PI-Servo with State-D Feedback Control for LTI Systems

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Abstract: This study proposes a new PI-servo with state-D feedback control design method for linear time-invariant systems of type-0. The method aims for simultaneous command following and disturbance rejection control objectives. The main theorem with proof and design procedures are presented. Two numerical examples serve to demonstrate the advantages of using the proposed method compared with the classic Ogata’s method.

Keywords: PI-servo, state-D feedback, type-0 LTI systems

INTRODUCTION

Control design via state-variable method has well established for many years. Many design approaches are available as described by Ogata (2002a), for instance. For LTI systems, the pole-placement method has been a fundamental design approach, primarily gain feedback through state(s) and output(s). The emerging concept of state-derivative (state-D) feedback was explained in the context of geometry Lewis and Syrmos (1991). Recently, stabilization and disturbance rejection via pole-placement using state-D feedback have been presented (Abdelaziz and Valasek, 2003, 2004; Moreira et al., 2010). The state-D feedback is fundamentally practical and has an advantage over the conventional state feedback in that it results in smaller gains. A practical example is that a derivative signal can be derived from an accelerometer output in a vibration control system (Kwak et al., 2002a; Reithmeier and Leitmann, 2003b). In studies such as Abdelaziz and Valasek (2005a), Duan et al. (2005b) and Kwak et al. (2002b), LQRs to achieve the state-D feedback were reported. A simple design based-on conventional state feedback to achieve the state-D feedback is also possible (Cardim et al., 2007). Other results of the pole-placement for multivariable systems with state-D feedback can be found in papers such Araujo et al. (2009a), Abdelaziz (2010, 2009, 2007), Abdelaziz and Valasek (2005 b, c) and Faria et al. (2009). Results on state-PID feedback for LTI systems have recently been reported by Sujitjorn and Wiboonjareon (2011). It is noticed that most of these results are aimed for stabilization; state regulation and disturbance reject objectives, not command following or servo objective. Servo design for LTI systems using state-variable approach is possible, for instance the materials found in Ogata (2002a) and Puwani et al. (2006). Design approaches for plants of type-0 and -1 are somewhat different. For a type-1 plant, the closed-loop system naturally exhibits no steady-state errors. In contrast, a type-0 plant needs an additional integral element in the forward path to achieve the same.

This study proposes a new design method for type-0 PI-servo system with state-D feedback controller by using the pole-placement approach. An integrator is augmented to the system so that the system will exhibit no steady-state errors in the response to step input. In order to apply the proposed method, the mathematical model of the system must be firstly linearized and converted into Frobenius canonical form. Then feedback gain matrix and proportional-integral gains can be obtained. The satisfied performances of the system controlled by the proposed controller are shown by simulations.

PROBLEM DESCRIPTION

A linear time-invariant dynamical system having single input and single output is completely controllable and described by

\[ \dot{x} = Ax + Bu, \quad x(t_0) = x_0 \]  
(1)

\[ y = Cx \]  
(2)

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) is the scalar control input, \( A(n \times n) \) is the system matrix, \( B(n \times 1) \) is the control gain vector, \( C(1 \times n) \) is the output matrix and \( y \) is the output. The system characteristic polynomial resulted from the A matrix is represented by:
The system states are fed back through the gain matrix $K_d$ and the error signal, i.e., the difference between the reference input ($r$) and the output ($y$), is fed to the proportional-integral (PI) controller. This error signal is denoted as $\xi$. The block diagram shown in Fig. 1 represents the control system. Correspondingly, Eq. (4) - (5) expresses the control signal and the error signal, respectively:

$$u = -K_d \dot{x} + k_p \dot{\xi} + k_i \xi$$  \hspace{1cm} (4)

$$\dot{\xi} = r - y = r - Cx$$  \hspace{1cm} (5)

where, $\xi$ is the output of the integrator, $k_p$ and $k_i$ are controller parameters. The design problem is to find the gains $k_p$ and $k_i$, the matrix $K_d$. To do this, the original system needs to be transformed into the Frobenius canonical form designated by:

