Compressive Channel Estimation for OFDM Cooperation Networks

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Abstract: In this study, we study compressive channel estimation for Orthogonal Frequency Division Multiplexing (OFDM) modulated Amplify-and-Forward (AF) relay networks. Based upon the mathematical channel model of P2P and sparseness measure function, by using Monte-Carlo runs, we show that the OFDM cooperation convoluted channels also exhibit sparsity. In this study, we propose two compressive channel estimation methods to exploit the inherent sparse structure in multipath fading channels of OFDM cooperation networks with Amplify-and-Forward (AF) relays. Simulation results demonstrate that the proposed compressive channel estimation methods provide significant improvement in mean square error (MSE) performance compared with the conventional channel estimation methods.

Key words: Amplify-and-Forward (AF), cooperation networks, Compressive Sensing (CS), multipath fading channel, Orthogonal Frequency Division Multiplexing (OFDM)

INTRODUCTION

Cooperative relaying has been researched intensively for wireless networks in recent years due to its capability of enhancing the transmission capacity and providing the spatial diversity for single-antenna wireless transceivers by employing the relay nodes as virtual antennas (Laneman and Wornell, 2003; Gao et al., 2008). Multiple-input multiple-output (MIMO) system is a well-known technique which can boost the capacity and diversity of wireless communications (Foschini et al., 1999). However, it is difficult to equip multiple antennas at mobile terminals because of cost and size limits. In order to reckon with this problem, relay-based cooperation communication networks have been proposed recently. It is expected that relay protocols will be a very promising method for future high data-rate radio communication systems.

In cooperative communication system, terminals can be usually divided into three parts: source, relay, and destination. Source and destination can be the base station and mobile station, respectively, or vice versa, and the relay receives signal from source and retransmits to destination. Generally, there are two kinds of protocols in cooperation networks: the Amplify-and-Forward (AF) scheme and Decode-and-Forward (DF) scheme. In the AF relaying, the relay amplifies and retransmits its received noisy signal without decoding it. In the DF relaying, the relay terminals demodulate and modulate again the received signals and then forward them to the destination (Laneman et al., 2004). Compared with the DF scheme, the AF scheme is more effective since the cooperative terminals do not need to decode their received signals. Therefore, we focus our attention on the AF relay scheme in this study.

Many relay studies focused on flat-fading channels, where single-carrier systems are of interest (Seyfi et al., 2010; Gao et al., 2008). However, the use of relays in frequency-selective broadband channels is important as well. It is well known that the popular solution against frequency-selective fading is OFDM. So, cooperative OFDM relay networks are increasingly important. Recently, several OFDM channel estimation methods for modulating cooperation communication have been proposed (Gao et al., 2011; Zhang et al., 2009; Chan et al., 2007) and these OFDM channel estimation methods using AF protocol are based on the assumption that the wireless channels between the terminals and the relay nodes have rich multipath (Cotter and Rao et al., 2002). However, recent channel measurements have demonstrate that the wireless channels tend to exhibit a sparse or cluster-sparse structure (Adachi et al., 2009) in delay-spread domain. In order to take advantage of OFDM cooperation channel sparsity, two novel compressive channel estimation schemes are proposed in this study. Simulation results validate the effectiveness of the proposed methods.
SYSTEM MODEL

 Relay transmission model: Consider a three-node OFDM cooperation network with two terminals S, D and one relay node R is shown in Fig. 1. We assume that all the terminals are equipped with only one antenna and work in the half-duplex mode, therefore, they cannot receive and transmit simultaneously.

 In a relay networks, data transmission is usually divided into two phases. The source broadcasts its own information and the relay forwards its received signal to the destination.

 Due to the limited transmit power and the multipath fading channel, we assume that there is no direct path between S and D as shown in Fig. 1. Therefore, frequency-selective multipath channels will generate multiple delayed and attenuated copies of the transmitted waveform. Source and relay are assumed to have average power constraints in $P_S$ and $P_R$ respectively. Assume that the channel impulse response between S and R is $h_1(t)$, which is constant within one transmission period and represented by

$$h_1 = \sum_{l=0}^{L_1-1} h_{1,l}(\tau - \tau_{1,l})$$  \hspace{1cm} (1)

 where $h_{1,l}$ is the complex-valued path and it satisfies $E[\sum_{l=0}^{L_1-1} |h_{1,l}|^2] = 1$, and $\tau_{1,l}$ denotes the symbol-spaced time delay of the lth path, and $L_1$ is the length of the channel between S and R.

