Economic Order Quantity Model with Two Levels of Delayed Payment and Bad Debt

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Abstract: The purpose of this study is to determine the optimal retailer's replenishment policies considering the customers' bad debt and delayed payment in the three-stage supply chain with the dominant retailer. The effect of bad debt is analyzed on the interest earned and interest charged to build the models of the retailer's decision in two cases. By analyzing the model, the retailer's optimal replenishment time and the optimal order quantity are obtained. Furthermore, analyze the effect of parameters on the retailer's optimal order policies. Finally, the numerical analysis is presented to demonstrate the conclusions. The results show that the delayed payment offered by the manufacturer becomes large, the retailer's optimal order cycle and the optimal order quantity increases or remains the same; When the delayed payment time offered by the retailer decreases, the retailer's optimal order cycle and the optimal order quantity increases or remains the same. When the fixed ordering cost is reduced, the retailer's optimal order cycle and the optimal order quantity decreases or remains the same. When the charged interest is greater than the earned interest, with the bad debt rate increasing, the retailer's optimal order cycle and optimal order quantity is converged to a certain value.

Keywords: Bad debt, delayed payment, EOQ, the dominant retailer

INTRODUCTION

Nowadays, the retailers have more bargaining power than the manufacturers in some supply chains. For example, in China market, SUNING and GOME have almost controlled the home appliances market; and in the medium-sized cities of the food and commodity markets, most of the market share is dominated by the Wal-Mart and Carrefour. Therefore, the strong retailers often required the suppliers to provide a longer extension of the payment period, while the retailers usually provide an extension of pay period to its customers for encouraging the customers' purchase behavior. Because the strong position of the retailers in the supply chain, the delayed payment period provided by the retailers is less than the credit period offered by the manufacturer. For the retailers facing the majority of customers for non-specific, there exist the un-receivable accounts that customers will have bad debt losses.

The delayed payment has been focused on by many scholars. In the traditional Economic Order Quantity model (EOQ), the manufacturer receives payment in full of the purchase price immediately, without taking into the delayed payment period. EOQ model with the delayed payment is proposed by Goyal (1985), which is extended by Chu et al. (1998) for considering the deteriorated products. The situation with the exponential rate of deterioration of the situation deteriorating items is analyzed by Aggarwal and Jaggi (1995). The order strategy is researched by Jamal et al. (1997) and Chang and Dye (2001) allowing the shortage in the case of delayed payment and deteriorating conditions. Teng (2002) further considered the unit selling price is not the same as the unit product costs in the model. The economic production quantity model is analyzed by Chung and Huang (2003) considering the manufacturer to the retailer under the deferred payment policy. In addition, some authors took into account cash discounts of the delayed payment, for example, (Huang, 2003, 2005; Chang, 2002; Ouyang et al., 2002).

As the development of research, there are some papers discussing the two levels of delayed payment. Huang (2003) first extended Goyal's model to analyze the two levels of the trade credit policy, the manufacturer's credit period available to retailers as M and the retailers provide the customers a credit period N, M>N. Qiu et al. (2007) considers the manufacturer to give retailers the delayed payment on purchases, the retailers give the customers a fixed period of delayed payment of the three-stage model of economic order quantity. Huang (2007) analyzes the optimal inventory updating strategy under the two levels of the delayed payment time, but the delayed payment time which is given to the customers by the retailers is less than the delayed payment time which is given to the retailers by the manufacturer. Also, the retailer requires the customers to pay the deposit as the part of the retailing price to get the products.

However, Huang (2007) did not consider that when the retailers provide the customers a credit period N, the retailer does not receive the payment during the time
interval [0, T]; the customers purchase the products at time point T, which will pay the purchasing price at the time point T+N. Therefore, the retailer receives payment during the time interval [T, T+N]. Jinn-Tsair and Chun-Tao (2009) amended the shortcomings of Huang (2007) to relax the constraints I_0 > I_e and M > N, discussing the optimal product quantity in the two levels of the delayed payment time.

The research on the delayed payment above did not consider the bad debt rate of the customer. When facing the non-specific customers, the retailers will have the risk that the accounts receivable can not be recovered. So the retailers solving the economic order quantity, we must consider the losses caused by the bad debts. But the research above in the construction of retailer cost function, does not take into consideration the bad debts to the function model. Qin (2011) have done some research to relax this assumption.