$$F = Tx, \quad x = T^{-1}F$$  \hspace{1cm} (6)

in which $F(t)(n \times 1)$ is the transformed state vector, $T(n \times n)$ is the transformation matrix, $A_d(n \times n)$ is the transformed system matrix and $B_d(n \times 1)$ is the transformed control gain vector. The matrices $A_d$ and $B_d$ can be simply obtained from (7):

$$A_d = TAT^{-1}, \quad B_d = TB$$  \hspace{1cm} (7)

where,

$$T = [q \ qA ... qA^{n-1}]^T$$  \hspace{1cm} (8)

$$q = h^T M^{-1}$$  \hspace{1cm} (9)

$q$ is a $(1 \times n)$ vector. $M$ is the controllability matrix of the system (1) and is expressed by:

$$M = [B \ AB \ A^2B ... A^{n-1}B]$$  \hspace{1cm} (10)

$M$ has rank $n$ and $h = [0 \ 0 ... 1]^T$ is a unit vector. One can obtain the Frobenius form of the system (1) as,

$$F = A_d F + B_d u, \quad \text{i.e.,}$$

$$F = \begin{bmatrix}
0 & 1 & 0 & ... & 0 \\
0 & 0 & 1 & ... & 0 \\
... & ... & ... & ... & ... \\
0 & 0 & 0 & ... & 1 \\
-a_n & -a_{n-1} & ... & -a_0
\end{bmatrix}
F + 0 h$$  \hspace{1cm} (11)

**MAIN RESULTS**

Referring to the system in Frobenius canonical form, the control signal for the state-D feedback can be written as:

$$u = -K_F \hat{F}$$  \hspace{1cm} (12)

in which $K_F = [k_1 \ k_2 ... k_n]$ and $K_d = K_F T$. Therefore, $F = A_d F - B_d K_d \hat{F}$ represents the system with the inner feedback loop. It is characterized by $\Delta_d(s) = \det[s(I + B_d K_d) - A_d]$, which is desired to be $\Delta_d(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + ... + \alpha_1 s + \alpha_0$, where $\alpha_0 = a_0$ and $a_0 \neq 0$. For an $n$ order system, the characteristic polynomial is expressed as:

$$\Delta_d(s) = (1 + k_n) s^n + (a_{n-1} + k_{n-1}) s^{n-1} + ... + (a_1 + k_1) s + a_0$$

Consider the system (1) subject to the pole-placement by the state-D feedback and PI-servo control having the control $u$ as in (4), the system can be described by:

$$\dot{x} = Ax + B \left[-K_d \dot{x} + k_p \dot{\xi} + k_i \xi\right]$$

$$\dot{x} = Ax - BK_d \dot{x} + Bk_p \dot{\xi} + Bk_p \dot{\xi}$$

$$\dot{x} = Ax - BK_d \dot{x} + Bk_p \xi + Bk_p (r - Cx)$$
\[ \dot{X} = (I + BK_d)^{-1}AX - (I + BK_d)^{-1}Bk_p \]
\[ Cx + (I + BK_d)^{-1}Bk_p + (I + BK_d)^{-1}Bk_p \]

Assume that the reference input is a unit-step function, for \( t > 0 \) the dynamical representation of the system in (5) and (13) can be rewritten in a matrix form as follows:

\[
\begin{bmatrix}
\dot{X}(t) \\
\dot{Z}(t)
\end{bmatrix} =
\begin{bmatrix}
(I + BK_d)^{-1}A - (I + BK_d)^{-1}BK_dC(I + BK_d)^{-1}Bk_p \\
(I + BK_d)^{-1}Bk_p
\end{bmatrix} x(t) - x(\infty)
\]

For an asymptotically stable system, \( y(\infty) = r; \xi(\infty), x(\infty) \) and \( u(\infty) \) must converge to constant levels including zero.

