int_{\theta+\omega}^{\beta_0+\omega} \tan(y) dy = -\cot\Big|_{\theta+\omega}^{\beta_0+\omega} \tag{5}$$

The theoretical kinetic friction angle ψ_K as formula 6 shows:

$$\psi_K = \arctan(\overline{\mu_{fc}}) \tag{6}$$

The theoretical kinetic friction coefficient of macro-rocks is the time and value average of the series asperities cell $\overline{\mu_{fc}}$. In mostly materials the series asperities maybe regarded as equal. So the theoretical kinetic friction coefficient of macro-rocks is also equal to $\overline{\mu_{fc}}$. From the formula 2, 3, 5, we may simplify further and get the formula 7:

$$\overline{\mu_{fc}} \approx \tan\omega + \frac{2}{3}\tan\beta_0 \tag{7}$$

When the initial inclination angle β_0 is very small, such as smaller than 8º. Because the mathematic limit of tan β_0 is β_0, so tan $\beta_0 \approx \beta_0$, so the kinetic friction coefficient of macro-rocks (also $\overline{\mu_{fc}}$) may simplify further and get the formula 8:

$$\overline{\mu_{fc}} \approx \tan\omega + \frac{2}{3}\beta_0 \tag{8}$$

The application value of formula 5, 7, 8 is characterization the change relationship of kinetic friction coefficient with the change of inclination angle of asperities semi-quantitatively. This is a coarse "dynamic contact" and "dynamic deformation friction". In the practice, those formula 5, 7, 8 should be selected according to the situation. Those simple relationships will be some important application value, because the relationship set up a bridge between the initial surfaces geometric configurations (can be detect) and the friction coefficient (cannot be detect). Those simple relationships avoiding the cutting depth and stress value, this should be uses conveniently.

Static friction coefficient model: Theory circle studies seldom on the static friction coefficient in the past years. The study holds that the static friction is the friction before or when the initial asperities tangential deformation starts in the Fig. 1. That is to say that the static friction coefficient is the maximum friction coefficient value μ_{fstmax} when the inclination angle is the initial angle β_0. So, the static friction coefficient μ_{fst} maybe be considered as also the μ_{fstmax} in Table 1 approximately and as well $\mu_{fst} \approx \mu_{fstmax}$. Why be considered as approximately? Because of this is not an exact model. Strictly speaking, the static friction coefficient not only is determined by the surface asperities, but also the time length and plastic deformation of static contact. But the approximate is also having more considerable value in theory and application.

Toward the sandstone, the friction coefficient of asperities inclined plane (equated with the contact face AB as Fig. 1) μ_{fs} is often 0.2-0.27. The kinetic friction angle is 11.4º-15.15º. We use the formula 4 to calculate the μ_{fsmax} and look it as the static friction coefficient μ_{fst}. The μ_{fst} and some other parameters as Table 1 shows.

Table 1: The relations between kinetic and static friction coefficient of different inclination angle of the asperities in sandstone surface

		Inclination angle β_0 of the asperities			
μ_{fs}	Content	5º	10º	15º	20°
11.40º	μ_{fst}	0.2943	0.3919	0.4964	0.6104
	θ	1.6700º	3.3600º	5.1000º	6.9200º
	μ_{fcmin}	0.2320	0.2634	0.2962	0.3309
	$\overline{\mu_{fc}}$	0.2632	0.3276	0.3863	0.4706
	λ	0.8942	0.8360	0.7982	0.7710
15.15º	μ_{fst}	0.3669	0.4694	0.5808	0.7041
	θ	1.6700º	3.3600º	5.1000º	6.9200º
	μ_{fcmin}	0.3021	0.3347	0.3688	0.4051
	$\overline{\mu_{fc}}$	0.3345	0.4021	0.4748	0.5546
	λ	0.9117	0.8564	0.8176	0.7877

From the Table 1, The Kinetic/Static Ratio of friction coefficient λ should between 0.771-0.9117. The Kinetic/Static Ratio of friction coefficient λ is the function of the β_0. The λ is inversely proportional to the β_0. The initial inclination angle β_0 in most material's (include rocks) asperities should between 5º and 20º. Although the friction coefficient of 20º asperities inclination angle is often smaller than the friction coefficient 0.6-1.2 (An-Ou, 1992) of sandstone. The study holds that the friction coefficient which part that larger than 0.5546 (Table 1) is because of the coupling of different scale asperities. The coupling theory should reference (Cheng-Yao *et al*., 2012).