Additionally, the mathematical models proposed in this study are based on the following assumptions:

- The retailer has more power than the manufacturer in the three echo supply chain, therefore, we have M ≤ N.
- The annual demand rate is known and constant.
- The time horizon is infinite.
- The retailing price is equal to the production Cost (Goyal, 1985; Chu et al., 1998). In the extended research, we can relax this assumption.
- The shortages are not allowed because of the time-based competition in today’s market. The manufacturer is offered by the manufacturer a trade credit of M periods to settle the entire purchase cost. On the other hand, the customers are offered by the manufacturer a trade credit of N periods. However, the customers have the credit risk, resulting in the uncollectible account receivables. Uncollectible accounts expense is recorded in the period from N to T+N, in which individual accounts receivable are determined to be worthless. The bad debt rate for the individual accounts is r. By the time T+N, the manufacturer will request the write-off of the uncollectible accounts.

Models: The inventory level I(t) is decreasing to meet the customers demand. If the customers’ demand rate is D, the inventory level over time is:

\[
\frac{dI(t)}{dt} = -D, \quad 0 \leq t \leq T
\]

And meeting the initial conditions I(t) = 0, then we can obtain the solution of Eq. (1):

\[
I(t) = -Dt + DT, \quad 0 \leq t \leq T
\]

The purchasing quantity of the retailer is:

\[
Q = I(0) = DT
\]

The annual cost associated with the retailer is TRC(T)

The ordering cost per year + the purchasing cost per year + the holding cost per year + the interest charged per year = the earned interest per year.

In which: The ordering cost per year equals A/T; the purchasing cost per year equals cD; the holding cost per year (not including the interest) H is as follows:

\[
H = \frac{h}{T} \int_{0}^{T} I(t) dt = \frac{hDT}{2}
\]

The retailer’s annual charged interest and annual earned interest are related to its action strategy. Therefore, we discuss the problem in the following two cases.

Case 1: N ≤ M ≤ T+N

The interest charged per year: Assuming an unlimited supplying rate of the product, the inventory level is shown in Fig. 1a. Without considering the impact of bad debts, the
At time point $M$, the retailer needs financing to pay for not un-receivable money. After time point $M$, the retailer repays the financing money with the sales revenue, hence, the interest expense for the unit cycle $Y_k$ is:

$$Y_k = \frac{I_k S_{ACC}}{T} = \frac{c l D}{2T} (T + N - M)$$

\[ (r+1)T - (1-r)(M - N) \]

If not considering bad debt, it means that $r = 0$.

The interest earned per year: The retailer start to selling the products at the time point 0 and begins to receive the receivable money from its customers at time point $N$. At time point $M$, the retailer pays money to the manufacturer for covering the purchase cost. Therefore, the retailers can invest the receiving money during the time interval $[N, M]$ with the earned interest $I_e$.

Under the influence of bad debts, the earned interest per year is:

$$Y_e = \frac{I_e S_{CAN}}{T} = \frac{c l D (1 - r)(M - N)^2}{2T}$$

when not consider the bad debt rate, it is $r = 0$.

The total annual cost: Under the influence of bad debt, the retailer's total cost is:

Case 2: $N < T + N \leq M$: In case 2, assuming the unlimited products' supplying rate, the inventory level is shown in Fig. 2a. Without considering the impact of bad debts, the retailer's total sales revenue is presented in Fig. 2b. Taking into account the customer's bad debt losses, the retailer's total sales revenue are shown in Fig. 2c. The retailer’s receivable accounts, after delayed payment time $N$ providing to the customers, exists a certain bad debt rate $r$.

By the assumptions we can see that at time $T+N$, the retailer has written off the bad debt losses of this cycle. At the time point $M$, the retailer pays the amount of financing with its own funds, its interest with zero. At time point 0, the retailer begins to sell the products; recover the receivable from the customers at time point $N$; pay the purchase cost to the
manufacturer at the time point $M$. Therefore, the retailers can obtain the interest income at the time interval $[N, M]$ with the earned interest $I_e$. With the bad debt, the earned interest per year is:

$$Y_e = \frac{I_e S_{BDMN}}{T} = \frac{cD(1 - r)(2M - 2N - T)}{2}$$  \hfill (7)

The annual total cost is:

$$TRC_1(T) = \frac{A}{T} + cD + \frac{hD(1 - r)(2M - 2N - T)}{2}$$  \hfill (8)

