New and Modified Equations for Planning Two-Buildup Directional and Horizontal Wells

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Abstract: Two-buildup directional and horizontal well is usually used for the type trajectory that the target zone is accurate but build rate is not. In the past and at present, the petroleum engineers use the Karlsson method and equations to design the two-buildup directional and horizontal well. But through rigid derivation and analysis, we have found some places that need to be improved. This study has put forward new and modified equations on the basis of Karlsson method. The new method considers the maximum and minimum build rates expected in the first and second curves and supposes that the upper and lower bound trajectory are all two-build up trajectory type. This study also has considered the difference of the maximum and minimum build rates expected in the second curve and the first curve. The new and modified method and equations are tested through two examples.

Keywords: Build rate, directional and horizontal wells, tangent angle, trajectory design, two-buildup

INTRODUCTION

Directional drilling techniques have been used for many years to reach subsurface objectives that have had inaccessible surface locations. Economic considerations and increased environmental concerns have increased the number of directional wells drilled in recent years. Another and rapidly growing area requiring directional drilling techniques is horizontal and extended reach drilling. In some reservoirs the horizontal well may improve drainage by increasing the area of the wellbore in contact with the reservoir (Wiggins and Juvkam-Wold, 1990).

Careful planning is the key to successful directional drilling. One of the first steps in planning the directional well should be the design of the wellbore trajectory (Gao, 2004). Wellbore trajectories can be categorized into three classes. The first type well is basically a “build and hold” trajectory where the wellbore is deflected from the vertical at some kickoff point and the angle built until a maximum angle is reached and then held until the target is intercepted by the wellbore. This type trajectory is appropriate for the condition that the build rate and target zone are all very accurate (Han, 2007). The second type well is a “build, hold, build” trajectory where the wellbore is deflected to some angle, the angle is held and then built in a manner such that the target is penetrated. In contrast to the first type, this trajectory adds a hold section between two buildup sections to adjust the trajectory deviation causing by the build error (Xiong, 2002). This type trajectory is appropriate for the condition that the build rate and target zone are all very accurate (Han, 1991).

Along with the progress of the geological exploration technology, the accuracy of target zone is high (Luo, 2006). So we should only consider the build error when we design the wellbore trajectory.

In the past, the major method utilized in planning the two build up horizontal well is the karlsson method (Karlsson et al., 1989). Karlsson has considered the effect of build rate error on the trajectory and put forward the design method and equations. Until now this method and those equations are in use. But through our research we have found out some insufficient. So we have made some amendments to the karlsson equations and introduced a new method to design the two build up horizontal well. Two examples are used to demonstrate the application of the equations.

METHODOLOGY

The insufficient of Karlsson method: In order to point out the insufficient we firstly show the trajectory design method and derivation process of equations that created by Karlsson et al. (1989). The design process is shown below:

- Based on the minimum expected build rate to design single build trajectory from $T_b$ to surface. So we can get the depth of kickoff point:

$$H_0 = T_b - \frac{1719}{k_{min}} (\sin \alpha_b - \sin \alpha_s)$$

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where,
\( T_b \) : The maximum allowable target depth
\( H_0 \) : The kick off point depth
\( \alpha_B \) : Planned final well inclination
\( \alpha_a \) : The initial well inclination
\( K_{\text{min}} \) : The minimum build up rate
\( K_{\text{max}} \) : The maximum build up rate

- The tangent section is designed into the well path to compensate the variance in build rate. The most efficient angle for the tangent must be calculated based upon the target thickness, as well as maximum and minimum build rates expected in the second curve. From Fig. 1 we can get:

\[
T_b - T_t = \frac{1719}{k_{\text{min}}} (\sin \alpha_B - \sin \alpha_{\text{tan}}) = \frac{1719}{k_{\text{max}}} (\sin \alpha_B - \sin \alpha_{\text{tan}})
\]

