Solution for the Nonlinear Multi-Objective Model in Construction Projects Using Improved Particle Swarm Optimization

Lianying Zhang, Qiong Wu, Chen Chen and Yan Yue
College of Management and Economics, Tianjin University, Tianjin, China 300072

Abstract: In project management, spending minimum time and cost while achieving maximum quality is of great significance to its success. Consequently, it is vital to find an optimal equilibrium between the three objectives of construction projects. To achieve this goal, this study presents an advanced nonlinear multi-objective model to solve the time-cost-quality trade-off problem. We assume that the quality of an activity is influenced by its duration and cost and quantify the quality of a project by calculating the mean of the quality coefficients of all the activities. The concepts of value management are introduced to formulate the evaluation function, so that the solutions are further optimized for project managers’ decision making. When solving the model, an improved Particle Swarm Optimization (PSO) is developed by introducing genetic operators and immune selection to the original PSO for higher efficiency and faster convergence. The efficiency and reliability of the proposed algorithm in generating optimal solutions for the trade-off problems are demonstrated through an application example.

Keywords: Construction project, improved particle swarm optimization, multi-objective optimization, time-cost-quality trade-off, value management

INTRODUCTION

Delivering a project with the least time, minimum cost and maximum quality is one of the essential requirements as to the success of project management. In order to balance these three interrelated and conflicting objectives, an amount of research has been carried out.

Traditional research has mainly focused on analyzing time and cost trade-off problems on the basis of the Critical Path Method (CPM) (James and Morgan, 1959), which is a fundamental quantitative technique developed for project management. And the solutions to the early time-cost trade-off models can be divided into two categories: the exact algorithms-dynamic programming and the LP/IP hybrid programming, etc. Adeli and Karim (1997) and Burns et al. (1996) and the heuristic algorithms-genetic algorithm, ant colony algorithm, simulated Annealing algorithm, etc. Chan et al. (1996), Feng et al. (1997), Feng et al. (2000), Hegazy (1999) and Li et al. (1999). However, with the emergence of the new construction contracts, project managers are challenged by the need of minimizing project time and cost while maximizing its quality. Therefore the trade-off between time, cost and quality of a project has been the subject of a quantity of research so far, which is more practical and significant than the two-dimensional models.

Models established by researchers on time, cost and quality trade-off can be classified into two distinct categories: continuous trade-off models (Babu and Suresh, 1996; Khang and Myint, 1999) and discrete models (El-Rayes and Kandil, 2005; Hamed and Seyyed, 2007; Iranmanesh et al., 2008; Ghodsi et al., 2009). In the continuous category, linear models and non-linear models are distinguished in line with the relations defined among time, cost and quality. Babu and Suresh (1996) presented the first linear programming models for the time-cost-quality trade-off problem assuming that not only cost but also quality was influenced by activity crashing and defined that the two entities varied as linear functions of completion time. The authors adopted the continuous scale from 0 to 1 to specify the quality level of each activity in accordance with its degree of crashing. Then the overall project quality level was obtained by calculating the average of the individual activity quality levels. Three mathematical models were developed by the authors, which optimized the time, cost and quality objectives respectively. Khang and Myint (1999) investigated Babu and Suresh’s models and applied them to a cement factory construction project to study their practicality. The practicality and feasibility of the time and cost optimization models were demonstrated. But the methodology of the quality measurement were said to be over subjective and inaccurate. In fact, the introduction of quality is of great significance to the research of the time-cost-quality trade-off problem, however it has to be pointed out that there lacks an applicable and relatively
more accurate method to quantify quality. Moreover, despite that the linear relations between time and cost, time and quality are accepted by some scholars in their research, these assumptions are not practical in real life construction projects, especially when the existence of budget threshold in the time-cost curve is taken into consideration.