### MODEL ANALYSIS

**Case: 1 N < M ≤ T+N:** To obtain the optimal solution, take the first order and the second order derivatives of the formula (6) with respect to $T$:

$$\frac{\partial TRC_1(T)}{\partial T} = -A + \frac{hD}{2} + \frac{cD(1 - r)(1 + r)T^2}{2} - \frac{cD(1 - r)(M - N)^2}{2T^2}$$

$$\frac{\partial^2 TRC_1(T)}{\partial T^2} = \frac{1}{T^2}[2A + cD(M - N)^2(1 - r)(I_e - I_a)]$$

If $2A + cD(M - N)^2(1 - r)(I_e - I_a) > 0$, we can obtain $\frac{\partial^2 TRC_1(T)}{\partial T^2} > 0$. It is clearly that $TRC_1(T)$ is strictly convex function in $T$. Therefore, we can obtain the optimal order cycle $T^{*11}$ based on $\frac{\partial TRC_1(T)}{\partial T} = 0$. We can obtain:

$$T^{*11} = \sqrt{\frac{2A + cD(M - N)^2(1 - r)(I_e - I_a)}{hD + cD(1 - r)}}$$  \hfill (9)

The corresponding optimal order quantity $Q^{*11}$ is:

$$Q^{*11} = D T^{*11} = \sqrt{\frac{2AD + cD^2(M - N)^2(1 - r)(I_e - I_a)}{h + cD(1 - r)}}$$  \hfill (10)

But in case 1, the retailer’s optimal order cycle should meet the condition that $M ≤ T^{*11} + N$, therefore, we can get the inequality as following:

$$\Delta_1 = \frac{2A + cD(M - N)^2(1 - r)(I_e - I_a)}{hD + cD(1 + r)} - \frac{hD(M - N)^2}{2I_a + (1 - r)I_e}$$

$$\geq 0$$  \hfill (11)

- If $2A + cD(M - N)^2(1 - r)(I_e - I_a) ≤ 0$, we can get $\Delta_1 < 0$ based on the formula (11).

In case $1N ≤ M ≤ T+N$, we can get:
Therefore, in case 1, when: \(2A+cD(M-N)2(1-r)(I_c-I_1)\geq0\), the function \(TRC_1(T)\) is increasing with \(T\). We can obtain that the optimal order cycle is \(T*_{12} = M-N\) and the optimal order quantity is \(Q*_{12} = D(M-N)\).

Based on the analysis (1) and (2), we can get the Theorem 1.

Theorem 1: in case 1 with \(N \leq M \leq T+N\),

- If \(\Delta_1 \geq 0\) then:
  \[T* = T*_{11} = \sqrt{\frac{2A+cD(M-N)^2(1-r)(I_c-I_1)}{hD+cl_cD(1+r)}}\]
  and \(Q*_{1} = Q*_{11} = Q*_{12} = D(M-N)\)

- If \(\Delta_1 < 0\), then:
  \[T* = T*_{12} = M-N\]
  and \(Q*_{1} = Q*_{12} = D(M-N)\)

Proof: From the analysis above, it is easy to obtain the Theorem 1. Based on which, we can get the retailer’s annual minimized cost \(TRC1(T*_{11})\).

Case 2: \(N < T \leq N \leq M\): To obtain the optimal solution, take the first order and the second order derivatives of the formula (8) with respect to \(T\), which is:

\[
\frac{\partial TRC_2(T)}{\partial T} = \frac{-A}{T^2} + \frac{hD}{2T} + \frac{cl_cD(1+r)}{2T}
\]

\[
\frac{\partial^2 TRC_2(T)}{\partial T^2} = \frac{2A}{T^3} > 0
\]

The function \(TRC_2(T)\) has the minimized value for \(\frac{\partial^2 TRC_2(T)}{\partial T^2} > 0\). Therefore, we can obtain the retailer’s optimal order cycle based on \(\frac{\partial TRC_2(T)}{\partial T} = 0\):

\[
T*_{2} = \sqrt{\frac{2A}{hD+cl_cD(1+r)}}
\]

The retailer’s optimal order quantity is:

\[
Q*_{2} = DT*_{2} = \sqrt{\frac{2AD}{h+cl_c(1+r)}}
\]

And the retailer’s annual minimized cost is \(TRC2(T*_{2})\).