Equation (2) can be rearranged as:

\[
\alpha_{\text{tan}} = \sin^{-1} \left[ \sin \alpha_B - \frac{k_{\text{max}}k_{\text{min}}(T_b - T_t)}{1719(k_{\text{max}} - k_{\text{min}})} \right]
\]

where,
\( \alpha_{\text{tan}} \) : The optimum tangent angle

- When the buildup rate (\( K_{\text{exp}} \)) of the first curve is acquired through drilling, the corresponding optimum target TVD (\( T_{\text{tgt}} \)) within the target zone can be calculated using the following equation:

\[
T_{\text{tgt}} = T_t + 1719\left( \frac{1}{k_{\text{exp}}} - \frac{1}{k_{\text{max}}} \right) (\sin \alpha_B - \sin \alpha_{\text{tan}})
\]

(4)

- Then the expected tangent length can be calculated using the following equation:

\[
l = \frac{(\cos \alpha_a - \cos \alpha_B)(1719 - 1719)}{\sin \alpha_{\text{tan}}} \frac{k_{\text{min}}}{k_{\text{exp}}}
\]

(5)

These are all four steps of the karlsson method to design the two build up horizontal well when considering the buildup error.

But we have found some doubtable places in the derivation process. First, the horizontal distance to the wellhead is not equal under the \( k_{\text{max}} \) and \( k_{\text{min}} \):

\[
S_1 - S_2 = \frac{1719}{k_{\text{min}}} (\cos \alpha_{\text{tan}} - \cos \alpha_B) - \frac{1719}{k_{\text{max}}} (\cos \alpha_{\text{tan}} - \cos \alpha_B) \neq 0
\]

(6)

The correct derivation process and the optimum tangent length, optimum tangent angle and the corresponding optimum target TVD (\( T_{\text{tgt}} \)) equations are shown below based on the Fig. 2.

Based on the minimum expected build rate to design single build up trajectory from \( T_b \) to surface:

\[
H_0 + \frac{1719}{k_{\text{min}}} (\sin \alpha_B - \sin \alpha_a) = T_b
\]

(7)

\[
H_0 \tan \alpha_a + \frac{1719}{k_{\text{min}}} (\cos \alpha_a - \cos \alpha_B) = S
\]

(8)

Based on the maximum expected build rate to design two build up trajectory from \( T_t \) to surface:

\[
H_0 + \frac{1719}{k_{\text{max}}} (\sin \alpha_{\text{tan}} - \sin \alpha_a) + i \cos \alpha_a + \frac{1719}{k_{\text{min}}} (\sin \alpha_B - \sin \alpha_{\text{tan}}) = T_t
\]

(9)

\[
H_t \tan \alpha_a + \frac{1719}{k_{\text{max}}} (\cos \alpha_a - \cos \alpha_{\text{tan}}) + i \sin \alpha_a + \frac{1719}{k_{\text{min}}} (\cos \alpha_B - \cos \alpha_{\text{tan}}) = S
\]

(10)

Subtracting Eq. (9) from (7) and Eq. (10) from (8) results in Eq. (11) and (12):
\[ l \cos \alpha_{\text{tan}} = \left( \frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{max}}} \right) \left( \sin \alpha_a - \sin \alpha_s \right) + T_b - T_s \]  
(11)

\[ l \sin \alpha_{\text{tan}} = \left( \frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{max}}} \right) \left( \cos \alpha_a - \cos \alpha_s \right) \]  
(12)

Dividing Eq. (12) by (11), the equation becomes:

\[ \tan \alpha_{\text{tan}} = \frac{1719}{k_{\text{min}} - k_{\text{max}}} \left( \cos \alpha_a - \cos \alpha_s \right) \] 
\[ \frac{T_b - T_s}{(1719)} \left( \frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{max}}} \right) \left( \sin \alpha_a - \sin \alpha_s \right) \]  
(13)

Equation (13) can be written in the following form:

\[ \alpha_{\text{tan}} = \arctan \left( \frac{1719}{k_{\text{min}} - k_{\text{max}}} \left( \cos \alpha_a - \cos \alpha_s \right) \right) \] 
\[ \frac{T_b - T_s}{(1719)} \left( \frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{max}}} \right) \left( \sin \alpha_a - \sin \alpha_s \right) \]  
(14)

