Based on the Force Deployment Model of Unascertained Expectation

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Abstract: In this study, we utilize the unascertained mathematics method to give the unascertained number of countermeasure of anti-terrorism strategic force deployment and unknown event. It has been defined the situation sets of force deployment, condition density and mathematical expectation of density model. It has been given the unascertained parameters Cij which decide and direct the force deployment. Find out the condition density matrix of force deployment, further get the conditional density of single target force deployment, using the maximum density mathematical expectation in order to get the optimal mathematical model of multiple target force deployment. Analyzing the coefficient of model and provide two kinds of discussed computing method. The model overcomes the limitation of past deterministic thinking method which study the force deployment and provide the decision maker a relative substantial theory evidence.

Keywords: Conditional density, force deployment, mathematical expectation, model, situation, unascertained number

INTRODUCTION

The problem of forces ‘deployment is a fundamental problem in military decision-making, in the past decision we can usually meet optimal result in a positive background and a result in stochastic condition .the practical anti-terrorism strategic forces’ deployment problem is usually anfractuosity, because it’s not a doubtless problem but not a Stochastic Process. It has a preordained fate relationship, it also has some complicated indeterminacy factors. In practice the system of anti-terrorism strategic forces’ deployment is a complicated "indeterminacy system. Thus it’s practical for us to use the Unascertained Mathematical ideal method to study the problem forces’ deployment. Bi and Yan (2010) study the research of military strategy thought in the countemporaryera. Li (2005) study the military strategy learns lectures. Shen et al. (2005) study the targets being misty to make policy troops an allotment model. Ren et al. (2007) have a research of the troops of the rate deployment model. Liu et al. (2010) analyze the troop disposition assessment Reconnaissance-Attack system.

In this study, we utilize the unascertained mathematics method to give the unascertained number of countermeasure of anti-terrorism strategic force deployment and unknown event. It has been defined the situation sets of force deployment, condition density and mathematical expectation of density model. It has been given the unascertained parameters Cij which decide and direct the force deployment. Find out the condition density matrix of force deployment; further get the conditional density of single target force deployment, using the maximum density mathematical expectation in order to get the optimal mathematical model of multiple target force deployment. Analysing the coefficient of model and provide two kinds of discussed computing method. The model overcomes the limitation of past deterministic thinking method which study the force deployment and provide the decision maker a relative substantial theory evidence.

DESCRIPTION OF THE PROBLEM

At present, the research of forces' deployment model consist of ascertain model and stochastic model. the problem of forces’optimized deployment in Military Operations Research is practically a ascertain designating problem. The model is expressed 0-1 planning:

\[
\begin{align*}
\text{max } p(s) &= \sum_{i=1}^{m} a_i x_i, \text{ s.t. } \sum_{i=1}^{m} x_i = 1; \sum_{j=1}^{n} x_{ij} = 1; \sum_{j=1}^{n} x_{ij} = 1 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)
\end{align*}
\]

The 0-1 variable \(x_{ij}\) is defined as:
when \(A_i\) is disposed \(B_j\), \(x_{ij} = 1\)
when \(A_i\) isn’t disposed \(B_j\), \(x_{ij} = 0\)

Although we can use parallelism method in parallelism to solve problem in order to get the result of forces ‘deployment (it is a 0-1 matrix in practice) to guide the military decision-making. Actually, it is based on the premise that the above-mentioned model is specific ascertain assumption. But in the practical application, the eruption of hostilities, even in the forces’ deployment process is full of many uncertain factors, thus many scholars use the probability method to study forces’ deployment so as to make forces’
deployment model transform studying ascertain problem to study stochastic problem:

$$\max p(S) = \sum_{j=1}^{n} p(B_j) p(S \mid B_j) = \sum_{j=1}^{n} p(B_j)$$

There, S is represented where we gain the victory. For the problem of studying probability is a stochastic process at the same time we need a larger specimen space, the eruption of hostilities, even in the forces' deployment process are not only a stochastic progress. It has a preordained fate relationship, it also has some complicated indeterminacy factors. Thus it's practical for us to use the Unascertained Mathematical ideal method to study the problem forces' deployment.