But in case 2, the retailer’s optimal order cycle should meet the condition \(T*_{2}+N \leq M\), so we can get the following inequality:

\[
\Delta_2 - 2A - (hD + cl_cD(1-r))(M-N)^2 \leq 0
\]

Also,

\[
\Delta_1 - \Delta_2 = cD(M-N)^2(1-r) - cl_cD(1+r)(M-N)^2 \geq 0
\]

Analysis of case 1 and case 2: Based on the analysis of case 1 and case 2, we can get the theorem 2:

Theorem 2: when \(M \leq N\),

- If \(\Delta_1 \geq 0\), the retailer’s optimal order cycle is \(T* = T*_{11}\); the optimal order quantity is:
  \[Q* = Q*_{11} = DT*_{11}\]
- If \(\Delta_1 < 0\) and \(\Delta_2 \leq 0\), the retailer’s optimal order cycle is \(T* = T*_{2}\); the optimal order quantity is:
  \[Q* = Q*_{2} = DT*_{2}\]
- If \(\Delta_1 < 0\) and \(\Delta_2 > 0\), the retailer’s optimal order cycle is \(T* = M-N\); the optimal order quantity is:
  \[Q* = D(M-N)\]

Proof:

- If \(\Delta_1 \geq 0\), from formula (15), we can get:
  \[\Delta_2 = 2A - (hD + cl_cD(1-r))(M-N)^2 \leq 0\]

In case 2, \(N \leq T+N \leq M\), for \(\Delta_2 \geq 0\), we can obtain that:
Therefore, in case 2, if $\Delta_2 \geq 0$, the function TRC(T) is decreasing with $T$.

In case 1 $N \leq T+N \leq M$ for $\Delta_1 \geq 0$, based on formula (11), we can obtain the optimal value of TRC(T) is $T^*_1$.

With Case 1 and Case 2 analysis, when $T$ is satisfied $N \leq T+N \leq M$, we can get:

$$\text{TRC}_1(T) = \text{TRC}_2(M-N) = \text{TRC}_1(M-N) \geq \text{TRC}_1(T)^*_1.$$  

Therefore, the Theorem 2(1) has been proved.

- If $\Delta_1 < 0$, in case 1 $N \leq M < T+N$, we can get that:

$$\frac{\partial \text{TRC}_1(T)}{\partial T} = \frac{-A}{T^2} + \frac{hD}{2} \frac{clD(1-r)}{T^2} \left(1+r\right) - \frac{\left(M-N\right)^2}{2T^2} \leq 0.$$  

Therefore, in case 1, the function TRC(T) is decreasing with $T$. In case 2, for $\Delta_2 \leq 0$, from formula (14), we can get $T^*_2$, the optimal solution of TRC(T).

Based on the analysis of case 1 and case 2, we can know that when $T$ meets $N \leq M < T+N$, then:

$$\text{TRC}_1(T) \geq \text{TRC}_1(M-N) = \text{TRC}_2(M-N) \geq \text{TRC}_2(T^*_2).$$  

Therefore, the theorem 1(2) is proved. If $\Delta_1 \leq 0$ and $\Delta_2 \leq 0$, the retailer’s original order cycle is $T^* = T^*_2$, and the optimal order quantity is $Q^* = DT^*_2$.

- Theorem 1(1) and 2(2) shows that in case 1 with $N \leq M < T+N$, if $\Delta_1 < 0$, the function TRC(T) is increasing with $T$; in case 2, $N < T+N \leq M$, if $\Delta_2 > 0$, $\text{TRC}_2(T)$ is decreasing with the $T$.

Therefore, when $\Delta_1 < 0$ and $\Delta_2 > 0$, the retailer’s original order cycle is $T^* = M-N$ and the optimal order quantity is $Q^* = D(M-N)$.

Order quantity are increasing with $M$ or unchanged; when the value of $N$ is gradually decreases, the retailer’s original order cycle and the optimal order quantity become larger or unchanged; when the value of $A$ is decreasing, the retailer’s original order cycle and the order quantity is decreasing or unchanged; when $I_1 \geq I_2$, with the value of $r$ increasing, the retailer’s original order cycle is increasing or decreasing to $(M-N)$ and keep unchanged.

In the traditional EOQ model, not considering the delayed payment and the bad debt, the retailer and the customer both pay the money instantly and $r = 0$. Therefore, the traditional EOQ model is the special situation of case 1 with $M = N = r = 0$. So it means $Q^* = 0 = \sqrt{2AD/h + clk}$.

