The Choice Models of the Mobilization Point about Diversified Mobilization Materials Demand

Pingzhou Tang, Mingfang Huang and Huiying Tian
School of Economics and Management, North China Electric Power University, Beijing, China

Abstract: In this study, we propose the algorithm which could solve the problem of the mobilization production task with little mobilization point after adding the adjustment conditions. Moreover, the model result is not only extremely meaningful to the scheduling problem during mobilization production process, but also useful in the field of studying diversified emergency material combination scheduling, etc. In all, with our method, litter mobilization point can increase the stability of the system and reduce the outside disturbance to the material mobilization.

Keywords: Bi-level planning, mobilization center, mobilization material, mobilization point choice

INTRODUCTION

At present, the research on emergency combination problems of continuous material consumption and saving point is based on constant material consumption. The literature (Dai and Li, 2000) established the constant material consumption emergency model using the research results of single resource material scheduling. The literature (Malone and Crowston, 1994) established multi-resources emergency system scheduling model. The literature (Xiong and Sin, 2011) built a multi-emergency point scheduling model of city emergency materials and gave the solution. The literature (Hu, 2010) built a multi-objective optimization model for emergency logistics distribution with multiple supply points and multiple resource categories. These studies have high practical value for the emergency materials scheduling. It is difficult to apply to materials mobilization practice because the hypothesis about the constant materials consumption was too simple. Therefore, this study establishes the unsteady materials consumption scheduling model according to the actual mobilization material consumption.

METHODOLOGY

The description of the problem: A mobilization production center (notes for $A$), totally need $m(m > 1)$ kinds of emergency materials and the value of $j$ kind of materials demand is $x_j > 0 (j = 1, 2, ..., m)$. Now there are $n(n > 1)$ mobilization points providing $A$ enough mobilization materials supply (notes for $A_1, A_2, ..., A_n$), among which the value of the $i$ point $A_i$ provide the $j$ kind of mobilization materials is $y_{ij} > 0 (i = 1, 2, ..., n; j = 1, 2, ..., m)$ and satisfy:

$$\sum_{i=1}^{n} y_{ij} \geq x_j \quad j = 1, 2, ..., m$$

Set the time from mobilization point $A_i$ to the mobilization center point $A$ is $t_i > 0, (i = 1, 2, ..., n)$, $t_1 < t_2 < ... < t_n$. The unsteady demand of the mobilization center can illustrate by a continuous curve or section function. Here we use the function $f(t)$ to show the $j$ kind material consumption rate curve.

The scheduling schemes of all kinds of materials are required. The mobilization point and the amount of their supplies should be determined. The number of mobilization point participation in the activities of mobilization production is the least under the premise of the continuous supply of mobilization materials and mobilization production start time the earliest. Assume mobilization materials of the mobilization center can be shipped continuously, mobilization center have not corresponding materials reserve and don't consider production delays, so under these assumption, mobilization materials consumption is consistent with the material consumption of the mobilization center. According to the above assumptions, some continuous consumption concepts under a variety of materials unsteady consumption rate for consumption condition are illustrated as follows (Liu and Shen, 2003).

Because the problem of diversified materials unsteady consumption rates exists when meeting war and emergency, mobilization organization must guarantee that the amount of the arrived material can meet the needs of materials consumption of mobilization center for continuous production at any time. That is, there can't appear the condition that the material is not enough to affect the production activities of mobilization center.
According to this principle, the continuous feasible definition of the scheme $F_j$ on mobilization production start time $T_{s_j}$ is continuous feasible, if $\forall t_i \in \left[T_{s_j}, T\right]$, then:

$$\sum_{k \in \mathbb{K}^{\leq 0}} y^k_j \geq \int_{0}^{t_i} [f_j(t)] dt \quad (2)$$

In the expression, $\sum_{k \in \mathbb{K}^{\leq 0}} y^k_j$ means the quantity of the $j$ kind arrived material in the $t$ moment, $\int_{0}^{t_i} [f_j(t)] dt$ means the material consumption quantity of the $j$ kind material in the $t$ moment. In accordance with the scheme $F_j$, the quantity of arrived materials at $t_i$ moment is no less than the quantity of requirement material from the mobilization production start time $T_{s_j}$ to the moment $t_i$, then we can say the scheme $F_j$ on mobilization production start time $T_{s_j}$ is continuous feasible.

Suppose the set of continuous feasible scheme about all start time $T_{s_j}$ is $y^*_j$, then the optimal scheme for solving the problem is:

$$\min_{C_{s_j}} y^*_j$$

The literature (Liu and Shen, 2003) set forth the theorem about the earliest emergence start time under the condition of single material steady consumption. The study sets forth the lemma of the earliest mobilization production start time under the condition of diversified material unsteady consumption.

