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Optimization of Shallow Foundation Using Gravitational Search Algorithm

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Abstract: In this study an effective method for nonlinear constrained optimization of shallow foundation is presented. A newly developed heuristic global optimization algorithm called Gravitational Search Algorithm (GSA) is introduced and applied for the optimization of foundation. The algorithm is classified as random search algorithm and does not require initial values and uses a random search instead of a gradient search, so derivative information is unnecessary. The optimization procedure controls all geotechnical and structural design constraints while reducing the overall cost of the foundation. To verify the efficiency of the proposed method, two design examples of spread footing are illustrated. To further validate the effectiveness and robustness of the GSA, these examples are solved using genetic algorithm. The results indicate that the proposed method could provide solutions of high quality, accuracy and efficiency for optimum design of foundation.

Key words: Gravitational search algorithm, minimum cost, optimization, shallow foundation

INTRODUCTION

A shallow foundation is designed for a building column in order to safely transmit the structural load to the ground without exceeding the bearing capacity of the ground and causing excessive settlements. A foundation design should address at least three basic requirements: geotechnical requirements, structural requirements and economic. The designed foundation must be safe from failure of its structural components and the surrounding geomaterial. It is mandatory that designs should satisfy both the geotechnical and structural requirements and, indeed, fulfillment of these requirements separates acceptable designs from unacceptable ones. The essentials of good engineering design hinges on the third requirement, economic. Several studies have been undertaken to develop methodologies for the analysis of foundations. However, limited work has been undertaken to develop methods for their economic design (Wang and Kulhawy, 2008; Wang, 2009; Khajehzadeh *et al*., 2011). Such an attempt has been made here. Traditional methods for design of spread foundation are based on trial and error approach, in which a trial design is proposed and is checked against the geotechnical and structural requirements, which is followed by revision of the trial design, if necessary. Moreover, there is no guarantee that final design is an economically design. While, in case of optimum design all requirements are considered simultaneously and there is a guarantee that the final design is optimized economically.

The conventional optimization technique is called a gradient algorithm, which is based on gradient information of the objective function and constraints. These methods are applicable mainly to continuous functions and are limited by the presence of the local minimum. In view of the limitations of the conventional optimization methods another kind of optimization techniques, known as heuristic algorithms, are not restricted in the aforementioned manner. The heuristic optimization algorithms do not require the objective function to be derivable or even continuous and can be employed directly on the fitness function to be optimized. These techniques have been successfully applied in treating various continuous and discrete optimization problems arising in various fields. However, there are only limited uses of these methods in optimum design of geotechnical structures.

Be different with other heuristic optimization algorithm based on swarm behaviors, such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA) is a newly developed heuristic optimization method based on the law of gravity and mass interactions (Rashedi *et al*., 2009). GSA is characterized as a simple concept that is both easy to implement and computationally efficient. The method has been confirmed higher performance in solving various

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nonlinear functions, compared with some well-know search methods (Rashedi *et al*., 2009).

In this study, the gravitational search algorithm is proposed to determine the optimum design of shallow foundation. The objective function considered is taken as the construction cost of the foundation, and design is based on ACI (2005). This function is minimized subjected to the structural and geotechnical design constraints. A numerical example is presented in order to illustrate the performance of the present algorithm.

Gravitational search algorithm: Gravitational Search Algorithm (GSA) is a newly developed stochastic search algorithm based on the law of gravity and mass interactions (Rashedi *et al*., 2009). In this approach, the search agents are a collection of masses which interact with each other based on the Newtonian gravity and the laws of motion, in which the method is completely different from other well-known population-based optimization method inspired by the swarm behaviors. In GSA, agents are considered as objects and their performance are measured by their masses. All of the objects attract each other by the gravity force, while this force causes a global movement of all objects towards the objects with heavier masses (Rashedi *et al*., 2009). The heavy masses correspond to good solutions of the problem. In other words, each mass presents a solution, and the algorithm is navigated by properly adjusting the gravitational and inertia masses. By lapse of time, the masses will be attracted by the heaviest mass which it presents an optimum solution in the search space.

To describe the GSA, consider a system with N agents (masses), the position of the agent i is defined by:

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^n) \text{ for } i = 1, 2, \ldots, N \tag{1}$$

where x_i^d presents the position of the agent i in the dimension d and n is the search space dimension.