**Theorem 3:** when $M \geq N$ and

$$2A+cD(M-N)^2(1-r)(I_k-I_i) > 0$$

- If $I_1(1-r) < I_k$, then $Q^*_2 > Q^*_0$
- If $I_1(1-r) > I_k$, then $Q^*_2 < Q^*_0$
- If $I_1(1-r) = I_k$, then $Q^*_2 = Q^*_0$

**Proof:**

- When $I_1(1-r) < I_k$:

$$Q^*_i = DT^*_i = \sqrt{\frac{2AD}{h+clk(1-r)}} < \sqrt{\frac{2AD}{h+clk}} = Q^*_0.$$  

Similarly, we can prove the (2) and (3).

- When $2A+cD(M-N)^2(1-r)(I_k-I_i) > 0$, then we get that:

$$Q^*_i = \sqrt{\frac{2AD}{h+clk(1-r)}}.$$  

Similarly, Theorem 3 shows that when the earned interest is higher than the charged interest, the retailers will take advantage of the upstream extension of the payment period; the economic order quantity is less than the traditional economic order quantity.
NUMERICAL ANALYSIS

To illustrate the feasibility of the models and theorems, we illustrate with the numerical analysis following. Assume the retailer makes its decision about the order strategy of the products: \( A = 200 \) dollars per time, \( D = 1500 \) units per year, \( c = 25 \) dollars per year, \( h = 15 \) dollars per year, \( I_e = 0.09 \), \( I_k = 0.05 \), \( M = 20/360 \) per year, \( N = 15/360 \) per year, \( r = 0.01 \).

Calculate to obtain \( \Delta_1 > 0 \) and \( \Delta_2 > 0 \)

\[ T_{*11} = 0.1192, \quad T_{*2} = 0.1244 \]

the retailer's minimixed cost is 40781; the optimal order cycle is \( T^* = 0.1192 \); the optimal order quantity.

\[ Q^* = 178.8 \]

Now, with the adjustment of model parameters \( M, N, A \), the retailer's optimal decisions are shown in Table 2, 3, 4 and 5. Through the numerical analysis, it is easier to understand the influence of the parameters on the retailer's optimal order cycle and the optimal order quantities.

<table>
<thead>
<tr>
<th>Table 2: The retailer's optimal decisions with the adjustment of parameter ( M )</th>
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<tbody>
<tr>
<td>( M )</td>
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<td>35/360</td>
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<tr>
<td>30/360</td>
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<tr>
<td>25/360</td>
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<td>15/360</td>
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<th>Table 3: The retailer's optimal decisions with the adjustment of parameter ( N )</th>
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<tr>
<td>( N )</td>
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<tr>
<td>15/360</td>
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<tr>
<td>10/360</td>
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<tr>
<td>5/360</td>
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<td>0</td>
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<th>Table 4: The retailer's optimal decisions with the adjustment of parameter ( A )</th>
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<td>( A )</td>
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<td>250</td>
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<td>200</td>
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<td>100</td>
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<th>Table 5: The retailer's optimal decisions with the adjustment of parameter ( r )</th>
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<td>( r )</td>
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<td>0.05</td>
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<td>0.10</td>
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CONCLUSION

In the retailer dominant supply chain with two levels of delayed payment, the optimal decisions of the retailer are obtained considering the bad debt. The retailer's cost function is established in two cases. By calculating the functions, obtain the optimal order cycle and the optimal order quantity.

The study found that when the delayed payment period offered by the manufacturer \( M \) becomes large, the retailer's optimal order cycle and the optimal order quantity increases or remains the same; when the retailer giving to the customers extended payment period \( N \) decreasing, the retailer's optimal order cycle and the optimal order quantity increases or remains the same. When the fixed ordering cost \( A \) is reduced, the retailer's optimal order cycle and the optimal order quantity decreases or remains the same. When the charged interest is greater than the earned interest, with the bad debt rate increasing, the retailer's optimal order cycle and optimal order quantity is converged, respectively, in \( (M-N) \) and \( D \) \( (M-N) \). Therefore, the results not only have a certain theoretical significance, but also can better guide decision-making for the retailer.

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REFERENCES


Qin, J., 2011. Optimal replenishment policies with the customer’s bad debt under delayed payment time. The International Conference on Management Science and Intelligent Control, Aug 24-26, Anhui, China.