**Lemma 1:** If $t^*_j < t^*_{j+1} < \ldots < t^*_m$, then the earliest mobilization production start time of the scheme $F_j$ is:

$$T_{s_j} = \max_{i \in \{1, 2, \ldots, p\} \setminus \{i\}} \left[ t^*_j - t^*_i \right] \quad (4)$$

where, $t^*_i$ is derived from the type $\sum_{k \in \mathbb{K}^{\leq 0}} y^k_j = \int_{0}^{t^*_i} [f_j(t)] dt$, so that $y^k_j = 0$, $t^*_i$ is the time that $j$ kind of mobilization material reach the mobilization center $A$ from the $k$-th mobilization point. One possible solution should meet.

$$\sum_{k=0}^{p_j-1} y^k_j < x_j \leq \sum_{k=0}^{p_j} y^k_j, \quad k = 1, 2, 3, \ldots, p - 1$$

Then the corresponding optimal solution is:

$$F_j = \left\{ (A_i, y^i_j), (A_{i+1}, y^{i+1}_j), \ldots, (A_{p-1}, y^{p-1}_j) \right\} \quad (5)$$

For the above conclusion, the literature (Liu and Shen, 2003) advanced strict mathematical proof under steady consumption rate and the proof of under the condition of diversified material unsteady consumption rate is omitted in this study.

**Mathematical models:** The study uses bi-level planning modeling for the diversified materials scheduling problem of mobilization center production.

Based on the above analysis, setting mobilization point number as the upper problem and mobilization production time as the lower problem, when mobilization center exists the requirement of diversified materials under the unsteady consumption rate, bi-level optimization mathematical model can be built as follows:

$$\min \sum_{i=1}^{m} t^*_j$$

$$\max_{i \in \{1, 2, \ldots, p\}} \left[ t^*_j - t^*_i \right]$$

$$s.t. \sum_{i=0}^{p_j-1} y^k_j < x_j \leq \sum_{i=0}^{p_j} y^k_j$$

$$s.t. t^*_j + z^j = 0$$

$$z^j \in \{0, 1, 2, 3, \ldots, n; \quad j = 1, 2, \ldots, m\}$$

where, $z^j$: The upper decision variable which can take 0 or 1, when the $i$-th mobilization point is in production, $z^j = 1$, otherwise $z^j = 0$.

$t^*_j$: The time needed for transforming $j$ kind of mobilization material from the $i$-th mobilization point $A_i$ to the mobilization center $A$, $t^*_j < t^*_{j+1} < \ldots < t^*_m$.

$t^*_j$: The sustainable time of materials in the mobilization center, which can be obtained from

$$\sum_{i=0}^{p_j-1} y^k_j \leq x_j \subseteq \int_{0}^{t^*_j} [f_j(t)] dt$$

$f_j(t)$: The consumption rate of $j$ kind material in mobilization center.

$x_j$: The $j$ kind of materials demand of mobilization center.

$y^k_j$: The available $j$ kind of materials of the $i$-th mobilization point.

First, set $p_j = \min \left\{ p_j \left| \sum_{k=0}^{p_j-1} y^k_j \geq x_j, 1 \leq p_j \leq n \right. \right\}$, $z^j = \sum_{k=0}^{p_j-1} y^k_j$, $z^j = 0$.
According to the constraint condition, the earliest arrival time to the mobilization center of \( j \) kind of mobilization material, that is, the earliest mobilization production start time is:

\[
T_{j} = \max_{i \in \{1, 2, ..., p_j\}} \left[ t_j' - t_j^i \right]
\]  

Then, the scheduling scheme of the \( j \) kinds of material is:

\[
F_j = \left\{ (A_1, y_1^h), (A_2, y_2^h), \ldots, (A_{p_j}, y_{p_j}^h) \right\}
\]  

Because the production begins after all the demand material arrived at the mobilization center, therefore, the earliest mobilization production start time under diversified material demand condition is different from the single material demand. While, the earliest mobilization production start time under diversified material demand condition should be:

\[
T_s = \max_{j \in \{1, 2, ..., p\}} T_j
\]  

Production finishing time is:

\[
T_f = \max_{j \in \{1, 2, ..., p\}} \left[ T_j + T_j^i \right]
\]

**Model algorithm:** The algorithm above will include some mobilization point which the arrival time to the mobilization center is relatively short but have less reserved materials, thereby increasing the number of emergency saving points. However, too many points will reduce the stability of the emergency system and increase the emergency cost and bring about unnecessary interference to the normal social life as well. Therefore, the number of mobilization point should be as small as possible, given the continuous consumption can be met. The following criterion is introduced so that the mobilization point with more reserved materials should be first selected when that mobilization point meet the earliest mobilization production start time.