DISCUSSION

The theoretical and application value of those models: The theoretical value of those models is as flowing:

- The study set up the relationship between the inclination angle of the asperities and the friction coefficient. The relationship set up a bridge between the initial surface geometric configuration (can be detect) and the configuration which after the deformation (cannot be detect). The average of initial point and end point make up of the macro-value. The relationship can help someone grasp the analytical friction theory theoretically but not use an exact numerical calculation model.
- The study attends that the friction should be a dynamic change process of asperities tangential deformation and how the dynamic geometric configuration and tangential deformation affect the friction coefficient.
- Preliminary study and set up a coarse but heuristic "dynamic deformation friction" theory which different to the static asperities contact process that no tangential deformation.

The application value of those models is as flowing:

- The study set up an analytical friction theory which suitable for most materials and would be a widely use in tribology fields.

- Inclination angle of the asperities data maybe obtain from friction surface photoelastic experiment and the study on fractal dimension. But the "photoelastic statistics" can only detect the asperities inclination angle which without tangential deformation. But model in this study can help get the friction repose angle and friction coefficient after tangential deformation.
- In the study practice, the stress, the cutting depth, a deffective distance of asperities all are difficult to get, the model can calculate the friction coefficient without the stress and the cutting depth.
- We should pay attention to the value of the "friction coefficient of smooth surface" which is the minimum friction coefficient after smoothing or running-in and may be considered as the pure plane friction coefficient μ_{fs}. The average inclination angle of the asperities and fractal dimension can be back-stepping and reconstruct when know the "friction coefficient of smooth surface" and the friction coefficient before smoothing or running-in. The "back-stepping method or reconstruct solving approach" maybe becomes a substitution of "photoelastic statistics" to study the friction coefficient.

The scientific issues and logical system:

- **Asperities inclination angle and cutting depth, who dominates whom?** The cutting depth δ must be realized through deformation. If the friction wants to become a smooth friction coefficient, the "friction repose angle" must dominate the cutting depth. The cutting depth will change with the stress change, but the "friction repose angle" may be not changes. The situation that exist the friction force jumping in some friction face must be some larger asperities which cannot achieve the deformation to guarantee the "friction repose angle". The "Jumping" is the change of the cutting depth but the "friction repose angle".
- **Discussion on "dynamic contact" and "dynamic deformation friction":** That is still seldom studies on the "dynamic contact" and "dynamic deformation friction". The model in the study will a simplest "dynamic deformation tribology". In the future, the exact and complicated "dynamic deformation tribology" model should be a complicated system include not only the contact number, stress, but also the average cutting depth, valley-peak corresponding relation, peak-peak contact time, tangential deformation, hysteretic loop and rebound rate, friction repose angle and so on. The "dynamic deformation tribology" model ("dynamic contact" and "dynamic Friction") must promote the development of the Tribology.

CONCLUSION

- The instantaneous kinetic friction angle is a periodical change process in a friction process; the inclination angle of two contact face should become smaller and reach the minimum angle-friction repose angle. The theoretical kinetic friction coefficient of macro-rocks is the time and value average of the series asperities cell.
- The tangent of the initial inclination angle of two asperities is three time of the tangent of the "friction repose angle". The relationship set up a bridge between the initial surface geometric configuration (can be detect) and the configuration which after the deformation (cannot be detect).
- The static friction coefficient is the maximum friction coefficient value, the Kinetic/Static Ratio of friction coefficient should between 0.771-0.9117. And the Kinetic/Static Ratio is a inversely proportional to the inclination angle.
- The "friction repose angle" must dominate the cutting depth. The cutting depth will change with the stress change, but the "friction repose angle" not changes. The "Jumping" is the change of the cutting depth but the "friction repose angle".
- The sandstone friction coefficient which part that larger than 0.5546 is because of the coupling of different scale asperities.
- The average inclination angle of the asperities and fractal dimension can be back-stepping and reconstruct when know the "friction coefficient of smooth surface" and the friction coefficient before smoothing or running-in. This maybe becomes a substitution of "photoelastic statistics" to study the friction.

Variable symbol definition:

$\tan\psi$: Friction coefficient
β : (Average) inclination angle of the asperities
β_0 : Initial inclination angle of the asperities
θ : Friction repose angle
ω : Friction angle of asperities inclined plane
ψ_K : Kinetic friction angle
δ : Cutting depth
μ_{fg} : Friction coefficient of sand particles surfaces
μ_{fs} : Friction coefficients asperities inclined plane, equal to $\tan\omega$
μ_{fc} : Instantaneous kinetic friction coefficient in a friction process
$\mu_{fc\,max}$: The maximum value $\mu_{fc\,max}$ of instantaneous friction coefficient μ_{fc} in a friction process, approximately equal to μ_{fst}
$\mu_{fc\,min}$: Minimum value μ_{fcmin} of instantaneous friction coefficient μ_{fc} in a friction process

$\overline{\mu_{fc}}$: Kinetic friction coefficient is the integral average value of μ_{fc}

μ_{fst} : Static friction coefficient, approximately equal to μ_{fcmax}

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