**Definition 1:** All troops countermeasures of deployment problem research of troops are called countermeasures collection. Called $$ A = (A_1, A_2, \ldots, A_m) $$, Among them, $$ A_i (i = 1, 2, \ldots, m) $$ is the i-th deployment countermeasure troops. If mean the $$ x_i $$ grow the troops number of troops deployment countermeasures (when $$ i=j $$, have $$ x_i < x_j $$), if mean $$ \phi(x_i) = \frac{x_i}{\sum_{i=1}^{n} x_i^2} $$.

Then call $$ [[x_1, x_n], \phi(x)] $$ for the unascertained number of countermeasures.

**Definition 2:** All events that may occur of deployment problem of troops are called events collection. Called $$ B = (B_1, B_2, \ldots, B_n) $$, Among them, $$ B_j (j = 1, 2, \ldots, n) $$ is the j-th event. The $$ b_j $$ is the possibility size of the occurrence of the event $$ B_j $$ when $$ x = j $$, $$ h(x) = b_j $$.

We called $$ [[1, n], h(x)] $$ as the unascertained number of the event.

**Definition 3:** Cartesian product of countermeasures collection $$ A = (A_1, A_2, \ldots, A_m) $$ and events collection $$ B = (B_1, B_2, \ldots, B_n) $$, $$ A \times B = \{(A_i, B_j) \mid A_i \in A, B_j \in B\} $$ are called the situation in collection, called $$ S = A \times B $$.

For arbitrarily $$ A_i \in A, B_j \in B $$, we called $$ (A_i, B_j) $$ as situation. Make $$ S_{ij} = (A_i, B_j) $$, Then event $$ B_j $$ occur and $$ A_i $$ deploy troops and carry out a task with $$ B_j $$.

**Definition 4:** For situation $$ (A_i, B_j) $$, Use $$ C_{ij} $$ to mean the reliability of a defence $$ B_j $$ for countermeasures $$ A_i $$, then we called $$ C_{ij} $$ as the unascertained density of situation.

**Definition 5:** For situation $$ (A_i, B_j) $$, Use $$ b_{ij} $$ to mean the reliability of performance the task and obtaining a victorious while using countermeasure $$ A_i $$ when event $$ B_j $$ occur. Then we called $$ b_{ij} $$ as the unascertained factor density of situation. Then we called:

$$ \{b_{ij}\}_{n\times m} = \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1m} \\ b_{21} & b_{22} & \ldots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \ldots & b_{nm} \end{bmatrix} $$

as the unascertained factor density array.

**Definition 6:** Set $$ [[a, b], \phi(x)] $$ as an unascertained number, $$ \phi(x) $$ is an factor density, Then we called $$ E = \int_a^b \phi(x) dx $$ as the unascertained density expectation. If $$ [[a, b], \phi(x)] $$ is an unascertained rational, then $$ E = \sum_{i=1}^{n} x_i \phi(x_i) $$.

**Definition 7:** Suppose there are R experts analyse the density of situation $$ (A_i, B_j) $$, order $$ b_{ij}^{(k)} $$ to be density $$ b_{ij} $$ generalized residual result which the k-th expert deal with situation $$ (A_i, B_j) $$, thus $$ (b_{ij}^{(1)}, b_{ij}^{(2)}, \ldots, b_{ij}^{(R)}) $$ are $$ b_{ij} $$ generalized residual vector. If the weight of k-th expert is $$ w_k $$, $$ 0 < w_k \leq 1 $$ (k = 1, 2, …, R) that $$ \sum_{k=1}^{R} w_k b_{ij}^{(k)} $$ is the generalized residual vector $$ b_{ij}^{(\ast)} $$ of $$ b_{ij} $$ which is density of $$ (A_i, B_j) $$.