\[
T_{j} + t_j^i \geq t_j^i, i < j \leq p_j, (i = 1, 2, ..., p_i - 1) \]  

\( T_s \) is the mobilization production start time, \( t_j^i \) is the sustainable time of the \( j \) kind materials \( y_j^i \) (from the \( i \)-th mobilization point) in the mobilization center under the consume rate \( f_j(t) \). \( t_j^i \) is the time needed for transforming materials from the next available mobilization point to the mobilization center.

\[
T_{s1} = 12.3
\]

**EXAMPLES AND THE RESULTS COMPARISON**

A certain place is in emergencies, mobilization \( A \) advancing mobilization production need three kinds of emergency spare parts, respectively for \( x_1 = 260, x_2 = 160, x_3 = 170 \). There are eight candidate mobilization points, \( A_1, A_2, ..., A_8 \). The arrival time and the available materials of the mobilization point are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
<th>( A_6 )</th>
<th>( A_7 )</th>
<th>( A_8 )</th>
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<tr>
<td>( r )</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>11</td>
<td>12</td>
<td>14</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>( y_1^i )</td>
<td>40</td>
<td>30</td>
<td>30</td>
<td>120</td>
<td>20</td>
<td>110</td>
<td>90</td>
<td>130</td>
</tr>
<tr>
<td>( y_2^i )</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>20</td>
<td>120</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>( y_3^i )</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>40</td>
<td>120</td>
<td>70</td>
<td>60</td>
</tr>
</tbody>
</table>

Set \( i = h_1 = 1 \) and choose:

\[
h_2 = \max \left\{ h \left| T_{s1} + t_j^i \geq h > h_1 \right. \right\}
\]

Set \( i = h_2 \) and choose:

\[
h_3 = \max \left\{ h \left| T_{s1} + t_j^h, h > h_2 \right. \right\}
\]

Through recurrence, the optimal solution of the upper planning problem in Eq. (6) can be solved. Marking the optimum number of mobilization points as \( q_s \), it can be obtained the \( z_j^h = z_j^{h+1} = \ldots = z_j^{h+2} = 1, z_j^{h+3} = z_j^{h+4} = 0 \).

Excluding some mobilization point satisfying the time required but with short reserved materials, then the new optimum scheme is:

\[
F_j = \left\{ (A_1, y_1^h), (A_2, y_2^h), \ldots, (A_{p_j}, y_{p_j}^h) \right\}
\]

Repeating \( j \) times to the above algorithm, the optimum solution of all \( j \) kind mobilization materials after scheduling adjusting can be obtained. After classifying the corresponding point, the mobilization point which provides the mobilization materials can be determined ultimately.
According to the Eq. (7), the earliest start time of mobilization production is:

\[ T_{s1} = 11.5 \]

\[ T_{s2} = 17.7 \]

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\[ T_{s1} = 11.5 \]

\[ T_{s2} = 17.7 \]

At the same time, the feasible scheduling solution of the three kinds of mobilization material is:

\[ F_1 = \{ (A_1, 40), (A_2, 30), (A_3, 30), (A_4, 120), (A_5, 20), (A_6, 20) \} \]

\[ F_2 = \{ (A_1, 30), (A_2, 30), (A_3, 40), (A_4, 20), (A_5, 20), (A_6, 10) \} \]

\[ F_3 = \{ (A_1, 20), (A_2, 20), (A_3, 40), (A_4, 20), (A_5, 20), (A_6, 10) \} \]

According to the solution, the three kinds of mobilization materials need six mobilization points, \( A_1, A_2, A_3, A_4, A_5 \) and \( A_6 \).

Introducing the criterion (11), the mobilization point solution of the three kinds of mobilization materials is:

\[ F_1^* = \{ (A_1, 40), (A_2, 30), (A_3, 30), (A_4, 120), (A_6, 40) \} \]

\[ F_2^* = \{ (A_1, 30), (A_4, 30), (A_6, 100) \} \]

\[ F_3^* = \{ (A_1, 20), (A_4, 40), (A_6, 110) \} \]

The mobilization point solution after adjustment is \( A_1, A_2, A_3, A_4 \) and \( A_6 \). That is, five mobilization points can meet the mobilization requirements. Thus it can be seen that the scheme after adjustment is better than the former.

**CONCLUSION**

Setting the emergency material consumption as unsteady rate is more suitable to the actual materials mobilization. The unsteady consumption rate increased the difficulty of solving algorithm. The model result is not only extremely meaningful to the scheduling problem during mobilization production process, but also useful in the field of studying diversified emergency material combination scheduling, etc. After adding the adjustment conditions, the algorithm can solve the problem of the mobilization production task with less mobilization point than that of literature (Liu and Shen, 2003). Less mobilization point can increase the stability of the system and reduce the outside disturbance to the material mobilization.

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