Meanwhile, the discrete models were investigated and several algorithms have been developed to solve them. El-Rayes and Kandil (2005) initially formulated the first discrete time-cost-quality trade-off model. The researchers suggested that each activity of the project could be performed by different feasible resource utilization options. And each option decided corresponding time, cost and quality of the activity. To estimate the quality of the activities, a series of quality indicators were introduced, which allowed practical and objective measurement of the activities’ performance. The holistic project quality was afterwards calculated by the weighted sum of the quality levels of all the activities in the project. Meanwhile Genetic Algorithm (GA) was employed to search for the Pareto optimal solutions, which provided new visions to solve the models. In the case project which consisted of 18 activities, 305 Pareto optimal solutions were generated by Hamed and Seyyed (2006) also developed a discrete time-cost-quality trade-off model. The authors assumed that the duration and quality of project activities were discrete, non-increasing functions of a single non-renewable resource. Three inter-related binary integer programming models were developed by the authors which optimized the time, cost and quality objectives respectively. To better solve the model, the authors developed a meta-heuristic solution procedure called electromagnetic scatter search. The capability of the algorithm was tested on a randomly generated large and complex problem having 19,900 activities. Additionally, different forms of quality aggregations and effect of activity mode reductions were investigated. Then a new meta-heuristic multi-colony ant colony algorithm was developed by Afshar et al. (2007) for the optimization of the three objectives in the time-cost-quality trade-off analysis. To more efficiently solve the discrete model, Particle Swarm Optimization (PSO), initially proposed by Kennedy and Eberhart (1995), was for the first time adopted by introducing Dynamic Ideal Point method (Rahimi and Irannanesh, 2008). As the problem is NP-Hard (De et al., 1997; Irannanesh et al., 2008) developed another meta-heuristic named FastPGA based on a version of genetic algorithm. In addition, Jose Ramon San Cristobal (2009) used the 0-1 Integer Programming model to solve the discrete TCQT problem. The applicability of the model was verified by applying it to a road building project. And based on the same methodology, two alternative versions of the model which optimized the other two objectives were shown. The discrete models are more practical since in most real-life projects, resources are available in discrete units, such as a number of machines, a number of workers and so on (Demeulemeester and Herroelen, 2000). Nevertheless, as pointed out by Ghodsi et al. (2009), in case that the total number of the project modes was very high (either because of high number of activities, or because of high number of mode per activity, or a combination), the discrete model generated gigantic complexity and became unrealistic. Additionally, the quality of a project was determined not only by time, but also by cost. Thus Ghodsi et al. (2009) formulated a continuous non-linear time-cost-quality trade-off model with 6 axioms on the basis of the existing models. A sample project was investigated and the Pareto optimal front has been found using a recent version of the ε constraint method.

Above all, the existing literature provides a broad vision for research of the time-cost-quality trade-off problem however, the time-cost-quality trade-off problems are not well solved because there has not been a universal and generalized applicable method to quantify the quality objective and what’s more, the existing quantifying methods still need to be modified. Based on the existing research, this paper presents a multi-objective optimization model which enables the project managers to minimize construction time and cost, while maximizing its quality with more practical quantified definition of project quality. To improve the practicality of the research and enhance the efficiency of the model solution, we formulate a three-dimensional non-linear optimization model and develop an improved PSO algorithm to solve it. The concepts of value management are introduced to formulate the evaluation function, so that the solutions are further optimized for project managers’ decision making. An application example is analyzed afterwards to verify the efficiency and reliability of the proposed algorithm in generating optimal solutions for the trade-off problems.

MODEL FORMULATION AND IMPLEMENTATION

Quantifying construction quality level: Internationally, quality is defined as the degree to which a set of inherent characteristics fulfills requirements. As to construction projects, it has been commonly accepted that the features of satisfying construction quality can be summarized as functionality, durability, reliability and maintainability, safety, economy and sustainability. Since the overall construction quality mainly depends on the quality of each activity, the quantification of construction quality can be conducted in the following two steps:

- Quantifying construction quality of each activity
- Calculating the quality level of the whole project based on the activity quality levels
For the purpose of facilitating the measurement, the data of inspection lot acquired by project inspection serves as an indication of the activity quality level. In the practice of construction projects, the planners can judge the quality of the inspection lot by testing on or examining dominant items and general items. Dominant items have major influence on project safety, sustainability and the public interest. For example, dominant items of brick masonry inspection lot include strength grade of brick, strength grade of mortar and so on, while general items consist of the masonry methods, thickness of horizontal mortar, etc. In order to integrate construction quality theory with the quantification of construction quality, each activity can be considered as an inspection lot. Consequently, the calculation of the activity quality level is presented in Eq. (1) based on the result of the inspection.

\[ Q_i = x_{id}Q_{id} + x_{ig}Q_{ig} \]  

(1)

where, \( Q_i \) is the quality coefficient of activity (i). \( x_{id} \) represents weight of quality level of dominant items in activity (i), \( x_{ig} \) is the weight of the quality level of general items in activity (i), i.e., \( x_{id} + x_{ig} = 1 \). \( Q_{id} \) denotes the quality level of dominant items in activity (i). \( Q_{ig} \) is the quality level of general items in activity i. As to different activities, the weights of dominant items and general items are determined by conducting the Delphi Method. For simplicity, in this paper \( x_{id} \) and \( x_{ig} \) are set 0.8, 0.2 respectively. \( Q_{id} \) and \( Q_{ig} \) are obtained from the records of the inspection lot. The quality of the whole project is calculated as the average quality level of all the project activities.