Therefore, at steady-state:

\[ \dot{x}(\infty) = -C \]
\[ \dot{Z}(\infty) = 1 \]

and (16) follows:

\[
\begin{bmatrix}
\dot{x}(t) & \dot{Z}(t)
\end{bmatrix} =
\begin{bmatrix}
(I + BK_d)^{-1}A - (I + BK_d)^{-1}BK_dC(I + BK_d)^{-1}Bk_p \\
(I + BK_d)^{-1}Bk_p
\end{bmatrix} x(t) - x(\infty)
\]

or in short form:

\[
\begin{bmatrix}
\dot{x}_c(t) \\
\dot{Z}_c(t)
\end{bmatrix} =
\begin{bmatrix}
(I + BK_d)^{-1}A & 0 \\
(I + BK_d)^{-1}B & C
\end{bmatrix} x_c(t) + \begin{bmatrix}
0 \\
Bk_p
\end{bmatrix} u_c(t)
\]

where \( x_c(t) = x(t) - x(\infty) \), \( \dot{x}_c(t) = \dot{x}(t) - \dot{x}(\infty) \) and

\[ u_c(t) = u(t) - u(\infty) = -k_p C x_c(t) + k_d \dot{x}_c(t) \]

Define \( \dot{e} = \begin{bmatrix} \dot{x}_c(t) \\ \dot{Z}_c(t) \end{bmatrix} \) as an error vector of \( m \) order \((m = n+1)\). Equation (17) can be rewritten as:

\[ \dot{e} = A_1 e + B_1 u_c \]

where, \( A_1 = \begin{bmatrix} \tilde{A} & 0 \\ -C & 0 \end{bmatrix}, B_1 = \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix} \)

and the control signal:

\[ u_c = K_p e \]

where, \( K_p = [-k_p C k_i] \). The error dynamic represented by (18) is characterized by the characteristic polynomial \( \Delta_o(s) = \beta_0 s^n + \beta_{m-1} s^{m-1} + ... + \beta_1 s + \beta_0 \) in which \( \beta_0 = 1 \). From (18) and (19) one can obtain (20) with its associated characteristic polynomial in (21):

\[ \Delta_o(s) = \det[sI - A_1 - B_1 K_p] \]

**Theorem 1:** For a completely controllable LTI system \((1)\) of order \( n \), the state-D feedback to achieve the desired characteristic polynomial \( \Delta(s) = \alpha_1 s^n + \alpha_{n-1} s^{n-1} + ... + \alpha_1 s + \alpha_0 \) employs the control \( u = -K_d x \) where, \( K_d = K_d T \); the PI-servo to achieve the desired characteristic polynomial \( \Delta_d(s) = \beta_0 s^n + \beta_{m-1} s^{m-1} + ... + \beta_1 s + \beta_0 \) in order of the output feedback system, employs the control \( u_e = -K_{pe} e \) where, \( K_p = [-k_p C k_i] \). The gain matrices can be calculated from:

\[ K_d = \begin{bmatrix} \alpha_1 - a_1 & \alpha_2 - a_2 & \ldots & \alpha_{n-1} - a_{n-1} \end{bmatrix} \]

or in short:

\[ K_{pi} = -k_p C k_i \]

**Proof:** \( \hat{F} = A_0 + B_1 [K_d \hat{X}] \) represents the inner closed-loop system in Frobenius form. For an \( n \)-order system, its characteristic polynomial is:

\[ \Delta_d(s) = \det[s(I + B_1 K_d) - A_d] = a_0 + (a_1 + k_i) s^{n-1} + \ldots + (a_{n-1} + k_{n-1}) s + k_n s^n \]

Through the coefficient matching of (24) and the desired polynomial \( \Delta(s) \) one can obtain. \( K_d = [\alpha_1 - a_1, \alpha_2 - a_2, \ldots, \alpha_{n-1} - a_{n-1}, 1]T \). The formula (22) is concluded. For proof of this part see Proposition 2.1 in Suijitoorn and Wiboonjareon (2011).