After evaluating the current population fitness, the mass of each agent is calculated as follows:

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{2}$$

where,

$$m_i(t) = \frac{fit_i(t) - Worst(t)}{best(t) - Worst(t)} \tag{3}$$

where $fit_i(t)$ represent the fitness value of the agent i at time t. best(t) and worst(t) are the best and worst fitness of all agents, respectively and defined as follows:

$$\begin{aligned} best(t) &= \min_{j \in \{1,\ldots,N\}} fit_j(t) \\ Worst(t) &= \max_{j \in \{1,\ldots,N\}} fit_j(t) \end{aligned} \tag{4}$$

To evaluate the acceleration of an agent, total forces from a set of heavier masses applied on it should be considered based on a combination of the law of gravity according to:

$$F_i^d(t) = \sum_{j \in kbest, j \neq i} rand_j \, G(t) \frac{M_j(t) \times M_i(t)}{R_{i,j}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{5}$$

where $rand_j$ is a random number in the interval [0, 1], $G(t)$ is the gravitational constant at time t, M_i and M_j are masses of agents i and j, ϵ is a small value and $R_{i,j}(t)$ is the Euclidean distance between two agents, i and j. *kbest* is the set of first K agents with the best fitness value and biggest mass, which is a function of time, initialized to K_0 at the beginning and decreased with time. Here K_0 is set to N (total number of agents) and is decreased linearly to 1.

By the law of motion, the acceleration of the agent i at time t, and in direction d, $a_i^d(t)$, is given as follows:

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j \in kbest, j \neq i} rand_j \, G(t) \frac{M_j(t)}{\left\| X_i(t), X_j(t) \right\|_2 + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{6}$$

Finally, the searching strategy on this concept can be described by following equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{7}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{8}$$

where x_i^d, v_i^d and a_i^d represents the position, velocity and acceleration of ith agent in dth dimension, respectively. $rand_i$ is a uniform random variable in the interval [0, 1]. This random number is applied to give a randomized characteristic to the search.

It must be pointed out that the gravitational constant $G(t)$ is important in determining the performance of GSA and is defined as a function of time t:

$$G(t) = G_0 \times \exp\left(-\beta \times \frac{t}{t_{max}}\right) \tag{9}$$

where G_0 is the initial value, β is a constant, t is the current iterations, t_{max} is the maximum number of iteration. The parameters of maximum iteration t_{max}, population size N, initial gravitational constant G_0 and constant β control the performance of GSA (N, G_0, β and t_{max}).

According to the description above, the whole workflow of the gravitational search algorithm is as follows:

Step 1: Define the problem space and set the boundaries, i.e., equality and inequality constraints
Step 2: Initialize an array of masses with random positions
Step 3: Check if the current position is inside the problem space or not. If not, adjust the positions so as to be inside the problem space
Step 4: Evaluate the fitness value of agents
Step 5: Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for $i = 1,2,. . ., N$
Step 6: Calculation of the total force in different directions and acceleration for each agent based on Eq. (5) and Eq. (6)
Step 7: Update the velocities according to Eq. (7)
Step 8: Move each agent to the new position according to Eq. (8) and return to Step 3
Step 9: Repeat Step 4 to Step 8 until a stopping criteria is satisfied

In (Rashedi *et al.*, 2009), GSA has been compared with some well known heuristic search methods. The high performance of GSA has been confirmed in solving various nonlinear functions. As an excellent optimization algorithm, GSA has the potential to solve a broad range of optimization problems. In this paper, the method is applied for optimization of shallow foundation.

Shallow foundation optimization: The supporting part of a structure generally is referred to as the foundation. The function of the foundation is to transfer the load of the structure to the soil on which it is resting. Depending on the structure and soil encountered, various types of foundations generally classified as deep or shallow foundations are used. Shallow foundations transfer structural loads to the soil very near the surface, while deep foundations transfer to a subsurface layer or a range of depths. Shallow foundations include spread footing foundations, mat-slab foundations, and slab-on-grade foundations. A foundation must be designed to satisfy strength requirements (such as bearing capacity to avoid catastrophic failure), serviceability requirements (such as excessive settlement to avoid undesirable performances) and economic requirements. The main objective of the current study is to propose an effective method for optimization of spread shallow foundation.

In optimal design problem of spread foundation the aim is to minimize the construction cost of the foundation under constraints. This optimization problem can be expressed as follows:

$$\begin{aligned} &\text{Minimize:} && f(X) && \\ &\text{Subject to:} && g_i(X) \le 0 && i = 1, 2, ..., p \\ & && h_i(X) = 0 && j = 1, 2, ..., m \\ & && L_k \le X_k \le U_k && k = 1, 2, ..., n \end{aligned} \tag{10}$$

where $f(X)$ is the objective function $g_i(X)$, $h_j(X)$ are inequality and equality constraints respectively and L_k,U_k are lower and upper bound constraints.

