The Realization of a Discrete-Time Deterministic Regulator for Energy Renewable Multivariable Systems

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Abstract: This study proposes an optimal control law for a linear constant energy renewal multivariable system supplying power to protection and signaling circuits for switchgears and substation operations. The energy storage mechanism creates a control source that dictates the frequency of the power supply voltage. The entire analysis was based essentially on the use of a vector of inputs and outputs, and a matrix of operations characterizing the storage mechanism, assuming the system is both controllable and observable under arbitrary state feedback. The property of the output regulator which was desirable from the viewpoint of feedback control design was clarified.

Key words: Decouplable linear systems, energy renewal multivariable system, optimal discrete regulators, switchgears and substation operations

INTRODUCTION

Electric power generation has rapidly increased in recent years due essentially to the pressure of limited energy resources and environmental policies. When the load demand of the system is less than generation, there is the need to store the excess energy in an efficient system. However, due to the intermittent nature of energy renewal, use of energy storage such as batteries becomes inevitable to compensate for the fluctuations of power when there is shortage of generation. Electric power generation is categorized into generation, transmission and distribution. The system is a complex multi-machine interconnection with voltage and frequency dependent loads; the main components being generating stations, transmission lines and distribution systems. Substations are necessary for change of voltage (up or down) between the generating, transmission, primary distribution and secondary distribution systems, and for providing switching for control of the network under both normal and fault conditions. In cases where no voltage change is involved the substation is called a switching station. Batteries are used as auxiliaries for switchgears and substations (Fig. 1) for supplying power to control protection and signaling circuits. They require a charging system, usually a rectifier which converts a mains (A.C.) supply to D.C. supply. The usual mode is the standby parallel operation, i.e., with mains supply available, the charger unit supplies the D.C. control voltage requirement while at the same time charging the batteries. In the event of mains failure the batteries take over to supply the control voltage requirement.

One of the major features of linear constant multivariable systems is that each input variable affects the output variables. This has led to representations for multivariable systems based on state or output feedback and the use of a vector of inputs, a vector of outputs, and a matrix of operations to characterize the input-outputs relation. It has been shown (Gao and Wang, 2004; Wang, 1970) that for a subclass of decouplable linear systems satisfying the criterion, \( m + \sum_{i=1}^{n} d_i = n \) where \( m \) is the number of inputs and outputs and \( n \) the dimension of the state space, a minimum order realization can be found such that the set \( \{d_i\}; i = 1, ..., m \) defines a necessary and sufficient condition for decoupling the system by output feedback. This condition implies that the system is both controllable and observable under arbitrary state feedback. For this class of systems, it can be easily seen that the set \( \{d_i\}; i = 1, ..., m \) is a complete set of independent invariants, which thus defines a unique canonical form (Silverman and Payne, 1971; Sato and Lopresti, 1971) for finding a control law such that the subsets of the output vector of the closed-loop system are controllable by subsets of the input vector in an independent manner. The possible variations of this approach pose the problem of finding existence of a decoupling control law and characterization of the class of decoupling control law (Majumdar and Choudhury, 1973; Cremer, 1971; Morse and Wonham, 1970), thereby satisfying the necessary conditions for acceptable dynamic response namely, steady state accuracy and asymptotic decoupling (Hirsch and Smith, 2005).
The discrete-time deterministic optimal control problem is well established in the existing literature for linear constant dynamic systems (Hirsch and Smith, 2005; Anderson and Moore, 1968; Shaked, 1976). For this system, the performance criterion is related to the smallness of a norm represented by the relation:

\[ J = \frac{1}{2} \int_{t_0}^{t_1} \left( x^T Q x + u^T R u \right) dt + \frac{1}{2} x^T(t_1) G x(t_1) \]

\[ = \int_{t_0}^{t_1} f_\psi(x,u) dt + \frac{1}{2} x^T(t_1) G x(t_1) \]

\[(1)\]

The requirement here was to determine an optimal control function \( u(t) \), \( t \in [t_0, t_1] \) to minimize \( J \), where the matrices \( Q(t) \), and \( R(t) \) are assumed systematic and nonnegative, and positive definite respectively, and \( G \) is a nonnegative definite matrix.

For optimal solution, the Hamiltonian:

\[ H(x,p,t,u) = \frac{1}{2} \left( u^T R u + x^T Q x \right) + p^T(Ax + Bu) \]

\[ = p^T(t)f(x,u) + f_\psi(x,u) \]

\[(2)\]

was defined, where \( p \), represent the adjoint or costate variables.

In this study, we propose an optimal control law for a linear constant energy renewal system utilizing the degrees of freedom available in the closed-loop input vectors, as well as the associated pant description using state feedback where the process is assumed detectable and controllable.