For the error dynamic (20), \( \Delta_o(s) = \beta_0 s^n + \beta_{m-1} s^{m-1} + ... + \beta_1 s + \beta_0 \) represents the desired characteristic polynomial. From the Cayley-Hamilton theorem, we can write:

\[ \Omega(A) = A^n + \beta_1 A^{n-1} + \ldots + \beta_{m-1} A + \beta_m I = 0 \]
Consider a dynamic of third-order, we can express:

\[ I = I \]
\[ \tilde{A} = A_1 + B_1 K_{\text{pi}} \]
\[ \tilde{A}^2 = (A_1 + B_1 K_{\text{pi}})^2 = A_1^2 + A_1 B_1 K_{\text{pi}} + B_1 K_{\text{pi}} \tilde{A} \]
\[ \tilde{A}^3 = (A_1 + B_1 K_{\text{pi}})^3 = A_1^3 + A_1^2 B_1 K_{\text{pi}} + A_1 B_1 K_{\text{pi}} + \tilde{A} + B_1 K_{\text{pi}} \tilde{A}^2 \]

and (26) follows for \( \beta_3 = 1 \):

\[ \beta_4 I + \beta_4 \tilde{A} + \beta_3 \tilde{A}^2 + \tilde{A}^3 = \varnothing(\tilde{A}) = 0 \]

Due to (25) \( \beta_3 I + \beta_4 A_1 + \beta_4 A_1^2 + A_1^3 = \varnothing(A_1) \neq 0 \), one can write:

\[ \varnothing(A_1) = -B_1 \left( \beta_4 K_{\text{pi}} + \beta_4 K_{\text{pi}} \tilde{A} + B_1 K_{\text{pi}} \tilde{A}^2 + \beta_4 A_1 K_{\text{pi}} + A_1 B_1 K_{\text{pi}} + \beta_4 A_1^2 K_{\text{pi}} \right) \]

Since \( \varnothing(\tilde{A}) = 0 \):

\[ \varnothing(A_1) = -B_1 \left( \beta_4 K_{\text{pi}} + \beta_4 K_{\text{pi}} \tilde{A} + B_1 K_{\text{pi}} \tilde{A}^2 + \beta_4 A_1 K_{\text{pi}} + A_1 B_1 K_{\text{pi}} + \beta_4 A_1^2 K_{\text{pi}} \right) \]

Equation (27) can be rewritten as:

\[ -[B_1 A_1 A_1^2 B_1]^{-1} \varnothing(A_1) = \begin{bmatrix} \beta_4 K_{\text{pi}} + \beta_4 K_{\text{pi}} \tilde{A} + B_1 K_{\text{pi}} \tilde{A}^2 \\ \beta_4 A_1 K_{\text{pi}} + A_1 B_1 K_{\text{pi}} + \beta_4 A_1^2 K_{\text{pi}} \end{bmatrix} \]

Multiplying both sides of (28) by \([0 0 1]\), we obtain:

\[ -[0 0 1] [B_1 A_1 A_1^2 B_1]^{-1} \varnothing(A_1) = \]

\[ -[0 0 1] \begin{bmatrix} \beta_4 K_{\text{pi}} + \beta_4 K_{\text{pi}} \tilde{A} + B_1 K_{\text{pi}} \tilde{A}^2 \\ \beta_4 A_1 K_{\text{pi}} + A_1 B_1 K_{\text{pi}} + \beta_4 A_1^2 K_{\text{pi}} \end{bmatrix} = K_{\text{pi}} \]

Therefore, the formula (23) is concluded for the system of an m-order. This completes the proof.

The followings are design procedures:

**Step 1:** Transform the system (1) in Frobenius form using Eq. (6) - (8).

**Step 2:** Assign poles for the inner loop of state-D feedback and express the desired characteristic polynomial of the form \( \Delta(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0 \).

**Step 3:** Calculate the gain matrix \( K_{\text{d}} \) using (22).

**Step 4:** Assign poles for the PI-servo loop and express the desired characteristic polynomial of the form \( \Delta_{\text{o}}(s) = \beta_n s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0 \).