Similar to other heuristic optimization methods, the GSA algorithm was developed to solve unconstrained optimization problems. A number of approaches have been taken in the evolutionary computing field to do constraint handling. However, the penalty function method has been the most popular constraint-handling technique due to its simple principle and ease of implementation. Penalty methods add a penalty to the objective function to decrease the quality of infeasible solutions. In the current study, penalty function defined by the following equation is used:

$$F\,(f\,X) = (X) + r.\sum max[0,\, g(X)]^2 + \text{r}.\sum |\,h(X)|^2 \tag{11}$$

where $f(X)$, $g(X)$ and $h(X)$ are objective function, inequality and equality constraints which according to Eq. (10) and r is a penalty factor.

A description of design variables, design constraints and objective function to economic design of footing are presented in the following sections.

Design variables: Figure 1 shows the cross-section of a spread foundation. As it is shown in this figure, the eight design variables are include; length of foundation (X_1), width of foundation (X_2), thickness of foundation (X_3), depth of embedment (X_4), long direction reinforcement (X_5) and short direction reinforcement (X_6).

Design constraints: According to the Bowles (1982) and ACI (2005), the design constraints may be classified as geotechnical and structural requirements. These requirements represent the failure modes as a function of the design variables.

The geotechnical constraints are imposed as:

Settlement of foundation: Settlement of foundation should be within a permissible limit according to the following inequality:

$$\delta \le \delta_{all} \tag{12}$$

where δ_{all} is allowable settlement and δ is immediate settlement of foundation (Budhu, 2006).

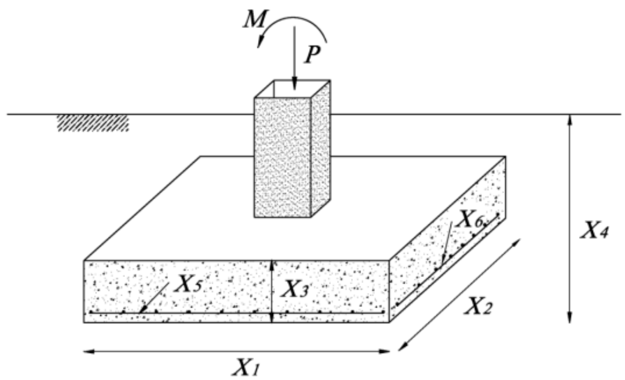

Fig. 1: Cross section of reinforced spread foundation

Bearing capacity: The bearing pressure is the contact pressure between the bottom of a foundation and the underlying soil. To have a safe design the imposed stress should be less than safe bearing capacity of soil as follows:

$$q_{\max} \le \frac{q_{ult}}{FS} \tag{13}$$

in which q_{ult} is ultimate bearing capacity of foundation on cohesion less soil for eccentric loading (Bowles, 1982) and *FS* is factor of safety.

Structural constrains requirements for the spread foundations consist of:

Punching shear failure mode: To avoid such a failure, the upward ultimate shearing force V_u increased by applying the strength reduction factor f_V must be lower than the nominal punching shear strength according to Eq. (14).

$$\frac{V_u}{\phi_V} \le \min\left\{(1+\frac{2}{\beta_c})/6, \left(\frac{a_s d}{b_0}+2\right)/12, \frac{1}{3}\right\} \times \sqrt{f_c'}\, b_0 d \tag{14}$$

where: b_0 = perimeter of critical section taken at $d/2$ from face of column, d = depth at which tension steel reinforcement is placed, β_c = ratio of long side to short side of column section and α_s = 40 for interior columns.

One way shear failure mode: In order to avoid such a failure, Eq. (15) must be satisfied. V_u is the upward ultimate shearing force and f_V is taken to be 0.85. V_c is taken in accordance with the ACI (2005) as:

$$\frac{V_u}{\phi_V} \le \frac{1}{6} \times \sqrt{f_c'}\, bd \tag{15}$$

Flexure failure mode: A moment is acting at the face of column must be lower than or equal to the nominal strength of the concrete section having an effective depth d, a width b, and reinforced with tension steel A_s. Thus:

$$M_u / \Phi_M \le M_n \tag{16}$$

Minimum steel reinforcement: The minimum steel reinforcement placed in structural slabs of uniform thickness to be (ACI, 2005):

$$A_{smin} = 0.002\, X_1 X_3 \tag{17}$$

Limitation of depth of embedment: Finally, the depth of embedment (X_4) should be greater than a minimum depth to prevent frost damage and should be limited to a maximum depth to minimize disturbance to adjacent structures. So:

$$0.5 \le X_4 \le 2 \tag{18}$$

Objective function: The total construction cost of the spread foundation is considered as the objective function in the analysis. The cost function may be expressed in the following form:

$$f(X) = P_c V_c + P_e V_e + P_b V_b + P_f A_f + P_s W_s \tag{19}$$

where, P_c, P_e, P_b, P_f and P_s show the unit price of concrete, excavation, backfill, formwork, and reinforcement respectively. In addition, V_c ,V_e and V_b denote the volume of concrete, excavation and backfill, A_f shows the area of formwork and W_s indicates the weight of steel. The main objective function may be obtained by substituting the objective function of Eq. (19) and inequality constraints presented in section 3.2, into Eq. (11).

