Leaf Mass Estimation Affected by Leaf Shape and Tree Height based on Simulation and Four-Layer Hidden-node Neural Network

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Abstract: Leaf mass, a vital parameter of forest ecology and physiology, is estimated in this study. According to the simulation result in this study, leaf shape makes little contribution to the exposure area. Moreover, a four-layer hidden-node neural network model is used in the new model, analyzing correlation between leaf mass and height of trees. Leaves of various areas are tested as sensitivity analysis for the simulation which studied the influence of leaf shape on exposure area. Finally, leaf mass is estimated by a statistical model based on the regression relationship of sapwood area and leaf mass.

Keywords: Four-layer hidden-node neural network model, leaf mass, shape, simulation

INTRODUCTION

As leaves play a pivotal role in the growth of a tree, the description of leaves has been a crucial issue remains to be further researched.

Based on studying KNOX genes in its initial cells, (Jihyun and Sarah, 2011) Hake provided an explanation why leaves shape differs from each other. In the research, different growing processes of leaves are considered. Hassan et al. (2010) and Etienne et al. (2011), concentrated on general framework for leaf folding as well, who outline various packing structure in buds and the corresponded folding process of leaves. Etienne et al. (2011) meanwhile, the estimation of leave mass calculated by volume of the leaves has been figured out. Taking Maize as an example, the deviation between leave mass estimation and the actual values is of 50 percent, a little bit low though. Thus, what emphasized in this study is the accurate estimation of leaves mass, since neglect of it.

The description of leaves is considered with particular attention to the shapes sand the accurate estimated mass of the leaves. This question can be broken down into two more main detailed questions.

Does the shapes of leaves affect their exposure?
How to estimate the leaf mass of a tree?

OVERLAP AREA INFLUENCED BY SHAPES

Overlap occurs on a tree with average number of leaves as is shown in Fig. 1.

As different shapes of leaves have different overlapping shapes. Does leaf shapes affects their overlapping areas? What is specific influence the leaf shapes have on the overlapping areas if it does?

A simulation of overlapping areas was made, whose result is shown in Fig. 1. Plenty of leaves, which are simplified as geometric figures without consideration of their thickness, are generated randomly in a specific space, which represents the space taken by leaves in reality. Then, calculate the area exposed directly in the sun, after projecting the leaves to the horizontal plane. By comparing these areas, relationship between leaf shapes and overlapping area was finally figured out.

In Fig. 2, the green ovals represent the leaves while the yellow ones represent the shadows of the leaves.

Shape selection and image processing: Due to the variety of leaf shapes, leaves from three kinds of trees,
which are Ovate, Lobed and Palmate, are selected due to their typical shapes.

For calculating the area of leaves, colors of leaves are neglected here, which are made black (Zhang and Wen-Ping, 2011). Then, the area uniformity was achieved for getting new images of the leaves. New images are 100 x 100 pixel each. By amplifying and narrowing the shape in the image (André and Odemir, 2010), black pixels of three shapes were figured out in the images of 2458 pixel as is in Fig. 3, which means the assumption that the areas of the three kinds of leaves are the same.

Program implementation: In the simulation, leaves with random revolving angles project on a 3000 x 3000 horizontal square. As a result, the area exposed in the sun could be figured out as is shown in Fig. 4.

RESULT ANALYSIS

The areas exposed in the sun are not exactly the same since the overlapping area of the three shapes differ from each other. However, the difference is not big enough in Fig. 4. The similarity, according to the inference, results from that all these kinds of leaves remain in natural selection, which enables to avoid overlap. Following causes leading to the similarity of the exposure areas are inferred.
In the figure, almost no overlap occurs at the beginning due to the sparsity of leaves. Therefore, there is no relationship between overlapping areas and leaf shapes, since the overlapping area is a constant, approaching to 0.

When the number of leaves reaches the threshold, overlap, which differs from shapes, occurred. The result of simulation shows that the leaves with the shape of Lobed have the biggest exposure area, with Palmate and Ovate followed.

Once the leaves become too serried in the specific space, where there are little interspaces left and the simulating results shows that the exposure area is of great proximity.