**MATERIALS AND METHODS**

The energy renewal multivariable system considered was a linear, time-invariant process assumed controllable and represented by the state equations:

\[ \dot{x}(t) = Ax(t) + Bu(t), x(t) \in \mathbb{R}^n \quad (3) \]

\[ \dot{p}(t) = Cx(t), p(t) \in \mathbb{R}^m \quad (4) \]

The process \( S(A,B,C) \) was also assumed to be square since the actual plant outputs and those defined by Eq. (4) need not necessarily coincide (Chow et al., 1989). The matrices \( B \) and \( C \) have full rank since the system has independent inputs and outputs. In addition \( CB \) has full rank, that is, rank \( CB = m \) so that the system is therefore clearly output controllable.

The initial condition was given by:

\[ x(0) = x_0 \]

\[(5)\]
For controllability, it was desired to take the system to a terminal state $x(t_f) = 0$, where $t_f$ is the terminal time. This was done by applying a suitable control $u(t) = f(x, t)$ which is optimal in some sense. Consequently, a quadratic performance criterion of the form:

$$J = \frac{1}{2} \int_0^{t_f} \left[ x^T(t) Q(t)x(t) + u^T(t) R(t)u(t) \right] dt$$  \hspace{1cm} (6)

was obtained and minimized. Without loss of generality, $Q(t)$, $R(t)$ and $S_f$ were assumed to be symmetric matrices. Further $S_f$, $Q(t)$ and $R(t)$ are respectively, positive semi-definite, positive definite matrices with $Q(t)$ and $R(t)$ being continuous in $t$. An appropriate choice of $S_f$, $Q(t)$ and $R(t)$ must be made for obtaining acceptable performance of the system (Krichman et al., 2001; Lehtomaki et al., 1981; Grimble, 1978). To obtain a linear feedback law a cost functional of the form given in (6) was desired to keep the problem mathematically tractable.

For optimal solution, the Hamiltonian:

$$H[x(t), u(t); t] = \frac{1}{2} x^T Q(t)x + \frac{1}{2} u^T R(t)u$$  \hspace{1cm} (7)

was defined, and by the maximum principle of Pontryagin (Krichman et al., 2001) the following was obtained:

$$\frac{\partial H}{\partial u} = 0 = R(t) u(t) + B^T(t) A(t)$$  \hspace{1cm} (8)

$$\frac{\partial H}{\partial x} = \dot{A} = Q(t)x(t) + A^T(t) A(t)$$  \hspace{1cm} (9)

Terminal condition:

$$\dot{A}(t_f) = S_f x(t_f)$$  \hspace{1cm} (10)

Hence from (8),

$$u(t) = -R^{-1}(t)B^T(t) A(t)$$  \hspace{1cm} (11)

Assuming that the solution to $\dot{A}(t)$ has the same structure as (10), $\dot{A}(t)$ was represented by:

$$\dot{A}(t) = S(t) x(t)$$  \hspace{1cm} (12)

Hence:

$$\dot{x} = A(t)x(t) - B(t) - B(t) R^{-1}(t) A^T(t) x(t)$$  \hspace{1cm} (13)

$$\dot{A} = S(t) x(t) - A^T(t) S(t) x(t)$$  \hspace{1cm} (14)

Combining (13) and (14), the following equation was obtained:

$$S(t) x(t) + S(t) x(t) = Q(t)x(t) - A^T(t) S(t) x(t)$$  \hspace{1cm} (15)

The result (15) holds for all nonzero $x(t)$ and hence $S(t)$ is an $n \times n$ symmetric matrix satisfying the matrix Riccati equation.

$$S = -g(t) A(t) + A^T(t) S(t) B(t) R^{-1}(t) B^T(t) S(t) + Q(t)$$  \hspace{1cm} (16)

With the terminal condition:

$$S(t_f) = S_f$$  \hspace{1cm} (17)

the optimal control law becomes:

$$u(t) = -A(t)x(t)$$  \hspace{1cm} (18)

where $K(t)$ is the $n \times n$ feedback gain matrix given by:

$$K(t) = R^{-1}(t) B^T(t) S(t)$$  \hspace{1cm} (19)

The corresponding optimal cost function was obtained as:

$$J^* = \frac{1}{2} x_0^T S(t_0) x_0$$  \hspace{1cm} (20)

An optimal regulator (Xu and Yu, 2006; Tempo et al., 1997) was then required to bring a subset of the system states to given non-zero constant set points. Consequently, we described the linear constant energy renewal multivariable system by the general representation (4). The and matrices were defined from Obinabo (2008) as:

$$\begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ b \end{bmatrix}$$

and respectively using the performance index of the form:

$$J = \int_0^\infty (x^2 + ru^2) dt$$

The model was shown to be reducible to an equivalent interconnection of subsystems, where each representation for a given system has the same input-output relations. The steady state Riccati equation was obtained as follows:
The state equations were obtained as:

\[
\dot{x} = -x + u, \quad x(0) = 1
\]

\[
J = \int_0^\infty \left[ x(t)^2 + b(u(t))^2 \right] dt, \quad b > 0
\]

The state equations were obtained as:

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 \\
    -1 & 0
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    1
\end{bmatrix} u
\]

\[
Q = \begin{bmatrix}
    0 & 1 \\
    0 & 0
\end{bmatrix}, \quad R = b
\]