**Time coefficient and cost coefficient:** As to an activity, its duration and cost have direct effects on its quality level. Traditionally, time and cost are chosen as the two dimensions in time-cost trade-off analysis with time and cost being measured by different units respectively. In this paper, the two dimensions are uniformed by transforming time to time coefficient and cost to cost coefficient so that we can discuss the two entities simultaneously. The method of transformation is discussed as follows: a certain quantity of time units, suppose D working hours or days etc. is set as a unit of time coefficient. If the duration of activity i ranges from \( T_{i_1} \) to \( T_{i_2} \), the time coefficient of activity i is noted as:

\[ T_i = \left[ \frac{T_{i_2} - T_{i_1}}{D} \right] \]

In real world projects, if completion date of the project is earlier, Net Present Value (NPV) will increase because the project will start to earn profit earlier. However, if the project is delayed, the contractor is forced to pay for liquidated damage for delay. Suppose the rate of liquidate damage for delay is \( I \), the ideology is that if the project operate smoothly, it creates a certain amount of profit I per unit of time. Therefore, if the cost of activity i is increased by \( \Delta C_i \), it can be taken as that the project delayed for \( \Delta C_i/I \), which means the time coefficient of the activity is increased by \( \Delta C_i/I* D \). Consequently if the cost of activity i varies from \( C_{i_1} \) to \( C_{i_2} \), cost coefficient is noted as \( C_i \in [C_{i_1}/I* D, C_{i_2}/I* D] \). In this way, we successfully unify time and cost in a uniform dimension.

**Quality model:** Time-cost-quality model is the key to solve the time-cost-quality trade-off problem. In order to simplify the model, we put forward the following assumptions on the relation between time, cost and quality of an activity:

- To keep the quality level while decrease the complete time of an activity, more expenditure is needed and vice versa
- It is costlier to reduce the duration of an activity with short complete time than to compress an activity with a relatively long time span for the same amount of time
- Compared with a low quality activity, improving the quality of activities which are at high quality levels is relatively difficult and expensive

Based on the above assumptions, the quality model is developed in the following equation:

\[ Q_i = (\log_{a_i} C_i + 1) + \log_{b_i} (T_i + 1) \]  

(2)

where, \( Q_i \) is the quality level of activity i. \( C_i \) is the cost coefficient of activity i. \( T_i \) represents time coefficient of activity i. \( a_i \) represents the influence factor of cost coefficient on the quality level of activity i and \( b_i \) represents the influence factor of time coefficient on the quality level of activity i. The two parameters are calculated based on the end values of the time, cost and quality coefficients of the activity according to Eq. (2). (We obtain one set of the values by the recording the values of \( C_i \) and \( T_i \) when the activity is executed at the minimum quality level \( Q_{min} \) regulated in practice; the other set is obtained by examining the value of \( Q_i \), when \( C_i \) and \( T_i \) reach maximum value \( C_{max} \) and \( T_{max} \) specified in project contracts or related documents).

**The nonlinear multi-objective model:** Based on the above assumptions and the quality model we have just formulated, an advanced nonlinear multi-objective model is developed as follows. The model incorporates three major objective functions to evaluate project performance.
in terms of construction time, cost and quality, respectively.

Minimize project time:

\[
\text{Minimize } T = \sum_{i \in S_c} T_i' \quad (3)
\]

where, \( T_i' \) denotes the duration of activity \( i \) on the critical path and \( S_c \) represents the set of activities on critical path.

Minimize project cost:

\[
\text{Minimize } C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[ C_i^c + \gamma_i (T_i - T_i^c)^2 \right] \quad (4)
\]

where, \( \gamma_i \) is the factor of marginal cost. \( C_i^c \) and \( T_i^c \) denote cost coefficient and time coefficient of activity \( i \) in normal state.

Maximize project quality:

\[
\text{Maximize } Q = \frac{1}{n} \sum_{i=1}^{n} Q_i \quad (5)
\]

where, \( Q_i \) is calculated with reference to Eq. (2).