**Sensitivity analysis:** In the simulation above, numbers of black pixels represent the leaf area. The number of pixel used is 2458. However, in this section, the size of the image were changed, from 2458 pixels to 1579, 2458 and 3579 pixels respectively, for analysis, as is shown in Fig. 5.

In Fig. 5, verification of the result was obtained according to this graph.

**ESTIMATION OF THE DRY WEIGHT**

To estimate the dry weight of leaves, it is defined that the weight proportion of carbon in leaves is $\theta$. The $\theta$ differs from trees to trees. Therefore, related measurement for parameters are required for estimating specific leaf mass of a tree.

Parameters are evaluated here, leading to the following results in Fig. 6.

In Fig. 6, the dry weight of leaves changes as time passing under the condition of enough carbon and nitrogen offered in Graph a. Graph b describes the leave mass changes under the condition without enough carbon, while Graph c is the description in the condition without enough nitrogen.
Methods of calculating the sapwood area were discussed indirectly due to the difficulty of measuring the sapwood area. One of the methods is to figure out the leaf mass after getting the relationship between the height and sapwood mass. Thus, the problem will be discussed as followed. The relation between leaf mass and height of the tree was researched by mechanism analysis at first. Then, the improved model using improved BP neural network would be mentioned to analyze the specific relationships.

**Estimation of leaf mass:** The mass of leaves can be divided into two types, dry weight and wet weight. Dry weight is the weight after the drying treatment. Due to the large water occupation in a leaf, which is greatly affected by external environment, such as climate, the dry weight of leaves on a tree was estimated for a more accurate value of the mass. The method of leaf mass estimation mentioned in this study is commonly used recently.

Two estimations were presented for the prediction of leaf dry weight. The former one is Naïve Solution based on the statistics principle while the latter one is a mechanism-estimating model according to analysis of the plant growth.

**Naïve solution:** Leaf is the only vegetative organ to most of the trees. Thus the nutrients created by leaves would be delivered to the root via leaf veins, branches and trunk as is shown in Fig. 7. Based on the transference, the relationship between leaf mass and the size of the trunk is positive correlation according to the inference.

Then, what is the specific relation between leaves and the size of the trunk? In a research, it was figured out that the relationship between basal area and foliage mass of Chinese fir and the relationship between sapwood area and foliage mass of Chinese fir Qian (1989). The sapwood is marked in Fig. 8.

The correlation coefficient between basal area and foliage mass is 0.8, while that between sapwood and foliage mass is up to 0.976. As a result, leaf mass could be calculated by the sapwood area of a specific kind of trees. Take Chinese fir as an example, the leaf mass of it is:

$$ m = 0.0591 \cdot S_{\text{sapwood}} + 0.1279 \quad (1) $$

**LEAF MASS AND HEIGHT**

Fig. 7: Transference of nutrients

Fig. 8: Sapwood of Chinese fir

Fig. 9: Relation between sapwood area and leaf area
Model improvement: As little research is found for calculating the leaf mass though the relation between sapwood area and leaf area was proposed by many papers. Weighing the leaves after drying treatment and figure out the mass per leaf area is improved in this study. The specific example is shown in Fig. 9.

In addition, the sapwood area could be obtained without cut the tree down, which is destruction to the trees being measured. The description of the two methods is shown below.

Before calculating the sapwood area with simple geometric knowledge, obtain the fluoroscopy of the trunk with modern technique.

After getting the relation between the height and sapwood area of the trees with statistical regression techniques, calculate the mass with the height. As the height is not easy to measure directly, geometric knowledge is used here to figure it out as is in Fig. 10.

Mechanism analysis solution: The growth process of plants is very complex, containing respiration, photosynthesis and mineral absorption. However, Botanists found several fundamental facts out according to their observation (Benjamin et al., 2011). First of all, carbon is absorbed by photosynthesis while routes absorb the nitrogen. Secondly, carbon and nitrogen could be transmitted from leaves to other part of the plant. Thirdly, the demands of the two elements are proportioned.

Meanwhile, complex energy conversion is involved on growth of plants. Carbohydrate, a synthetic compound during photosynthesis, which is recognized as energy source, has the two applications followed. One is to offer the energy for mineral absorption and inner transport. The other is to maintain the growth of the plant itself.