When the buildup rate (K_{\text{exp}}) is known in the drilling process, we can get:

\[ H_s \tan \alpha_s + \frac{1719}{k_{\text{exp}}} \left( \cos \alpha_a - \cos \alpha_s \right) + l \sin \alpha_{\text{tan}} + \frac{1719}{k_{\text{exp}}} \left( \cos \alpha_a - \cos \alpha_s \right) = S \]  
(15)

\[ H_s + \frac{1719}{k_{\text{exp}}} \left( \sin \alpha_s - \sin \alpha_a \right) + l \cos \alpha_{\text{tan}} + \frac{1719}{k_{\text{exp}}} \left( \sin \alpha_a - \sin \alpha_s \right) = T_{\text{tp}} \]  
(16)

Subtracting Eq. (8) from (15) results in Eq. (17):

\[ \left( \cos \alpha_a - \cos \alpha_s \right) \left( \frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{exp}}} \right) \] 
\[ \frac{l}{\sin \alpha_{\text{tan}}} \]  
(17)

Based on the Eq. (7), the kick off point depth can be expressed as:

\[ H_s = \frac{T_b - \frac{1719}{k_{\text{min}}} \left( \sin \alpha_s - \sin \alpha_a \right) \]  
(18)

Substituting Eq. (14), (17) and (18) into Eq. (16) yields:

\[ T_{\text{tp}} = \frac{T_b + \frac{1719}{k_{\text{exp}}} \left[ \cos \alpha_a - \cos \alpha_s \right] \left( \frac{\tan \alpha_{\text{tan}}}{\sin \alpha_{\text{tan}}} \right) \left( \sin \alpha_a - \sin \alpha_s \right)]}{\frac{1719}{k_{\text{min}}} - \frac{1719}{k_{\text{exp}}} \left( \cos \alpha_a - \cos \alpha_s \right) \]  
(19)

So the optimum tangent angle, optimum tangent length and the corresponding optimum target TVD (T_{\text{tp}}) can be got through solving the Eq. (14), (17) and (19).
\[ H_0 \cos \alpha_x + \frac{1719}{k_{\text{min}}} (\sin \alpha_{\text{tan}} - \sin \alpha_x) + l_1 \cos \alpha_{\text{tan}} + \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_{\text{tan}}) = T_b \] (20)

\[ H_0 \sin \alpha_x + \frac{1719}{k_{\text{min}}} (\cos \alpha_x - \cos \alpha_{\text{tan}}) + l_1 \sin \alpha_{\text{tan}} + \frac{1719}{k_{\text{min}}} (\cos \alpha_y - \cos \alpha_{\text{tan}}) = S \] (21)

\[ H_0 \cos \alpha_x + \frac{1719}{k_{\text{max}}} (\sin \alpha_{\text{tan}} - \sin \alpha_x) + l_2 \cos \alpha_{\text{tan}} + \frac{1719}{k_{\text{max}}} (\sin \alpha_y - \sin \alpha_{\text{tan}}) = T_r \] (22)

\[ H_0 \sin \alpha_x + \frac{1719}{k_{\text{max}}} (\cos \alpha_x - \cos \alpha_{\text{tan}}) + l_2 \sin \alpha_{\text{tan}} + \frac{1719}{k_{\text{max}}} (\cos \alpha_y - \cos \alpha_{\text{tan}}) = S \] (23)

Subtracting Eq. (22) from (20) results in Eq. (24):

\[ \frac{1719}{(k_{\text{min}} - k_{\text{max}}) (\sin \alpha_{\text{tan}} - \sin \alpha_x)} + (l_1 - l_2) \cos \alpha_{\text{tan}} + \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_{\text{tan}}) = T_b - T_r \] (24)