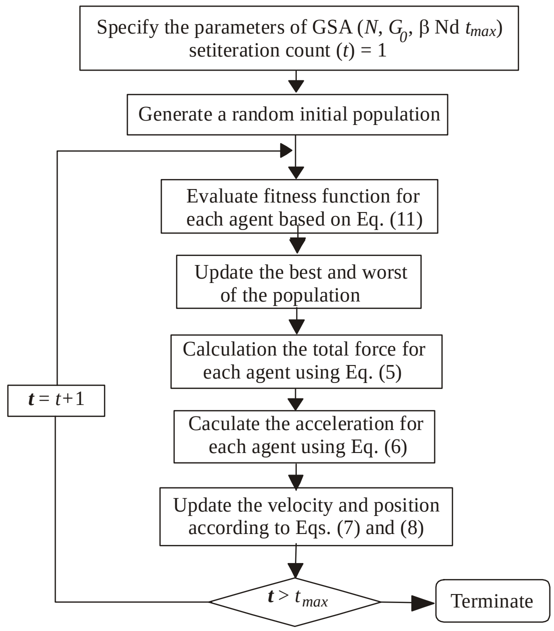

Fig. 2: Optimization of spread foundation using GSA

RESULTS AND DISCUSSION

This section investigates the validity and effectiveness of the proposed algorithm to optimum design of spread foundation. The implementation procedure of the GSA for the economic design of the spread footing is shown as a flowchart in Fig. 2. To verify the good performance of the proposed algorithm, two numerical examples of spread foundations will be solved by the present method. The procedure has been carried out using a computer program was developed in MATLAB. In our study, the GSA algorithm parameters were selected based on experimental studies and also previous literature as follows: population size is 50; maximum iteration number is 500; G_0 = 100; β = 20. The optimization procedure was terminated when the maximum number of iterations is reached. Moreover,
to verify the efficiency of as follows: population size is 50; maximum iteration number is 500; G_0 = 100; β = 20. The optimization procedure was terminated when the maximum number of iterations is reached. Moreover, to verify the efficiency of the proposed algorithm, the result of the GSA is compared with the results of genetic algorithm (GA). For the purpose of optimization by GA the routine from genetic algorithm optimization toolbox of MATLAB is used (The Math works Inc., 2009).

Example 1: The first example presents the optimum design of spread foundation in dry sand. The input parameters for this problem are given in Table 1.

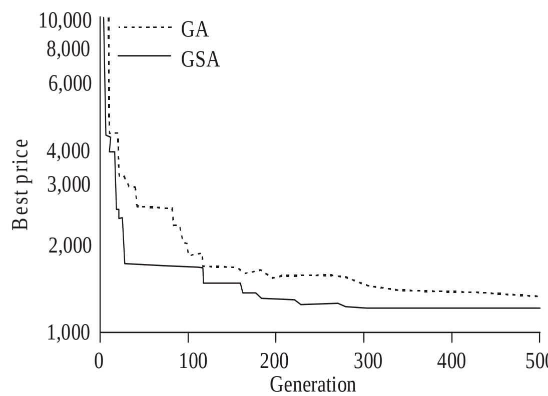

Fig. 3: Convergence rate of the algorithms for example 1

This case is solved using the proposed GSA method. Moreover, the problem is analyzed using genetic algorithm and the results of both methods (GSA and GA) are summarized in Table 2. As it can be seen from the results of Table 2, the minimum price of the foundation obtained by GSA is 1198 USD which is lower than best price obtained by GA (1325 USD) and GSA improved the results more than 9% .