The steady state Riccati equations were computed as follows:

\[
\begin{bmatrix}
    0 & -1 \\
    1 & 0
\end{bmatrix}
\begin{bmatrix}
    p_1 & p_2 \\
    p_2 & p_3
\end{bmatrix} +
\begin{bmatrix}
    p_1 & p_2 \\
    p_2 & p_3
\end{bmatrix}
\begin{bmatrix}
    0 & 1 \\
    -1 & 0
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
    p_1 & p_2 \\
    p_2 & p_3
\end{bmatrix}
\begin{bmatrix}
    0 & 1 \\
    \frac{1}{b} & 0
\end{bmatrix}
\begin{bmatrix}
    p_1 & p_2 \\
    p_2 & p_3
\end{bmatrix} +
\begin{bmatrix}
    1 & 0 \\
    0 & 0
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
    -p_1 & -p_2 \\
    p_2 & -p_3
\end{bmatrix} +
\begin{bmatrix}
    -p_1 & -p_2 \\
    -p_2 & p_3
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
    p_1^2 & p_2 \\
    p_2 & p_3
\end{bmatrix}
\begin{bmatrix}
    1 & 0 \\
    0 & 0
\end{bmatrix} = 0
\]

\[
-2p_2 - p_3 \frac{2}{b} + 1 = 0
\]

\[
-2p_2 - p_3 \frac{2}{b} - b = 0
\]

\[
2p_2 = p_3 \frac{2}{b} = 0
\]

\[
p_2 = b + \sqrt{b^2 + b}
\]

\[
p_3 = \sqrt{-2b^2 + 2b\sqrt{b^2 + b}}
\]

\[
U = -R^{-1}B^T px = \frac{1}{b}
\begin{bmatrix}
    0 & 1 \\
    1 & 0
\end{bmatrix}
\begin{bmatrix}
    p_1 & p_2 \\
    p_2 & p_3
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\]

\[
= -\frac{1}{b}
\begin{bmatrix}
    p_2 & p_3
\end{bmatrix} x
\]

Using these data, the steady state error between the states and their set points was found to be zero.

**RESULTS AND DISCUSSION**

The common trend in the design of optimal regulators places special premium on some aspects of the performance in order to minimize energy losses. In this study, this was done by quantifying the cost savings using a defined performance index. The controller that minimizes this function gives the optimal solution to the problem. The advantage of the linear quadratic optimal control approach is that if the state feedback system is realizable, the resulting closed-loop configuration known as the optimal regulator is expected to have some desirable sensitivity and robustness properties as in the case of finite-dimensional systems. The work reported in the existing literature (Xu and Yu, 2006; Gorecki et al., 1989) presents fundamental results on the existence and characterization of optimal control by Riccati equations with delay in state. For finite values of \((t_f - t_0)\), the direct solution method was used to compute the matrix Riccati Eq. (12) from the terminal time \(t = t_f\) with the condition \(S(t_f) = S_f\). From the solution \(S(t_f)\), \(K(t)\) was computed and stored for \(t_f \leq t \leq t_0\) and used when required.

The regulator problem is a feedback control system in which the reference input was constant for long duration (Philips and Harbor, 2000), and often for the entire duration of system operation. Succinctly, the regulator corresponds to a terminal controller with \((t_f - t_0) \to \infty\). For general time varying systems it is impractical to solve for, and store \(K(t)\) for \(0 \leq t \leq \infty\) by the direct method considered in this study. For stationary systems and periodic systems the regulator problem was solved by considering the A and B matrices as constant. When \(R\) and \(Q\) were chosen as constant matrices satisfying all the conditions stated earlier, then the matrix Riccati Equation was obtained as (Payne and Silverman, 1973):

\[
S = -S(t)A - A^T S(t) + S(t)BR^{-1}B^T S(t) - Q
\]

As \((t_f - t_0) \to \infty, S = 0 = -SA - A^T S + SBR^{-1}B^T S - BR^{-1}B^T S(t) - Q = S(t) \to S^0\)

where \(S^0\) is a constant matrix. Hence \(S^0\) satisfies the equation:
\[ SA + A^T S - S(BR)^{-1} B^T S + Q = 0 \]  

(22)

which is the steady state algebraic matrix Riccati equation.

**CONCLUSION**

A generalized optimal regulator problem for application to an energy renewal multivariable system supplying power for substation operations was formulated. This enabled optimal regulators for the transient response to be computed for both open- and closed-loop configurations. Conditions were developed for the resulting state feedback to be realizable. The quadratic performance criteria determined for the process is the optimal regulator with the desirable sensitivity and robustness of the form associated with the finite-dimensional linear constant multivariable system. the property of the output regulator which was required for optimal performance was obtained by defining the weighting matrices. This enabled the pole location problem to be brought into the optimal regulator design, and it was shown that if the generalized weighting matrices were chosen appropriately, the generalized optimal regulator problem could be reduced to one which could be solved only by computing solutions to finite-dimensional Riccati equations (Burns, 1990; Gibson, 1983).

**REFERENCES**