Subtracting Eq. (23) from (21) results in (25):

\[ \frac{1719}{(k_{\text{min}} - k_{\text{max}}) (\cos \alpha_x - \cos \alpha_{\text{tan}})} + (l_1 - l_2) \sin \alpha_{\text{tan}} + \frac{1719}{k_{\text{min}}} (\cos \alpha_y - \cos \alpha_{\text{tan}}) = 0 \] (25)

The Eq. (24) and (25) can be rearranged as follows:

\[ (l_1 - l_2) \cos \alpha_{\text{tan}} = T_b - T_r - \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_x) \] (26)

\[ (l_1 - l_2) \sin \alpha_{\text{tan}} = \frac{1719}{k_{\text{max}}} (\cos \alpha_x - \cos \alpha_y) \] (27)

Dividing Eq. (27) by (26), the equation becomes:

\[ \tan \alpha_{\text{tan}} = \frac{(l_1 - l_2) (\cos \alpha_x - \cos \alpha_y)}{T_b - T_r - \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_x)} \] (28)

The Eq. (28) can be written as:

\[ \alpha_{\text{tan}} = \arctan \frac{(l_1 - l_2) (\cos \alpha_x - \cos \alpha_y)}{T_b - T_r - \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_x)} \] (29)

Equation (20) and (21) can be rearranged as:

\[ l_1 \cos \alpha_{\text{tan}} = T_b - \frac{1719}{k_{\text{min}}} (\sin \alpha_y - \sin \alpha_x) - H_0 \cos \alpha_x \] (30)
Dividing Eq. (31) by (30), the equation becomes:

\[
\tan \alpha_{\text{tan}} = \frac{S - \frac{1719}{k_{\min}}(\cos \alpha_a - \cos \alpha_b) - H_o \sin \alpha_a}{T_b - \frac{1719}{k_{\min}}(\sin \alpha_b - \sin \alpha_a) - H_o \cos \alpha_a}
\]

The Eq. (32) can be rearranged as:

\[
H_o = \frac{\tan \alpha_{\text{tan}} \times (T_b - \frac{1719}{k_{\min}}(\sin \alpha_b - \sin \alpha_a)) - S - \frac{1719}{k_{\min}}(\cos \alpha_a - \cos \alpha_b)}{(\tan \alpha_{\text{tan}} \cos \alpha_a \sin \alpha_a)}
\]

So the depth of kick off point can be determined by Eq. (33).

When the buildup rate (K_{\text{exp}}) is known, we can get:

\[
H_o \sin \alpha_a + \frac{1719}{k_{\exp}}(\cos \alpha_a - \cos \alpha_{\text{tan}}) + l \sin \alpha_{\text{tan}} + \frac{1719}{k_{\exp}}(\cos \alpha_{\text{tan}} - \cos \alpha_b) = S
\]

\[
H_o \cos \alpha_a + \frac{1719}{k_{\exp}}(\sin \alpha_{\text{tan}} - \sin \alpha_a) + l \cos \alpha_{\text{tan}} + \frac{1719}{k_{\exp}}(\sin \alpha_b - \sin \alpha_{\text{tan}}) = T_{\text{tg}'}
\]

Combining the Eq. (34) and (35) yields:

\[
l = \frac{S - H_o \sin \alpha_a - \frac{1719}{k_{\exp}}(\cos \alpha_a - \cos \alpha_b)}{\sin \alpha_{\text{tan}}}
\]

So the optimum tangent angle, optimum tangent length and the corresponding optimum target TVD (T_{\text{tg}'} *) can be got through solving the Eq. (29), (36) and (37).

