Mathematical Modeling of Age and of Income Distribution Associated with Female Marriage Migration in Rajshahi, Bangladesh

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Abstract: An effort has been made, in this study, to fit mathematical models to age and income distribution associated with female marriage migration in Rajshahi district, Bangladesh. For this, the data is taken under the project entitled “Strengthening the Department of Population Science and Human Resource Development” in collaboration with UNFPA, Bangladesh. It is found that marriage migration associated with age follows polynomial model and income distribution associated with female marriage migration follows two parameters positive exponential model. To verify the adequacy and steadiness situation of the model, Cross Validity Prediction Power (CVPP) and F-test are employed to these models. The contribution of this paper to knowledge is the fitted cubic polynomial model and positive exponential model to the migration data aggregate.

Keywords: Cross Validity Prediction Power (CVPP), exponential model, F-test, female marriage migration associated with age and income, polynomial model

INTRODUCTION

Models are very realistic and sophisticated tools to express data in mathematical structure in the era of globalization process. Model is of great help to demographers as well as social researchers for realizing the process to distinguish among various important and unnecessary variables to find out the functional relationships among various demographic phenomena and social indicators. In the current study, deterministic model is used to explain the functional association between the dependent and independent variables that can take exact values. A number of models were fitted to the data aggregate (Morril and Pitts, 1967; Perry, 1969a, 1969b; Samuel, 1994; Sharma, 1984; Yadava et al., 1988) related to migration in Bangladesh, India and other countries of this globe. Hossain attempted to develop the model proposed by Sharma (1984) and Yadava et al. (1988). But these models did not afford good fit. Hossain (2000) used the Pareto-Exponential model proposed by Morril and Pitts (1967). Although Pareto-Exponential model provided better approximation than the models of Sharma (1984) and Yadava et al. (1988), it was not significantly well to the utilized data. In India, Sharma (1984) and Yadava et al. (1988) fitted well for Hindu community. Yadavs et al. (2002) showed that exponential distribution provides a better fit to the distribution of marriage migration associated with distance than Pareto-Exponential as applied by Hossain (2000). Moreover, Yadavs et al. (2002) compared their findings with those of the Pareto-exponential function applied by Yadav et al. (1998). Ali et al. (2004) showed that age associated marriage migration followed exponential distribution.

In this study, an n degree polynomial model (Waerden and Van Der, 1948; Spiegel, 1992; Gupta and Kapoor, 1997) for age related marriage migration and exponential model for income distribution associated with marriage migration is chosen to be applied here. It is to be mentioned that polynomial model has also been applied by Islam et al. (2003), Islam and Ali (2004) and Islam (2005). Moreover, exponential model was fitted for age structure of male population of Bangladesh in 1991 census (Islam et al., 2003)

Therefore, the fundamental objectives of this study are to build up mathematical models for female marriage migration associated with age and income.

METHODOLOGY

Data sources of the study: To accomplish the objectives mentioned above the data on female marriage migration associated with age and income distribution in Rajshahi district, Bangladesh is taken under the project entitled “Strengthening the Department of Population Science and Human Resource Development” sponsored by UNFPA, Bangladesh. These data have been used to fit the proposed model and shown in Table 1 and Table 2, respectively.

Model fitting: Using the scattered plot of marriage migration associated with age (Fig. 1, 2 and 3), it is seen that marriage migration with respect to age can be fitted by polynomial model. So, an nth degree polynomial model is considered and the model is
Table 1: Observed and predicted of age associated with female migrants due to marriage

<table>
<thead>
<tr>
<th>Age</th>
<th>Rural Observed</th>
<th>Rural Predicted</th>
<th>Suburban Observed</th>
<th>Suburban Predicted</th>
<th>Total (rural and suburban jointly) Observed</th>
<th>Total (rural and suburban jointly) Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>151</td>
<td>158</td>
<td>56</td>
<td>65</td>
<td>207</td>
<td>223</td>
</tr>
<tr>
<td>20-24</td>
<td>285</td>
<td>270</td>
<td>266</td>
<td>239</td>
<td>551</td>
<td>510</td>
</tr>
<tr>
<td>25-29</td>
<td>303</td>
<td>298</td>
<td>281</td>
<td>303</td>
<td>584</td>
<td>601</td>
</tr>
<tr>
<td>30-34</td>
<td>237</td>
<td>264</td>
<td>286</td>
<td>288</td>
<td>523</td>
<td>552</td>
</tr>
<tr>
<td>35-39</td>
<td>202</td>
<td>194</td>
<td>232</td>
<td>227</td>
<td>434</td>
<td>422</td>
</tr>
<tr>
<td>40-44</td>
<td>126</td>
<td>113</td>
<td>159</td>
<td>154</td>
<td>285</td>
<td>267</td>
</tr>
<tr>
<td>45-49</td>
<td>39</td>
<td>46</td>
<td>96</td>
<td>99</td>
<td>135</td>
<td>145</td>
</tr>
</tbody>
</table>

Table 2: Observed and predicted of income distribution associated with female migrants due to marriage