 Due to the same channel property as Eq. (1), channel vector $h_2$ between R and D is given by the follows:

$$h_2 = \sum_{l=0}^{L_2-1} h_{2,l}(\tau - \tau_{2,l})$$  \hspace{1cm} (2)

 where $h_{2,l}$ and $\tau_{2,l}$ denote the complex-valued path and the lth path symbol-spaced time delay , respectively. $L_2$ is the length of the channel between R and D:

 Transmitted signal at the source: Suppose that each OFDM block contains N information symbols and denote the frequency domain training vector from S as $x = [\bar{x}_0, \bar{x}_1, ..., \bar{x}_{N-1}]$. The corresponding time-domain signal vector can be obtained from the normalized inverse discrete Fourier transformation (IDFT) as:

$$x = F^H \bar{x}$$  \hspace{1cm} (3)

 where $F$ is the discrete Fourier transformation (DFT) matrix with the $(m,n)$ -th entity given by $F_{mn} = \frac{1}{\sqrt{N}} \exp(-j2\pi mn/N), (m,n = 0, 1, ..., N)$. In order to avoid the Inter-block Interference (IBI), S inserts the cyclic prefix (CP) of length $L_p$ in the front of each OFDM block before the transmission, and $L_p$ should satisfy that $L_p \geq \max(L_1-1, L_2-1)$. After performing DFT and inserting CP, the transmitted signal vector can be written as:

$$S = [x_{N-L_p}, ..., x_{N-1}, x_0, ..., x_{N-1}]$$  \hspace{1cm} (4)

 The received signals at the relay node and the destination: In the AF relay transmission, the received signal at the relay is amplified by a relay factor $\alpha$. Over a doubly selective channel between the source S and the relay R, after removing CP, the received signal at the relay node can be represented as (Adachi et al., 2009)

$$y_1 = H_1 x + n_1$$  \hspace{1cm} (5)

 where $H_1$ is an N×N circulated channel matrix with $[h_1^T, 0_{(N-1)L}]^T$ as its first column, $n_1$ is the complex additive Gaussian white noise (AWGN) with zero mean and covariance matrix $E[n_1 n_1^H] = \sigma_n^2 I_N$.

 The received signal at the destination D, which is from relay R, after removing CP, the received signal can be represented as:

$$y_2 = \alpha H_1 y_1 + n_2 = \alpha H_1 x + n$$  \hspace{1cm} (6)

 where $n = \alpha H_1 n_1 + n_2$ is the composite noise with zero mean and covariance matrix $E[n n^H] = \alpha^2 \sigma_n^2 |H_2|^2 + I_N$.

 The amplified positive coefficient $\alpha$ is given by:
\[
\alpha = \sqrt{\frac{P_{g}}{\sum_{l=0}^{L-1} \sigma_{l1}^{-2} \sigma_{l1}^{-2} + \sum_{l=0}^{L-1} \sigma_{l2}^{-2} \sigma_{l2}^{-2}}} 
\]  
(7)

According to the matrix theory, matrices \(H_1\) and \(H_2\) can be de-composed as \(H_1 = F^H \Lambda_1 F\) and \(H_2 = F^H \Lambda_2 F\) (Gray, 2006), respectively. \(F\) is the discrete Fourier transformation (DFT) matrix. Thus, system model (6) can be rewritten as:

\[
y_2 = F^H \alpha \Lambda_1 \Lambda_2 FX + n \tag{8}
\]

We define the convolution channel vector \(h = (h_1^* h_2)\) with the length of \((L_1 + L_2 - 1)\). After left-multiplying by \(F\), Eq. (8) can be rewritten as:

\[
y = X W h + \hat{n} = \hat{X} h + \hat{n} \tag{9}
\]

where \(X = \text{diag} (Fx)\) denotes the training signal matrix, \(\hat{X}\) is a matrix taking the first \((L_1 + L_2 - 1)\) columns of \(\sqrt{N} F\) and \(\hat{n} = \Lambda_1 F n_1 + F n_2\) is a realization of a complex Gaussian random vector.

**COMPRESSION CHANNEL ESTIMATION**

**Overview of compressive sensing:** Compressed Sensing (CS) theory has recently gained a fast-growing interest in applied mathematics and signal processing communities. And it has been applied in various areas, such as imaging, radar, speech recognition, data acquisition, etc. In communications, an immediate application of CS is in wireless sparse multipath channel estimation (Gui et al., 2011; Bajwa et al., 2008).

In this study, we consider the linear model as (9). According to the CS, if an unknown signal vector satisfies sparse or approximate sparse, then a designed measurement matrix \(X\) can accurately capture most of its dominant information. Hence, these kinds of unknown signal can be reconstructed from observation signal \(y\). However, the sparsest solution always is a Non-deterministic Polynomial-time hard (NP-hard) problem. In this study, we also consider the LS channel estimator for comparison. The LS estimator is written as the follows:

\[
\hat{h}_{\text{Lasso}} = \begin{cases} 
\hat{X}_{T} y & T \subseteq \text{suup}(h) \\
0 & \text{others}
\end{cases} \tag{11}
\]

where \(\text{suup}(h)\) denotes the nonzero taps supporting the channel vector \(h\), \(X_T\) is the submatrix constructed from the columns of \(X\), and \(T\) denotes the selected subcolumns corresponding to the nonzero index set of the channel vector \(h\). The MSE of LS estimator \(\hat{h}\) is given by (Bajwa et al., 2008):

\[
\text{MSE}(\hat{h}) = \sigma_s^2 \text{Tr}(\{\hat{X}_{T} m_1\}) \tag{12}
\]