For the second condition, the derivation process is shown below:

\[
H_o \cos \alpha_a + \frac{1719}{k_{1\min}}(\sin \alpha_{\text{tan}} - \sin \alpha_a) + l_1 \cos \alpha_{\text{tan}} + \frac{1719}{k_{2\min}}(\sin \alpha_b - \sin \alpha_{\text{tan}}) = T_b
\]

\[
H_o \sin \alpha_a + \frac{1719}{k_{1\min}}(\cos \alpha_a - \cos \alpha_{\text{tan}}) + l_1 \sin \alpha_{\text{tan}} + \frac{1719}{k_{2\min}}(\cos \alpha_{\text{tan}} - \cos \alpha_b) = S
\]

\[
H_o \cos \alpha_a + \frac{1719}{k_{1\max}}(\sin \alpha_{\text{tan}} - \sin \alpha_a) + l_z \cos \alpha_{\text{tan}} + \frac{1719}{k_{2\max}}(\sin \alpha_b - \sin \alpha_{\text{tan}}) = T_i
\]

\[
H_o \sin \alpha_a + \frac{1719}{k_{1\max}}(\cos \alpha_a - \cos \alpha_{\text{tan}}) + l_z \sin \alpha_{\text{tan}} + \frac{1719}{k_{2\max}}(\cos \alpha_{\text{tan}} - \cos \alpha_b) = S
\]

\[
H_o \sin \alpha_a + \frac{1719}{k_{1\max}}(\cos \alpha_a - \cos \alpha_{\text{tan}}) + l_2 \cos \alpha_{\text{tan}} + \frac{1719}{k_{2\max}}(\cos \alpha_{\text{tan}} - \cos \alpha_b) = S
\]
Subtracting Eq. (40) from (38) results in Eq. (42):

\[
\frac{1719}{k_{1min}} - \frac{1719}{k_{max}} (\sin \alpha_{tan} - \sin \alpha_y) + (l_i - l_j) \cos \alpha_{tan} + \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\sin \alpha_y - \sin \alpha_{tan}) = T_y - T_i
\]

Subtracting Eq. (41) from (39) results in Eq. (43):

\[
\frac{1719}{k_{1min}} - \frac{1719}{k_{max}} (\cos \alpha_y - \cos \alpha_{tan}) + (l_i - l_j) \sin \alpha_{tan} + \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\cos \alpha_{tan} - \cos \alpha_y) = 0
\]

The Eq. (42) and (43) can be written as follows:

\[
(l_i - l_j) \cos \alpha_{tan} = T_y - T_i - \left(\frac{1719}{k_{1min}} - \frac{1719}{k_{max}}\right) (\sin \alpha_{tan} - \sin \alpha_y) - \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\sin \alpha_y - \sin \alpha_{tan})
\]

\[
(l_i - l_j) \sin \alpha_{tan} = \frac{1719}{k_{max}} - \frac{1719}{k_{1min}} (\cos \alpha_y - \cos \alpha_{tan}) + \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\cos \alpha_{tan} - \cos \alpha_y)
\]

Dividing Eq. (45) by (44), the equation becomes:

\[
\tan \alpha_{tan} = \frac{\frac{1719}{k_{max}} - \frac{1719}{k_{1min}} (\cos \alpha_y - \cos \alpha_{tan}) + \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\cos \alpha_{tan} - \cos \alpha_y)}{T_y - T_i - \left(\frac{1719}{k_{1min}} - \frac{1719}{k_{max}}\right) (\sin \alpha_{tan} - \sin \alpha_y) - \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\sin \alpha_y - \sin \alpha_{tan})}
\]

\[
\alpha_{tan} = \alpha - \frac{\frac{1719}{k_{max}} - \frac{1719}{k_{1min}} (\cos \alpha_y - \cos \alpha_{tan}) + \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\cos \alpha_{tan} - \cos \alpha_y)}{T_y - T_i - \left(\frac{1719}{k_{1min}} - \frac{1719}{k_{max}}\right) (\sin \alpha_{tan} - \sin \alpha_y) - \left(\frac{1719}{k_{2min}} - \frac{1719}{k_{2max}}\right) (\sin \alpha_y - \sin \alpha_{tan})}
\]