<table>
<thead>
<tr>
<th>Income (in taka)</th>
<th>Rural Observed</th>
<th>Rural Predicted</th>
<th>Suburban Observed</th>
<th>Suburban Predicted</th>
<th>Total (rural and suburban jointly) Observed</th>
<th>Total (rural and suburban jointly) Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-500</td>
<td>48</td>
<td>65.14</td>
<td>3</td>
<td>35.51</td>
<td>51</td>
<td>90.86</td>
</tr>
<tr>
<td>500-1000</td>
<td>75</td>
<td>98.01</td>
<td>20</td>
<td>63.96</td>
<td>95</td>
<td>151.61</td>
</tr>
<tr>
<td>1000-1500</td>
<td>152</td>
<td>147.46</td>
<td>101</td>
<td>115.23</td>
<td>253</td>
<td>252.96</td>
</tr>
<tr>
<td>1500-2000</td>
<td>228</td>
<td>221.85</td>
<td>219</td>
<td>207.57</td>
<td>447</td>
<td>422.06</td>
</tr>
<tr>
<td>2000-2500</td>
<td>321</td>
<td>333.79</td>
<td>343</td>
<td>373.93</td>
<td>664</td>
<td>704.22</td>
</tr>
<tr>
<td>2500-3000</td>
<td>519</td>
<td>502.20</td>
<td>690</td>
<td>673.61</td>
<td>1209</td>
<td>1175.00</td>
</tr>
</tbody>
</table>

Fig. 1: Observed and predicted values of age associated with female migrants due to marriage of rural area in Rajshahi, Bangladesh. X axis represents age and Y axis represents female migrants.

Fig. 2: Observed and predicted values of age associated with female migrants due to marriage of suburban area in Rajshahi, Bangladesh. X axis represents age and Y axis represents female migrants.

Fig. 3: Observed and predicted values of age associated with female migrants due to marriage of rural and suburban area jointly in Rajshahi, Bangladesh. X axis represents age and Y axis represents female migrants.

Fig. 4: Observed and predicted values of income distribution associated with female migrants due to marriage of rural area in Rajshahi, Bangladesh. X axis represents income (in Taka) and Y axis represents female migrants.

\[ y = a_0 + \sum_{i=1}^{n} a_i x^i + u \] in which x be the mid value of age, y indicates marriage migration, a_0 is the constant, a_i is the coefficient of \( x^i \) (i = 1, 2, 3, ..., n) and u is the stochastic error term of the model. Here a suitable n has been selected for which the error sum of squares is minimum.
If female marriage migration associated with income is plotted in the graph paper (Fig. 4, 5 and 6), then it appears that income distribution due to marriage migration is positively exponentially distributed. Therefore, a two-parameter positive exponential model is considered and the form of the model is:

$$y = e^{(a_0 + a_1 x + u)}$$

In which x represents the central value of income (in taka); y represents marriage migration; $a_0$, $a_1$ are unknown parameters and u is the stochastic disturbance term of the model. These mathematical models are fitted employing the software STATISTICA.

**Model validation:** To check the adequacy and soundness of the model, the CVPP is applied here. The mathematical formula for CVPP is given by:

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1 - R^2)$$

where, n is the number of cases, k is the number of predictors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). The positive value of the difference of $\rho_{cv}^2$ and $R^2$ is the shrinkage of the model. 1-shrinkage is the stability of $R^2$ of the fitted model. It
should be noted here that to clarify the validation of the model the CVPP was also employed by Islam and Ali (2004), Islam et al. (2003) and Islam (2005). The estimated CVPP analogous to their $R^2$ and information on model fittings are presented in Table 3.

F-test: To verify the overall measure of the significance of the fitted model as well as the significance of $R^2$, the F-test is employed here (Gujarati, 1998).

APPLICATION OF MODELS AND RESULTS

The polynomial model is assumed to fit the model for female marriage migration due to ages in Rajshahi, Bangladesh and the fitted models are as follows:

\[ y = -1263.79 + 139.8992x - 3.92714x^2 + 0.032889x^3 \text{ for Rural area ... (1)} \]
\[ y = -1893.34 + 188.8944x - 5.16238x^2 + 0.043556x^3 \text{ for Sub-urban area ... (2)} \]
\[ y = -3157.13 + 328.7937x - 9.08952x^2 + 0.076444x^3 \text{ for both Rural and Sub-urban area ... (3)} \]

The positive exponential model is considered to be fitted the model for female marriage migration associated with income in Rajshahi, Bangladesh and the fitted models are presented below:

\[ y = \exp(0.00082x + 3.97230) \text{ for Rural area... (4)} \]
\[ y = \exp(0.00118x + 3.27543) \text{ for Sub-urban area ... (5)} \]
\[ y = \exp(0.00102x + 4.25339) \text{ for both Rural and Sub-urban area... (6)} \]

It is seen from the statistics of Table 3 that the fitted models are highly cross-validated and their shrinkages are 0.11484, 0.1196; 0.08886, 0.0081, 0.01275 and 0.0082 for the models (1)-(6) respectively. And the fitted models are more than 86, 85, 89, 98, 97 and 98% stable respectively for the models (1)-(6). Moreover, all the parameters of the fitted models are also highly statistically significant with more than 97% proportion of variation explained. Furthermore, the stability of $R^2$ of these models (1) to (3) is also more than 88%. And the stability of $R^2$ of the models (4) to (5) is also more than 99%.

In this study the calculated value of F-statistic are 40.05, 38.42 and 52.05 for the models (1) to (3), respectively while with 3 and 3 degrees of freedom (d.f) at 1% level of significance the tabulated value of F is 29.5. The enumerated value of F-statistic for the models (4) to (5) are 462.75, 292.30 and 455.77, respectively where as with (1, 4) d. f. at 1% level of significance the tabulated value of F is 21.2. That is, the fitted models are highly significant. Thus these models provide good fit to the female marriage migration with age and income. The observed and fitted values of these models are shown in Fig. (1) to (6) correspondingly.

CONCLUSION

In this study it is found, on the one hand, that four parameters 3rd degree polynomial model is fitted for the distribution of female marriage migration associated with age in Rajshahi district, Bangladesh. On the other hand, distribution of female marriage migration associated with income follow two parameters positive exponential model.

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REFERENCES