**Model solution:** There are many ways to solve multi-objective optimization problems, such as evaluation function method, analytical hierarchy process, multi-objective programming approach and so on. Evaluation function method is the most widely used. Ideal point, weighted square sums, linear weighting model are common evaluation functions. Unlike single-objective optimization problems, a number of efficient solutions called Pareto optimal solutions form the Pareto front of the multi-objective problem. The Pareto solutions are significant in the theoretical analysis yet, in practice they need to be further optimized for project managers’ decision making. In order to effectively solve the application problem of the optimization model and enhance the practicality of Pareto solutions, value management theory is introduced to build the evaluation function of the model as one possible criterion during project managers’ decision making process. The basic objective of value engineering is to maximize the value of a project, namely using less money while gaining better or more functions. The time-cost-quality trade-off analysis can be considered in the same way. As the cost of a project involves both money and time, the value of a solution can be evaluated by executing Eq. (6):

\[
V = \frac{\sum_{i=1}^{n} Q_i}{\sum_{i=1}^{n} C_i + \sum_{i=1}^{n} T_i} \quad (6)
\]

where, \( V \) represents the value of the whole project. \( Q_i \) is the quality level of activity \( i \). \( T_i \) is the time coefficient of activity \( i \). \( C_i \) is the cost coefficient of activity \( i \).

According to theories of value engineering, the production reaches the best status when \( V \) is around 1.0 with the production cost equivalent to the socially necessary cost. However when the theory is introduced into the evaluation function of the model, as a result of the different dimensions of the \( Q, C \) and \( T \), the mechanism is that the higher the value of \( V \), the better the solution becomes. Thus the trade-off process is to achieve the largest \( V \).

**THE IMPROVED PSO ALGORITHM**

**The original PSO algorithm:** Particle Swarm Optimization (PSO) is a population based stochastic optimization technique. The researchers were inspired by the social behavior of bird flocking and fish schooling. According to what scientists have found, in order to search for food, each member in a flock of birds determines its velocity based on its individual experience and information gained through interaction with other members in the flock. The principle of PSO is mainly based on this scheme. Each “bird”, called particle, “flies” in the solution space of the optimization problem searching for the optimum solution and thus its position represents a potential solution for the problem. In Particle Swarm Optimization terminology, the available solutions at each of the iteration are called the “swarm” which is equivalent to “population” in genetic algorithms. The original PSO formulae define each particle as a potential solution to a problem in D-dimensional space, with particle \( i \) represented as \( X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \). Each particle also maintains a memory of its previous best position \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iD}) \) and a velocity along each dimension, represented as \( V_i = (v_{i1}, v_{i2}, \ldots, v_{iD}) \). If we let \( p_{id} \) be the best known position of particle \( i \) and \( p_{gd} \) be the best known position of the entire swarm. By the combined effect of \( p_{id} \) and \( p_{gd} \), the velocity along dimension \( d \) is adjusted and used to calculate a new position for the particle. The portion influenced by the best particle in the neighborhood is regarded as the social component portion of the adjustment and the velocity influenced by the individual’s previous best position is considered the portion of adjustment influenced by cognition component. In Kennedy’s early versions of the algorithm, these formulae are:

\[
v_{id}^{k+1} = v_{id}^k + c_1 r_1 (P_{id} - x_{id}^k) + c_2 r_2 (p_{gd} - x_{id}^k) \quad (7)
\]

\[
x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \quad (8)
\]
where, constants $c_1$, $c_2$ determine the relative influence of the cognitive and social components and are often set the same value to give each component an equal weight (In this paper, $c_1$ and $c_2$ are set 2). A constant $v_{max}$ is used to arbitrarily limit the velocities of the particles and improve the solution of the search.

$$v_{id}^{k+1} = \omega v_{id}^{k} + c_1 r_1 (p_{id} - x_{id}^{k}) + c_2 r_2 (p_{gd} - x_{id}^{k})$$  \hspace{1cm} (9)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1}$$  \hspace{1cm} (10)$$