**Step 5:** Calculate the gain matrix \( K_{\text{pi}} \) using (23).

The above design procedures are very simple to conduct. They offer an advantage of assuring stability of the inner state-D feedback loop and the PI-servo control. The effectiveness of the proposed method is demonstrated via the following numerical examples of a DC-motor speed control and a mechanical vibration control, respectively. The results are compared with those obtained from using Ogata’s method (Ogata, 2002a).
Numerical examples: This section presents two examples to show the effectiveness of the proposed method in comparison with Ogata’s method. The first example is speed control of a DC motor. Relevant calculations are presented in details in a step-by-step manner. The second example is vibration control of a mass-spring-damper system of 4th-order. Calculations are presented in brief.

Example 1: The diagram in Fig. 2 represents the speed control of a DC motor. The system dynamic can be described by a state Eq. (30):

\[
\begin{bmatrix}
\dot{\omega} \\
\dot{i}_A
\end{bmatrix} = \begin{bmatrix}
\frac{b}{J_A} & \frac{K_T}{J_A}i_A \\
-K_B & \frac{R_A}{L_A}
\end{bmatrix} \begin{bmatrix}
\omega \\
i_A
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{L_A}
\end{bmatrix} V_M
\]

where \(R_A\) is armature resistance (\(\Omega\)), \(L_A\) = Armature inductance (H), \(b\) = Coefficient of viscous friction (Kg-m²/s), \(J_A\) = Armature inertia (Kg-m²), \(K_B\) = Motor voltage constant (V-s/rad), \(K_T\) = Motor torque constant (Nm/A), \(\omega\) = Angular speed of armature shaft (rad/s), \(i_A\) = Armature current (A).

In (30), states \(x = [\omega \ i_A]^T\) and control input \(u = V_M\). Physically, \(V_M\) is a DC voltage fed to the armature circuit. The following parameters of the motor are used: \(R_A = 30\ \Omega, L_A = 0.00169\ H, b = 5.8\times10^{-6}\ \text{kg-m}^2/\text{s}, J_A = 1.06\times10^{-6}\ \text{kg-m}^2, K_B = 0.0283\ \text{V-s/rad}\) and \(K_T = 0.0283\ \text{Nm/A}\) (Oberstar, 2005e). As a result, Eq. (31) describes the motor:

\[
\dot{\hat{x}} = \begin{bmatrix}
-5.4717 & 26698 \\
-16.746 & -17751
\end{bmatrix} x + \begin{bmatrix}
0 \\
591.72
\end{bmatrix} u
\]

Speed of the motor is defined as output and hence the output equation:

\[
y = [1 \ 0] x
\]

Design calculations according to the proposed method are as follows:

Step 1: Transformation of the system (31) results in the Frobenius form of:

\[
\dot{\hat{F}} = \begin{bmatrix}
0 & 1 \\
-5.4421\times10^5 & -17757
\end{bmatrix} F + \begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

with the corresponding characteristic polynomial of:

\[
\Delta_d(s) = (1+k_2) s^2 +(17757+k_1) s+5.4421\times10^5
\]

Step 2: Assign the desired poles of \(-5.5\pm j2\); hence \(\Delta_d(s) = s^2+11s+34.25\) is the desired characteristic polynomial.

Step 3: Calculate the gain matrix \(K_d\) using (22) resulting in \(K_d = \begin{bmatrix} 0.0044367 & 26.851 \end{bmatrix}\). At this stage, we can write the equation describing the error dynamic as:

\[
\dot{\hat{e}} = \begin{bmatrix}
-0.00014985 & 55283 & 0 \\
-5.4717 & 26698 & 0 \\
32847 & 0 & 0.03724
\end{bmatrix} e + \begin{bmatrix}
3.2847\times10^{14} \\
0.003724 \\
0
\end{bmatrix} e_c.
\]

Step 4: Assign the desired characteristic polynomial \(\Delta_d(s) = s^3 + 9.5 s^2 + 25.75 s + 40.625\) to achieve the desired poles of \(-1.5 \pm j 2, -6.5\) for the PI-servo control.