Most plant could be divided into three parts, root, stem and leaf. In the model, the assumption was made that roots and leaves make up a plant, ignoring the stems. Moreover, content changes of carbon and nitrogen are the only elements concerned in this model.

It is defined $C(T)$ as the carbon concentration at the time $t$ while $N(T)$ is the nitrogen concentration at the time $t$. The cost velocity of carbon in the plant is $V\varphi(C,N)$. In this equation, $V$ represents the volume of the plant. It is assumed that the proportion of carbon and nitrogen in new organ is the same as that in old one, since the content of chemical elements in plant cells is almost the same. Then define the ratio of nitrogen and carbon is $\lambda$ to get the cost velocity of nitrogen, $\lambda V\varphi(C,N)$. $a$ is the energy-cost ratio of absorption Macromolecules to form their own organization. $r$ is the proportional coefficient that.

Based on the assumptions above, the growth of roots and leaves are described as the Eq. (2):

$$\begin{align*}
\frac{dW_l}{dt} &= \frac{raW_l}{\rho_l} \varphi(C_l,N_l) \\
\frac{dW_r}{dt} &= \frac{raW_r}{\rho_r} \varphi(C_r,N_r)
\end{align*}$$

In the equation, $W_l$ and $W_r$ represent the weight of leaves and roots at the time $t$. Density of leaves and roots are represented by $\rho_l$ and $\rho_r$.

Carbon transmits from leaf to root, whose speed relates to concentration difference of carbon between leaf and root. Definition was made that $\beta_1$ is the ratio of the concentration difference. Thus, the following Eq. (3) is the description of the carbon content in leaf and root:

$$\begin{align*}
\frac{d(W_l)}{dt} &= \rho_l a W_l - \rho_l \beta_1(C_l - C_r) - W_l \varphi(C_l,N_l) \\
\frac{d(W_r)}{dt} &= \rho_r \beta_1(C_r - C_l) - W_r \varphi(C_r,N_r)
\end{align*}$$

Similarly, the concentration-different ratio of nitrogen is $\beta_2$. The relationship could be described as (4):

$$\begin{align*}
\frac{d(W_l)}{dt} &= \rho_l a W_l - \rho_l \beta_2(N_l - N_r) - \lambda W_l \varphi(C_l,N_l) \\
\frac{d(W_r)}{dt} &= \rho_r \beta_2(N_r - N_l) - \lambda W_r \varphi(C_r,N_r)
\end{align*}$$

As a result, the model of the growth of plant is shown in the formula (5):

$$\begin{align*}
\frac{dW_l}{dt} &= \frac{raW_l}{\rho_l} \varphi(C_l,N_l) \\
\frac{dW_r}{dt} &= \frac{raW_r}{\rho_r} \varphi(C_r,N_r) \\
\frac{d(W_l)}{dt} &= \rho_l a W_l - \rho_l \beta_1(C_l - C_r) - W_l \varphi(C_l,N_l) \\
\frac{d(W_r)}{dt} &= \rho_r \beta_1(C_r - C_l) - W_r \varphi(C_r,N_r) \\
\frac{d(W_l)}{dt} &= \rho_l a W_l - \rho_l \beta_2(N_l - N_r) - \lambda W_l \varphi(C_l,N_l) \\
\frac{d(W_r)}{dt} &= \rho_r \beta_2(N_r - N_l) - \lambda W_r \varphi(C_r,N_r)
\end{align*}$$
With data searching, parameters could be evaluated to describe the changes of leaves weight. Meanwhile, the model would be tested with the data of the plant growth.

**Relationship between leaf mass and height** The trunk of a tree was supposed as a cone in order to simplify the model as is shown in Fig. 11.

Based on Fig. 11, $H$ is the height of the tree, while $S_{\text{sapwood}}$ is sapwood area. Hence, relationship between the height and sapwood area was described as the formula (6):

$$S_{\text{sapwood}} = kH^2$$

In the formula, $k$ is a coefficient whose value is unknown.