By utilizing CS recovery algorithms for compressive channel estimation, we propose CCS-Lasso, CCS-CoSaMP. The two methods for cooperation convoluted channel estimate are described as follows:

- **CCS-Lasso** \(\hat{h}_{\text{Lasso}}\): Given the received signal \(y\), the unitary DFT matrix \(F\) and \(W\), training signal matrix \(\hat{X} = \text{diag} (Fx)W\) the regularized parameter \(\lambda = \sigma_s \sqrt{\log N}\). The CCS-Lasso (Gui et al., 2011) channel estimator \(\hat{h}_{\text{Lasso}}\) given by:

\[
\hat{h}_{\text{Lasso}} = \text{arg min} \left\{ \frac{1}{2} \| y - \hat{X} h \|_2^2 + \lambda \| h \|_1 \right\} \tag{13}
\]

- **CCS-CoSaMP** \(h_{\text{CoSaMP}}\): Given \(y, F\) and \(W\) and training signal matrix \(\hat{X} = \text{diag} (Fx)W\) the
maximum number of dominant channel coefficients is assumed as $S$. The CCS-CoSaMP method (Gui et al., 2011) can be described as follows:

- Set $T_0 = \emptyset$, $r_0 = y$. $T$ is nonzero coefficient index, and $r$ is the residual estimation error. Set the initialize iteration index $k = 1$.
- Select a column index $n_k$ of $X$ which is most correlated with the residual:

$$n_k = \left\{ \begin{array}{l} \arg \min \| r_{t+1} - \tilde{X}_k \| \text{and } T_k = T_{k-1} \cup n_k \end{array} \right.$$  \hspace{1cm} (14)

- Here we use LS method to calculate a channel estimator as $T_{LS} = \arg \min \| y - \tilde{X}_k h \|$, and select $T$ maximum dominant taps $h_{LS}$. Where $T_{LS}$ denotes the positions of the selected dominant taps in this substep.
- Update the dominant taps by $T_k = T_{LS} \cup T_k$
- Channel estimation,

$$h_k = \arg \min \| y - \tilde{X}_k h \|$$  \hspace{1cm} (15)

- Find the dominant channel coefficients, and replace the left taps $T/T_k$ by zero:

$$h_k = [h]_S$$  \hspace{1cm} (16)

- Update the estimation residual:

$$r_k = y - \tilde{X}_k h_k$$  \hspace{1cm} (17)

Increment the iteration counter $k$. Repeat (13)-(16) until stopping criterion holds and then set $h_{CoSaMP} = h_k$.

**SIMULATION RESULTS**

In this study, we adopt 10000 independent Monte-Carlo runs for average. The length of training sequence is $N = 256$. All of the nonzero taps of sparse channel vectors $h_1$ and $h_2$ are generated by Gaussian distribution and subject to $\| h_1 \|_2^2 = \| h_2 \|_2^2 = 1$. The length of the two channels is $L_1 = L_2 = 32$, and the positions of nonzero channel taps are randomly generated. We set transmit power equal to AF relay power, that is $P_s = P_r = P$.

The Signal to Noise Ratio (SNR) is defined as $S$ as $10 \log P/\sigma_n^2$). When the number of nonzero taps in cooperation channels $h_n(i = 1, 2)$ is changed, the simulation results are shown in Fig. 2 and 3.

Channel estimators are evaluated by average Mean Square Error (MSE) which is defined by:

$$\text{averageMSE} = \frac{\| h - \hat{h} \|_2^2}{M(L_1 + L_2 - 1)}$$  \hspace{1cm} (18)

where $h$ and $\hat{h}$ denote channel vector and its estimator, respectively. $M$ is the number of Monte Carlo runs and $(L_1 + L_2 - 1)$ is the overall length of channel vector $h$. In Fig. 2, the number of nonzero taps of $h_n(i = 1, 2)$ is set to 2, the cooperation convoluted channel also has sparsity. Figure 2 shows that performance of the proposed CCS estimator methods outperforms LS estimator and it is close to the ideal LS estimator by using known position of the channel. In addition, under low SNR (less than 10dB), the proposed CCS-Lasso estimator achieves even better performance than the LS estimator (known position). In Fig. 3, the number of nonzero taps of $h_n(i = 1, 2)$ is set to be 4.

From the simulation results in Fig. 2 and 3, it can be seen that the proposed estimators can exploit the channel sparseness. And if channels are dense rather than sparse, all of the proposed estimators will have the same performance as LS estimator.

**CONCLUSION**

In this study, we introduce OFDM channel estimations for the AF cooperative channel. Distinct from the conventional linear channel estimation methods, we
proposed compressive channel estimation methods for the OFDM cooperation networks under AF protocol. Sparseness of OFDM cooperation convoluted channel was demonstrated by a measure function. The proposed methods have exploited the sparsity in OFDM cooperation channel. Simulation results have confirmed the performance superiority of the proposed method to the conventional linear LS method.

ACKNOWLEDGMENT

This study is funded by the National Natural Science Foundation of China (No. 61071175).

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