Step 5: Calculate the gain matrix \(K_p\) using (23) resulting in \(K_p = -0.00029415\) and \(k_i = 0.04086\).

Based-on Ogata’s method, one can obtain the gain matrix \(K = \begin{bmatrix}-0.0283 & -29.988\end{bmatrix}\) for state feedback and \(k_i = 3.6925\times10^6\) for I-servo control. Note that the same poles as in Step 4 above were assigned. One can observe that the magnitudes of the state-D feedback gains are smaller than those of the proportional gain feedback and the
magnitudes of the PI-servo gains are somewhat larger than the I-gain of Ogata’s method. Figure 3 illustrates the simulation results of the motor following a unit-step reference input and an external disturbance of 0.1 units occur at 6 s. The response curves indicate that the proposed method provides a smooth control, whilst Ogata’s method renders a tight control with a small overshoot. In terms of rise-time, the response obtained from the proposed method is slower, but it settles in 2.3 s while that obtained from Ogata’s method settles in 4 s. In effect, the proposed method provides a fast response to the reference command. Note that both methods provide about the same figures of disturbance rejection time of 2 s.

**Example 2:** A mechanical system proposed in Ogata (2004b) is now considered. Figure 4 shows the diagram representing this system. The linear model describing the system can be easily obtained as:

\[
\begin{bmatrix}
    0 & 1 & 0 & 0 \\
    -9 & -2 & 6 & 2 \\
    0 & 0 & 0 & 1 \\
    3 & 1 & -3 & -1
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0.5
\end{bmatrix} u = 0 \tag{34a}
\]

\[
y = [1000] x \tag{34b}
\]

In order to achieve the pole locations at \(-10 \pm j2, -15 \pm j2\) for the state-D feedback, the characteristic polynomial \(\Delta_i(s) = s^4 + 50 s^3 + 933 s^2 + 7700 s + 23816\) is specified. As a result of applying (22), the gain matrix \(K_d = [0.21355 -0.90066 -0.16241 -1.9992]\) is obtained. The design proceeded to describe the error dynamic of the PI-servo control results in (35):

\[
\begin{bmatrix}
    0 & 1 & 0 & 0 & 0 \\
    -9 & -2 & 6 & 2 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    -27863 & -19.667 & -788.67 & -48 & 0 \\
    -1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    e_x \\
    e_y \\
    e_u \\
    e_\alpha \\
    e
\end{bmatrix} = 0 \tag{35}
\]

\(\Delta_i(s) = s^5 + 54.8 s^4 + 392.36 s^3 + 8198.6 s^2 + 29753 s + 36040\) is the desired characteristic polynomial to achieve the desired poles at \(-2 \pm j, -0.4 \pm j\), \(-50\) for the PI-servo. By applying (23), we finally obtain:

\[K_p = [-0.40558 -0.8124 0.78067 -0.0036278 4.5398]\]

i.e., \(k_p = 0.40558\) and \(k_i = 4.5398\).

Using Ogata’s method to achieve the same pole locations as above, one can obtain the proportional gains \(K = [6651.9 524.01 -494.5 130.6]\) and the I-servo gain \(k_i = 12013\). Apparently, both gain sets have much larger magnitudes than those resulted from the proposed method. Figure 5 illustrates the response curves due to a unit-step input and an external disturbance of 0.1 units of magnitude. Noticeably, the proposed method provides very satisfactory responses, which contain neither overshoot nor oscillation. Moreover, the command following response settles in 4 s and the system recovers from the external disturbance in 2 s, whilst those obtained from Ogata’s method are highly oscillatory and settle more slowly.

**CONCLUSION**

This study has presented a new control design method via pole-placement approach for type-0 LTI systems. The proposed method incorporates state-D feedback and PI-servo to achieve disturbance rejection and command following control objectives. Calculations
via the proposed design procedures are very simple and effective. Two numerical examples have demonstrated the advantages of using the proposed method over the classic Ogata’s method in that the obtained gains are smaller in magnitude and the closed-loop system responds smoother and faster.

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