There is only one unknown number (\alpha_{tan}) in Eq. (46), so we can get it through solving the Eq. (46). Equation (38) and (39) can be rearranged as follows:

\[
l_i \cos \alpha_{tan} = T_y - H_y \cos \alpha_y - \frac{1719}{k_{1min}} (\sin \alpha_{tan} - \sin \alpha_y) - \frac{1719}{k_{2min}} (\sin \alpha_y - \sin \alpha_{tan})
\]

\[
l_i \sin \alpha_{tan} = S - \frac{1719}{k_{2min}} (\cos \alpha_{tan} - \cos \alpha_y) - \frac{1719}{k_{1min}} (\cos \alpha_y - \cos \alpha_{tan}) - H_y \sin \alpha_y
\]

Dividing Eq. (48) by (47), the equation becomes:

\[
H_y = \frac{\frac{1719}{k_{1min}} (\sin \alpha_{tan} - \sin \alpha_y) - \frac{1719}{k_{2min}} (\sin \alpha_y - \sin \alpha_{tan}) - S + \frac{1719}{k_{1min}} (\cos \alpha_y - \cos \alpha_{tan}) + \frac{1719}{k_{2min}} (\cos \alpha_{tan} - \cos \alpha_y)}{\tan \alpha_{tan} \times \cos \alpha_y - \sin \alpha_y}
\]

We can get the H_y through solving the Eq. (49) after we known \alpha_{tan}.

When the buildup rate (K_1) of the first curve is known in the drilling process, we can get:

\[
H_y \sin \alpha_y + \frac{1719}{k_1} (\cos \alpha_y - \cos \alpha_{tan}) + l \sin \alpha_{tan} + \frac{1719}{k_2} (\cos \alpha_{tan} - \cos \alpha_y) = S
\]

When the varieties of K_2 range from K_{2min} to K_{2max}, optimum tangent length l also will change, we can get the range l \in(l_i, l_a) based on the Eq. (51) and (52):
\[ H_0 \sin \alpha_a + \frac{1719}{k_1} (\cos \alpha_a - \cos \alpha_{\text{tan}}) + l_a \sin \alpha_{\text{tan}} + \frac{1719}{k_{2\text{max}}} (\cos \alpha_{\text{tan}} - \cos \alpha_B) = S \]  \hspace{1cm} (51)

\[ H_0 \sin \alpha_a + \frac{1719}{k_1} (\cos \alpha_a - \cos \alpha_{\text{tan}}) + l_a \sin \alpha_{\text{tan}} + \frac{1719}{k_{2\text{min}}} (\cos \alpha_{\text{tan}} - \cos \alpha_B) = S \]  \hspace{1cm} (52)

Because optimum tangent length \( l \in (l_c, l_d) \) must satisfy the constraint Eq. (53) and (54):

\[ H_0 \cos \alpha_a + \frac{1719}{k_1} (\sin \alpha_{\text{tan}} - \sin \alpha_a) + l_c \cos \alpha_{\text{tan}} + \frac{1719}{k_{2\text{max}}} (\sin \alpha_B - \sin \alpha_{\text{tan}}) > T_t \]  \hspace{1cm} (53)

\[ H_0 \cos \alpha_a + \frac{1719}{k_1} (\sin \alpha_{\text{tan}} - \sin \alpha_a) + l_c \cos \alpha_{\text{tan}} + \frac{1719}{k_{2\text{min}}} (\sin \alpha_B - \sin \alpha_{\text{tan}}) < T_t \]  \hspace{1cm} (54)

So the range of optimum tangent length \( l \) is shown below:

\[ \max(l_c, l_d) < l < \min(l_c, l_d) \]

When the tangent length \( l \) is took one specific value between the range, we can get the build rate \( k_2 \) through solving the Eq. (55):

\[ k_2 = \frac{1719(\cos \alpha_{\text{tan}} - \cos \alpha_B)}{S - H_0 \sin \alpha_a - \frac{1719}{k_1} (\cos \alpha_a - \cos \alpha_{\text{tan}}) - l \sin \alpha_{\text{tan}}} \]  \hspace{1cm} (55)

So the optimum target TVD (\( T_{\text{tgt}} \)) can be got through solving the Eq. (29), (36) and (37):

\[ T_{\text{tgt}} = H_0 \cos \alpha_a + \frac{1719}{k_1} (\sin \alpha_{\text{tan}} - \sin \alpha_a) + l_c \cos \alpha_{\text{tan}} + \frac{1719}{k_2} (\sin \alpha_B - \sin \alpha_{\text{tan}}) \]  \hspace{1cm} (56)

**Example:** Two examples are presented to show the application of new and modified equations for the wellbore trajectories.