Shi and Eberhart (1998) introduced an inertia weight $\omega$ to the calculation of the particles’ velocity as shown in Eq. (9). At first a fixed inertia weight is used in the equation. Later, a linearly decreasing inertia weight is deployed. By linearly decreasing the inertia weight from a relatively high value (0.9) to a low value (0.4) through the course of a PSO execution, PSO tends to have more global search ability at the beginning of the run and more local search ability near the end (Shi and Eberhart, 1999). It means that the near optimal space of solution can be more efficiently located and then searched more thoroughly. After that, Clerc (1999) developed a constraint factor PSO, as shown in Eq. (11):

$$v_{id}^{k+1} = K \left[ \omega v_{id}^{k} + c_1 r_1 (p_{id} - x_{id}^{k}) + c_2 r_2 (p_{gd} - x_{id}^{k}) \right]$$  \hspace{1cm} (11)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1}$$  \hspace{1cm} (12)$$

where, $K = \frac{2}{|2 - l - \sqrt{l^2 - 4l}|}$, $l = c_1 + c_2$, $l > 4$

According to Eberhart and Shi (2000) who compared the two modifications, the PSO algorithm with the constriction factor $K$ can be considered as a special case of the algorithm with inertia weight since the three parameters are connected through constraint conditions. Thus in this paper, the strategy of linearly decreasing inertia weight is deployed.

**The improved PSO algorithm:** To overcome the drawbacks in the traditional PSO algorithm, an improved PSO is developed in this paper which integrates genetic operators and immune selection. The introduction of genetic operators, namely crossover and mutation, to original PSO will help to get rid of local optima thus keep the diversity of the population. And the immune selection operation helps to select the superior particles by simulating the biological immune mechanism based on particles’ affinity and concentration. Equation (13) defines the concentration of the particles. Equation (14) calculates the immune selection rate of the particles.

$$H_i = \sum_{j=1}^{m} \left( 1 - \frac{A_{i,j}}{\max \{ A_{i,j} \}} \right) > \lambda / m$$  \hspace{1cm} (13)$$

$$S(i) = f (A_i, H_i) = A_i / H_i$$  \hspace{1cm} (14)$$

where, $S(i)$ represents the immune selection rate of particle $i$. $H_i$ is the concentration of the particle $i$. $A_i$ is denoted as the affinity of particle $i$ which is adapted to its fitness value. $A_{i,j}$ is the difference between particles $i$ and $j$’s fitness values. $\lambda$ is a threshold usually set 0.9 to guarantee that the positions of the two particles are similar enough and $m$ is the population size of the particles set in the initialization.

Consequently, when the concentration of particle $i$ is constant, the rate of the particle being selected becomes bigger when its objective fitness value increases; and in case that the objective fitness value of particle $i$ is constant, the selection rate becomes smaller when the concentration of the particle becomes denser. Such a scheme keeps the particles with the best fitness values and at the same time inhibits the population from high concentration of similar particles, forming a strategy to maintain the diversity of the population while selecting the best particles.
Fig. 2: The activity-on-node (AON) network of the case project

There are 3 main phases in this improved PSO algorithm. The algorithm flow is illustrated as follows and shown in Fig. 1.

- Set the population size $m$ ($m = 100$ in our algorithm) and initialize the algorithm by randomly generating other PSO parameters, including position and velocity of each particle and maximum velocity.
- Calculate the fitness value of each particle
- If the results satisfy the termination conditions (reasonable error or reaching maximum iteration which is set 500 to generate the best particles in this paper), stop searching; if not, go to step 4
- For each particle, compare its current fitness value with its personal best value (Pbest). If the former is better, then replace Pbest with current fitness value. Likewise, if current fitness value of the particle is better than its global best value Gbest, then replace Gbest with its current fitness value. Afterwards record the particle that change the Gbest in a memory base
- Update the position and velocity of each particle according to Eq. (9) and (10)
- Judge whether it exist local optimum, if so, turn to step 7; (Some features are beneficial for the judgment: for instance, global optimum remains almost the same for 20 consecutive generations); If not, turn to step 2
- Randomly generate $u$ particles
- For the $m+u$ newly formed particles, immune selection is conducted
  - Calculate the concentration of each particle according to Eq. (13)
  - Calculate the immune selection rate of each particle with reference to Eq. (14)
  - Rank the particles according to their selection rate and select the first $m$ particles as the new generation.
- Cross the particles based on crossover rate which is set 0.9 in this study
- Mutate the newly generated particles on the basis of their mutation rate which is set 0.1. And then turn to step 2

**CASE STUDY**

Sample problem and model formulation: To verify the efficiency and reliability of the proposed model and algorithm, a case project which consists of 12 construction activities is cited here. The activity-on-node network of the project is shown in Fig. 2.