Due to the linear relationship between sapwood area and leaf mass, relationship between leaf mass and height was described as the following Eq. (7):

$$m = Ah^2 + B$$

In the equation, $A$ and $B$ are both coefficient whose values are unknown. Take Chinese fir as an example, leaf mass of it is:

$$m = 0.0591 \cdot kH^2 + 0.1279$$

In the equation, $k$ is a coefficient whose value is unknown. In Fig. 12, the value of leaf mass was discussed when $k$ is 0.1, 0.15 and 0.2.

According to Fig. 12, the leaf mass is well influenced by the height of trees, which means the height could be used to estimated the tree height.

**Back propagation artificial neural network:**

Although mechanism analysis could be used for the relationship between the leaf mass and the height and volume of a tree as is presented before, the influence to leaf mass with a single value cannot reflect the relationship completely. As a result, Back Propagation artificial neural network was mentioned here for improvement of the model.

The network is a special four-layer-network structure mode, which is different from the traditional three-layer-network structure mode. The weight was adjusted with gradient descent algorithm, which was in detailed description with the following Eq. (9):

$$\begin{align*}
    u_{kl} &= -\eta \frac{\partial E}{\partial u_{kl}} \\
    w_{jk} &= -\eta \frac{\partial E}{\partial w_{jk}} \\
    v_{ij} &= -\eta \frac{\partial E}{\partial v_{ij}}
\end{align*}$$

where, $u_{kl}$ is the weight adjustment value $k$, neuron on the second hidden layer and the neuron on $l$, the output layer? $w_{jk}$ is the weight adjustment value of $j$, neuron on the first hidden layer and $k$, neuron on the second hidden layer. $v_{ij}$ is the weight adjustment value of $i$, neuron on input layer and $j$, neuron on the first hidden layer.

As the convergence rate decreases when the activation function Sigmoid approaches to 0.1, training sample, test sample and sample output values of artificial neural network were put in the interval from 0.1 to 0.9, which is described as formula (10):

$$X_{new} = 0.1 + 0.8 \frac{X - X_{min}}{X_{max} - X_{min}}$$

Hidden notes were used to determine the neuron. Here are the basic steps.

**Step 1:** Initialization Determine the input and output layer, the number of hidden nodes, random weights initialization, learning coefficient, synthetic error $e_1$, target error $e_2$, threshold $\xi_1$ and $\xi_2$. Learning degree $k$ is given as 1 here.
Step 2: BP learning BP algorithm rules were used here for weight modification of neural network. Then, the training error and hidden point output sequences for all modes were calculated.

Step 3: Compound and stopping judgment If the current training error $E > e_1$ and $k = k + 1$, then turn to Step 2. If $e_2 < E < e_1$, turn to Step 4. If $E < e_2$, stop learning.

Step 4: Calculations calculate the standard deviation of all the hidden point output sequences.

Step 5: Compose the hidden nodes with high correlation. If the correlation coefficient of hidden nodes, $i$ and $j$, have the following relationship, compose the hidden nodes $(i, j)$.

Step 6: Generating offset node if a hidden nodes fit the relation of $S_i > S_j$, combine the hidden node with the offset node. Make $k = k + 1$ and turn to Step 2.

The problem required solution is to determine the influence of height, mass and volume defined by the profile to leaf mass. The input element was describes as follows:

- Height of the tree (as it proves that the positive correlation between the height and leaf mass)
- Mass of trees
- Volume defined by the profile

Since there is not enough data found to run the learning programmer of BP neural network. Parameters have to be debugged for the best learning efficiency.

**CONCLUSION**

According to the simulation, the shape may influence the exposure area when leaves are uniformly distributed though the effect is little. Our model is particularly appropriate for simulation of the shadow area by overlap of shadow in the sun with consideration of gyrator of leaves, to which the problem leads its way. Thus, the result of the model is objective and veritable.

More than one method was used for estimation of leaf mass in a tree. One method is estimation by relationships between sapwood area and leaf mass, analyzing several methods for value of sapwood area. The other is mechanism analysis, which is a reasonable description of the model.

Based on the BP neural network model due to its accuracy, it is improved to a four-layer hidden-node neural network model, which is used for analyzing the correlation between leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile).

**REFERENCES**