Figure 3 shows the variation in the best price obtained by different methods through the optimization procedure. It is obvious that the proposed GSA converges quickly toward the global optima and requires less iteration and computational time when compared with

Table 1: Input parameters for optimum design of spread foundation

Input parameter	Unit	Input values for example 1	Input values for example 2
Effective friction angle of base soil	degree	32	35
Unit weight of base soil	kN/m^3	16.5	18.5
Young's modulus	Mpa	35	50
Poisson's ratio	-	0.3	0.3
Vertical load (*P*)	kN	2000	4300
Moment (*M*)	kN-m	0.0	600
Concrete cover	cm	7.0	7.0
Yield strength of reinforcing steel	Mpa	140	140
Compressive strength of concrete	Mpa	28	30
Allowable settlement	mm	40	40
Factor of safety	-	3.0	3.0
Shear strength reduction factor	-	0.85	0.85
Flexure strength reduction factor	-	0.9	0.9
Unit price of excavation	m^3	25.16	25.16
Unit price of formwork	m^2	51.97	51.97
Unit price of reinforcement	kg	2.16	2.16
Unit price of concrete	m^3	173.96	173.96
Unit price of compacted backfill	m^3	3.97	3.97

Table 2: Optimization result for spread foundation

Design variable	Unit	Example 1		Example 2	
		GSA	GA	GSA	GA
X_1	m	1.81	1.69	1.35	1.53
X_2	m	2.00	2.40	4.76	5.04
X_3	m	0.42	0.42	0.74	0.78
X_4	m	1.81	1.68	2.00	1.23
X_5	cm^2	76	106	338	341
X_6	cm^2	64	57	43	54
Best price (USD)		1198	1325	2995	3378

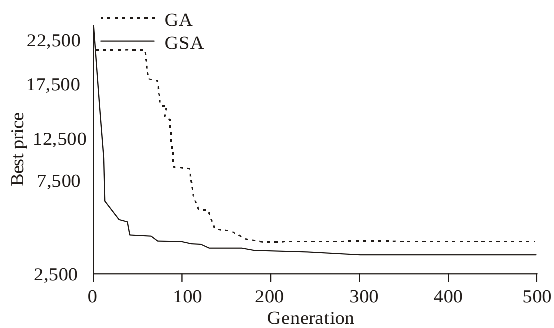

Fig. 4: Convergence rate of the algorithms for example 2

GA. Overall speaking, the proposed algorithm has been shown to perform extremely well for solving economic design of foundation.

Example 2: The second example considers a reinforced spread foundation under an eccentric load in dry sand. Input parameters for this example are shown in the last column of Table 1. This case also solved using both the GSA and GA methods and the optimization results for this case are presented in Table 2. The results show that for the optimization problem considered, the best price evaluated by GSA is 2995 USD which is lower than 3378 USD evaluated by GA and GSA produced 11% cheaper foundation than GA.

Figure 4 presents a performance comparison of two algorithms for the optimum design of the problem considered. As can be seen, in addition to generating superior results, the GSA has a very fast convergence rate in the early iterations and performed significantly better than GA. Therefore, GSA is an effective method for optimization of spread foundations.

CONCLUSION

An effective optimization method proposed to economic design of shallow foundation. The algorithm is based on Gravitational Search Algorithm (GSA). For optimization of foundation the objective function was considered as total construction cost of the foundation. During the optimization procedure, six design variables are treated, which vary within the ranges of geotechnical and structural requirements. A computer program is developed in MATLAB and the user is only required to feed the input parameters like soil and material properties, allowable settlement and safety factors. The effectiveness of the proposed algorithm has been shown through two numerical example of spread foundation. To further validate the accuracy and efficiency of the presented strategy, the results of GSA are compared with the results of GA. For the studies carried out here, it is found that gravitational search algorithm is a suitable technique for optimization of spread foundation and the method is able to find a better optimal solution compared with genetic algorithm.

REFERENCES

ACI Committee 318, 2005. Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05). American Concrete Institute, Farmington Hills, Mich.

Bowles, J., 1982. Foundation Analysis and Design. McGraw-Hill, New York.

Budhu, M., 2006. Soil Mechanics and Foundations. John Wiley & Son, New York.

Khajehzadeh, M., M.R. Taha, A. El-Shafie and M. Eslami, 2011. Modified particle swarm optimization for optimum design of spread footing and retaining wall. J. Zhejiang Univ. Sci. A Appl. Phys. Eng., 12(6): 415-427.

Rashedi, E., H. Nezamabadi-Pour and S. Saryazdi, 2009. GSA: A gravitational search algorithm. Inform. Sci., 179(13): 2232-2248.

The Math works Inc., 2009. Genetic Algorithm and Direct Search Toolbox for use with Matlab user's guide. Version 7.8.0.347, Release R2009a.

Wang, Y., 2009. Reliability-based economic design optimization of spread foundations. J. Geotech. Geoenviron., 135: 954-959.

Wang, Y. and F.H. Kulhawy, 2008. Economic design optimization of foundations. J. Geotech. Geoenviron., 134: 1097-1105.