**Definition 8:** Order $$ b_{ij}^{(k)} $$ be density $$ b_{ij} $$ generalized residual result which the k-th expert deal with situation $$ (A_i, B_j) $$, thus:

$$ \{b_{ij}^{(\ast)}\}_{n\times m} = \begin{bmatrix} b_{11}^{(\ast)} & b_{12}^{(\ast)} & \ldots & b_{1m}^{(\ast)} \\ b_{21}^{(\ast)} & b_{22}^{(\ast)} & \ldots & b_{2m}^{(\ast)} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1}^{(\ast)} & b_{n2}^{(\ast)} & \ldots & b_{nm}^{(\ast)} \end{bmatrix} $$

The generalized residual matrix of $$ b_{ij} $$ which is made by the k the expert (i = 1, 2, …, n, j = 1, 2, …, m) If the weight of the k-th expert is $$ w_k $$, $$ 0 < w_k \leq 1 $$ (k = 1, 2, …, R) then $$ \sum_{k=1}^{R} w_k b_{ij}^{(k)} $$.

Is be density $$ b_{ij}^{(\ast)} $$ generalized residual result which is the unascertained condition density $$ b_{ij} $$ of situation $$ (A_i, B_j) $$, thus $$ b_{ij}^{(\ast)} = \sum_{k=1}^{R} w_k b_{ij}^{(k)} $$.

The troops deployment problem can describe for our armies have the troops of total amount M, constitute $$ m $$ battle groups $$ A_1, A_2, \ldots, A_m $$ of independence according to the amount organic combination of the weapon material, true war technique and the personnel's amount, probably produce the point region of the riot activity of the terror raid as $$ B_1, B_2, \ldots, B_n $$.

At some a particular period, the region of $$ B_n $$ takes place the possibility of terrible activity as $$ b_n $$ and regard
as war the group \( A_i \) to set up defense at the \( B_j \), setback terror assault riot activity of the credibility is \( b_{ij} \), how to set up defense from the overall situation, namely with how of the situation set up defense formation situation to gather, or choice which \( c_{ij} \) to make the possibility of setback enemy biggest for our army, so we can obtain warlike victory.

THE MODEL OF TROOPS DEPLOYED

The uncertain number of troops deployed: Suppose that independent battle group number is \( m \) that is as \( A_1, A_2, ..., A_m \), the independent battle group \( A_i \) consist of \( x_i \) members, Suppose \( x_1 < x_2 < ... < x_m \). The main defiance areas to have possible terrorism war are \( B_1, B_2, ..., B_n \). The possibility that \( B_j \) take place terrible riot activity is \( b_{ij} \). Then we can get following 2 uncertain number:

\[
[[x_1, x_m], \varphi(x)] \text{ where } \varphi(x_i) = \frac{x_i}{\sum x_i} \left[ [1, n], h(y) \right]
\]

where \( h(y = j) = b_{ij} \) (1)

The \( b_{ij} \) means that the possibility size of the war to happen in \( B_j \) region.

The condition density of troops deployment:

Suppose \( b_{ij} = b_j(x, x = x_i) \)  

\( (i = 1, 2, ..., t; j = 1, 2, ...n) \)  

\( 0 \leq b_{ij} \leq 1 \)

\( b_{ij} \) is defined to the condition density of troops deployment, The \( b_{ij} \) means the credibility to obtain overall victory, which happen in \( B_j \) region, when \( A_i \) troops is organized to defiance \( B_j \) region to carry out the task of battling .

so get the uncertain condition density matrix of troops deployment:

\[
(b_{ij})_{mn} = \begin{bmatrix}
  b_{11} & b_{12} & ... & b_{1n} \\
  b_{21} & b_{22} & ... & b_{2n} \\
  ... & ... & ... & ... \\
  b_{m1} & b_{m2} & ... & b_{mn}
\end{bmatrix}
\]

(3)

The jth list \( (b_{1j}, b_{2j}, ..., b_{mj})^T \) of in the matrix show the credibility distribution that when battle group \( A_1, A_2, ..., A_m \) is organized the defence to \( B_j \), respectively, \( A_i \) defeated the enemy to get overall victory. The \( c_{ij} \) Mean the credibility of the battling task to carry out when war happens in \( B_j \) region , the \( A_i \) troops is deployed in \( B_j \). So then \( (c_{1j}, c_{2j}, ..., c_{mj}) \) means respectively the possibility size that battling group \( A_1, A_2, ..., A_m \) is organized a defiance to \( B_j \).