**Example 1:** It is desired to drill a well to a maximum allowable target depth of 2800 m and minimum allowable target depth of 2790 m under a local golf course. The required horizontal displacement is 800 m and the maximum angle desired at total depth is 86°. The initial inclination angle is 0°. The maximum and minimum allowable build rates are 7°/30 m and 5°/30 m.

For this well:

\[ T_t = 2790, T_b = 2800, S = 800, \alpha_a = 0°, \alpha_B = 86°, k_{\text{min}} = 5°/30 m, k_{\text{max}} = 7°/30 m \]

Substituting these values into Eq. (29) will allow the determination of the optimum tangent angle \( \alpha_{\text{tan}} \). The angle is determined to be 46.1°. Use Eq. (33) to get the depth of kick off point \( H_0 \). The depth is determined to be 1994.7 m. When the buildup rate is known in the drilling process, such as \( k_{\text{exp}} = 6.5°/30 m \), we can get the optimum tangent length \( l \), optimum target \( T_{\text{tgt}} \) based on the Eq. (36) and (37). The \( l \) and \( T_{\text{tgt}} \) are determined to be 769 and 2792 m.

**Example 2:** It is desired to drill a well to a maximum allowable target depth of 2800 m and minimum allowable target depth of 2790 m under a local golf course. The required horizontal displacement is 800 m and the maximum angle desired at total depth is 86°. The initial inclination angle is 0°. The maximum and minimum allowable build
rates of first curve are 7°/30 and 5°/30 m, respectively. The maximum and minimum allowable build rates of second curve are 6°/30 and 8°/30 m, respectively.

For this well:

\[ T_1 = 2790, \quad T_b = 2800, \quad S = 800, \quad \alpha_a = 0^\circ, \quad \alpha_B = 86^\circ, \]
\[ k_{1\text{min}} = 5°/30 \text{ m}, \quad k_{1\text{max}} = 7°/30 \text{ m}, \quad k_{2\text{min}} = 6°/30 \text{ m}, \]
\[ k_{2\text{max}} = 8°/30 \text{ m} \]

Substituting these values into Eq. (46) will allow the determination of the optimum tangent angle \( \alpha_{\text{tan}} \). The angle is determined to be 42.8°. Use Eq. (49) to get the depth of kick off point \( H_0 \). The depth is determined to be 1916.2 m. When the buildup rate of first curve is known in the drilling process, such as \( k_{\text{exp}} = 6.5°/30 \text{ m} \), we can get the range of optimum tangent length \( l \) based on the Eq. (51), (52), (53) and (54). The range is \( l \in (822.2, 863.3) \). In the drilling process we should control the optimum tangent length within the scope of the permit. When the tangent length is known in the drilling process, such as \( l = 850 \) m, we can get the buildup rate of second curve \( k_2 \) and the target TVD (\( T_{\text{tgt}} \)):

\[ k_2 = 7.52°/30 \text{ m}, \quad T_{\text{tgt}} = 2792 \text{ m} \]

**CONCLUSION**

This study finds some insufficient of Karlsson method, so this study gives the correct derivation process and presents the modified equations. The study has put forward new design method on the basis of Karlsson method, the new method supposes that the upper and lower bound trajectory are all two-build up trajectory type and considers the difference of build rate between the first and second build sections. The new and modified method and equations are tested through two examples. The results show that the new and modified method and equations are very suitable for designing the two-buildup directional and horizontal wells.

**ACKNOWLEDGMENT**

The financial support received from the national project (Grant No.: 2010ZX05026-001) is gratefully acknowledged.

**REFERENCES**