The detailed data of the project case is shown in Table 1, where $t_i^1$, $c_i^1$ and $q_i^1$ represent the activity time, cost and quality coefficient in normal state and $t_i^c$, $c_i^c$ and $q_i^c$ represent the activity time, cost and quality coefficient after crashing.

According to empirical analysis of construction project, the unit of time coefficient $D$ can be set the mean maximum of the activity durations in normal state. And in this case projects, the rate of liquidate damage for delay $I$ is set 1.07. Based on the data in Table 1, it is possible to calculate time coefficient ($T_i$), cost coefficient ($C_i$), factor of marginal cost ($i$), influence factor of cost coefficient on quality level ($a_i$), influence factor of time coefficient on quality level ($b_i$).

### Table 1: Detailed data of the case project

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<th>Activity</th>
<th>$t_i^1$</th>
<th>$c_i^1$</th>
<th>$q_i^1$</th>
<th>$t_i^c$</th>
<th>$c_i^c$</th>
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### Table 2: Date processed for implementation

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<td>0.4015</td>
<td>1.0037</td>
<td>2.9897</td>
<td>29.5198</td>
<td>2.4003</td>
</tr>
<tr>
<td>9</td>
<td>1.1811</td>
<td>0.6412</td>
<td>0.3535</td>
<td>1.3160</td>
<td>2.4289</td>
<td>42.7253</td>
<td>2.5953</td>
</tr>
<tr>
<td>10</td>
<td>0.9213</td>
<td>0.5906</td>
<td>0.2788</td>
<td>0.7361</td>
<td>4.1808</td>
<td>42.7253</td>
<td>2.1869</td>
</tr>
<tr>
<td>11</td>
<td>0.8268</td>
<td>0.4252</td>
<td>0.3346</td>
<td>0.9368</td>
<td>4.3745</td>
<td>13.0300</td>
<td>2.0532</td>
</tr>
<tr>
<td>12</td>
<td>1.1811</td>
<td>0.6614</td>
<td>0.4461</td>
<td>1.3383</td>
<td>3.3305</td>
<td>46.6836</td>
<td>2.4406</td>
</tr>
</tbody>
</table>
The best solutions in the last iteration generated by the improved PSO algorithm are obtained as the Pareto optimal solutions, as shown in Fig. 3. Fig. 4 to 6 shows the projections of these solutions on C - Q, T - Q and T - C dimensions for analysis. Some of the optimal results are singled out from the Pareto front as example solutions of the TCQT model in Table 3.

As a number of solutions have been generated by the improved PSO, decision makers need to select from them the best execution mode for the project. Based on the decision method of our model, the best solution to the case project should possess the highest V. In this case project, it is when the number of generations reaches...
Table 4: Optimum solution of the TCQT model

<table>
<thead>
<tr>
<th>Quality</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Time</th>
<th>Cost</th>
<th>Quality coefficient</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>39</td>
<td>27</td>
<td>29</td>
<td>33</td>
<td>37</td>
<td>53</td>
<td>40</td>
<td>48</td>
<td>31</td>
<td>30</td>
<td>41</td>
<td>222</td>
<td>258.59</td>
<td>0.8752</td>
<td>0.9537</td>
<td></td>
</tr>
</tbody>
</table>

189 and the fitness function achieves its convergence value 0.9537, as shown in Fig. 7 the optimal process curve of project value V. This optimum solution of the TCQT model is presented in Table 4.

**CONCLUSION AND DISCUSSION**

In this paper we present a time-cost-quality trade-off model which is developed in two key stages:

- Quantifying construction quality by testing on dominant items and general items based on Delphi Method
- Unifying time and cost through adopting the concept of time and cost coefficients

To solve the model, an improved PSO is developed to improve the search ability of the original algorithm by introducing genetic operators and immune selection. The test experiment based on the illustrative case project verifies that the improved PSO provides an effective mechanism to get rid of the local optimum in the late iterations and demonstrates greater capability for practical use.

In order to enhance the practicality of the nonlinear multi-objective model, the model needs to be further improved by investigating the definiteness and fuzziness of construction quality. And we will study the better ways to apply value management theory in the time-cost-quality trade-off optimization models so that the principles of value engineering can be better consisted with. Moreover, we will further explore the theoretical basis and mathematical proof of the improved PSO algorithm developed in this paper to justify its implementation.

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**REFERENCES**