The density model of troops deployment: Make

\[
\left[ c_{1j}, c_{2j}, ..., c_{mj}\right] \left[ b_{ij} \right] b_{ij} - b_m = \sum_{i=1}^m b_i c_{ij} = g(b_j) \quad (4)
\]

The \( g(b_j) \) means the credibility that \( A_1, A_2, ..., A_m \) defeated of enemy war that happen in \( B_j \) region, when \( B_j \) being defended with \( A_1, A_2, ..., A_m \) \( b_{ij} = \min \{b_i\} b_j = -\max \{b_i\} (j = 1, 2, ..., n) \) Combine to line up a preface according to the size of \( b_{ij} \). Get a condition density type uncertain number:

\[
\left[ b_{ij}, b_{ij} \right], g(b_j)
\]

Define:

\[
E(b) = \sum_{i=1}^n b_i g(b_i) \quad (5)
\]

as uncertain mathematics expectation of density type.

The \( E(b) \) means the uncertain mathematics expectation that our army acquires overall victory. The different troops deployment gets the different mathematics expectation. Comprehensive analysis on the above to know: troops deployment problem of the counter-terrorism strategic is look for reasonable troops deployment, To obtain reasonable \( c_{ij} \) \( (i = 1, 2, ..., t; j = 1, 2, ..., n) \) Make the \( E(b) \) maximize, namely:

\[
\max E(b) = \sum_{j=1}^n b_j g(b_j) = \sum_{j=1}^n b_j \sum_{i=1}^m b_i c_{ij} \quad (6)
\]

That means under Guiding of the target to look for reasonable uncertain density distribution, make the biggest possibility obtain the victory of war, this \( c_{ij} \) is a comprehensive information characteristic that mean the credibility when \( A_i \) is deployed to \( B_j \). All of \( b_{ij} \) and \( b_{ij} \) are uncertain condition density, which relies on politics, culture, diplomacy and military...etc of nation and region of terrible influence place, they are analyzed and calculated by the various factors that induct war.

The model (6) is also abstract into a problem as follows:

\[
\max f(x) = \sum_{j=1}^n \sum_{i=1}^m a_i x_{ij}
\]

\[
\sum_{i=1}^m x_{ij} = 1 \quad 0 \leq x_{ij} \leq 1 \quad (i = 1, 2, ..., t; j = 1, 2, ..., n)
\]
THE ANALYSIS AND ASCERTAINMENT OF MODEL COEFFICIENT

The coefficient $a_{ij}$ in the model (7) and coefficient $b_{ij}$ in the model (6) are the promise of optimizing deployment. Next, we carry on analysis and discussion. Both $b_{ij}$ and $b_{ij}$ are unknown condition density and depending on the comprehensive situation which constituted by the politics, culture, diplomacy, military strength of terrible nation and region.

Suppose these comprehensive situations can be divided into $k$ kind of state $z_1, z_2, ..., z_k$ and form into comprehensive situation $SZ$:

$$z_i \cap z_j = \Phi, (i \neq j)$$

Regard $SB$ as the appearance space of war occurrence. Obviously the SB has two elements. (Occur war or don't). Regard B as the event of war occurrence.

Only being causing the inducement appear just may cause war. Make $(B_j, F_i)$ as war situation caused by remote cause $F_i$. $f_{ij}$ is under the predisposing appearance just may cause war. Make $(B_j, F_i)$ as war situation caused by remote cause $F_i$. $f_{ij}$ is under the predisposing appearance just may cause war.

Method one: The result has been given through the unascertained measure of expert estimation, under the predisposing of $F_i$ that caused the event occurs in war situation $B_j$, this constitute the predisposing factors matrix:

$$\begin{bmatrix}
(B_j, F_i) & (B_j, F_i) & \ldots & (B_j, F_i) \\
(B_j, F_i) & (B_j, F_i) & \ldots & (B_j, F_i) \\
\vdots & \vdots & \ddots & \vdots \\
(B_j, F_i) & (B_j, F_i) & \ldots & (B_j, F_i)
\end{bmatrix}$$

The reason that a war occurs is able to be caused by one factor, or also can be caused by some factors at mean time; here we only discuss the circumstance of single factor independence that caused the war.

Suppose there are $R$ experts, the $K$ experts think under the predisposing of $F_i$, the possibility which a war occurs in the place of $B_j$ is $f_{ij}^{(k)}$, the evaluation set is consist of predisposing which caused the war from $R$ experts:

$$\begin{bmatrix}
(b_{ij}^{(1)}, b_{ij}^{(2)}, \ldots, b_{ij}^{(R)})
\end{bmatrix}$$

The weight vectors of $K$ experts are $[w_1, w_2, \ldots, w_R]$, thus get the $f_{ij}^{(s)}$ which is the prior estimate value of $f_{ij}$:

$$f_{ij}^{(s)} = \left[w_1 w_2 \ldots w_R \right] \begin{bmatrix} a_{ij}^{(1)} \\ a_{ij}^{(2)} \\ \vdots \\ a_{ij}^{(R)} \end{bmatrix} = \sum_{s=1}^{R} w_s a_{ij}^{(s)}$$

$(i=1,2,\ldots; j=1,2,\ldots,n)$

It is the same for the $b_{ij}$ which is able to be find out through the past experience and expert forecast method.

Method two: Use the probability, because $z_1, z_2, \ldots, z_s$ and it has become general trend $SZ$.

$$z_i \cap z_j = \Phi, (i \neq j)$$

thus:

$$p(F_i) = \sum_{k=1}^{K} p(F_i / Z_j) p(Z_j), (i=1,2,\ldots,I)$$

thus: $b_j = p(B_j)$ should be comprehend as probability:

$$b_j = \sum_{m=1}^{M} p(B_j / F_i)$$

while:

$$p(B_j / F_i) = \frac{P(B_j, F_i)}{P(F_i)} = \frac{p(F_i / B_j) p(B_j)}{\sum_{i=1}^{K} p(F_i / Z_j) p(Z_j)}$$

$(i=1,2,\ldots; j=1,2,\ldots,n)$

here:

$$p(B_j) = \sum_{m=1}^{M} p(B_j / Z_j) p(Z_j), (j=1,2,\ldots,n)$$

DISCUSSION

In fact, Still return to produce one some problems which stay at the further study to comprehension or useful property from the research of the $b_{ij}$ and the $b_{ij}$. Thus produce the model canning and solve method to be advantageous to analysis more and solve. If we can understand $\sum_{i=1}^{R} b_{ij} \leq 1$, under the some condition or not. Here"=" establish to mean inevitable occurrence war feeling, "\<" establish to express uncertain occurrence war feeling:

$$0 \leq \sum_{i=1}^{R} b_{ij} \leq 1, (i=1,2,\ldots,m) 0 \leq \sum_{j=1}^{n} b_{ij} \leq 1, (j=1,2,\ldots,n)$$
Moreover for $c_{ij}$ to speak, when area $B_j$ has war feeling, -1 battle group must defend the war feeling. Namely $\sum_{i=1}^{m} c_{ij} = 1, (j = 1, 2, \ldots, n)$, But for any deployment, each war area's hasing war feeling is an indetermination affairs, namely:

$$\sum_{j=1}^{n} c_{ij} \leq 1, \quad (i = 1, 2, \ldots, m)$$

Then we can give model's (6) stipulation condition of increment then get new model thus.

**CONCLUSION**

According to unascertained condition density deployment model of troops, use unascertained thought method to analysis deploy environment, lead to deploy into the troops of unascertained number and condition density matrix. We can get the best deployment model of troops though the unascertained number expect of biggest worthy, avoided using the limit of the dispose troops for battle of the thought method of assurance before. The key of this model is indeed to make sure that 3 unascertained numbers and condition density matrix.

Along with the development of war, the deployment model of troops by all means faces a new topic and the deployment model of troops also needs a further research.

**REFERENCES**


