Estimation of Velocity Profile Based on Chiu’s Equation in Width of Channels

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Abstract: Distribution of velocity in channel is one of the most parameters for solution of hydraulic problems. Determination of energy coefficient, momentum and distribution of sediment concentration depend on distribution of velocity profile. The entropy parameter of a channel section can be determined from the relation between the mean and maximum velocities. A technique has been developed to determine a velocity profile on a single vertical passing through the point of maximum velocity in a channel cross section. This method is a way in order to quick and cheap estimating of velocity distribution with high accuracy in channels. So that in this research the power estimation of Chiu method base on entropy concept was determined. Also Chiu’s equation that is based on entropy concept and probability domain, has compared with logarithmic and exponential equations to estimation of velocity profile in width of channel in various depths. The results show that Chiu’s equation better than logarithmic and exponential equations to estimation of velocity profile in width of channel.

Keywords: Channel, Chiu’s equation, entropy, velocity profile

INTRODUCTION

Knowledge of the average velocity distribution active into a natural river cross section assumes a precious role to describe flow resistance and sediment transport processes. To determine the mean velocity and discharge from velocity samples, the conventional method requires a great amount of time and measurements, and, hence, is unsuitable for unsteady flows and (Kalman) filtering schemes used to reduce uncertainties in flow forecasting (Chiu, 1987). Recent studies outline the opportunity to relate the local energy budget to the informational content hold into the point velocity measurements through a velocity distribution profile derived by an entropy-probabilistic approach (Chiu, 1985, 1991, 1995). The regularities are natural laws governing the flows, and their detection can be aided by theoretical analysis. Chiu and Said (1995) reported that the mean value of the ratio of the mean and maximum velocities of flow in a Channel section is constant. This follow-up paper is mainly concerned with the regularity about the maximum velocity in fluid flows, its information content, and potential applications. Civil engineers discovered a long time ago that the maximum velocity in open channel flow often occurs below the water surface and wondered why (Francis, 1878; Stearns, 1883). The velocity distribution has been investigated using deterministic as well as probabilistic approaches. An important probabilistic formulation was developed by Chiu (1987), introducing the formulation of the velocity distribution in the probability domain by considering the random sampling of flow velocity in a channel section. However, as such data are usually not available, Chiu proposed a link between the probability domain and the physical one. He derived possible expressions of the cumulative probability distribution function in terms of the coordinates in the physical space. Estimation of two-dimensional velocity distribution is not always simple and may require as many as six parameters (Chiu and Chiou, 1983). The probability density function of the velocity was then obtained by applying the maximum entropy principle (Chiu, 1987, 1988, 1989). Using this probabilistic formulation, the mean velocity, $u_m$, can be expressed as a linear function of the maximum velocity, $u_{max}$, through a dimensionless entropy parameter M (Chiu, 1991). The M value is a fundamental measure of information about the characteristics of the channel section, such as changes in bed form, slope, and geometric shape (Chiu and Murray, 1992). In this research the power estimation of Chiu method base on entropy concept was determined.

MATERIALS AND METHODS

Entropy velocity profile: The Experiments were carried out in central laboratory of university of Tehran in December 2008. The use of simple relationship able to describe the velocity distribution inside a cross section and based on few synthetic parameters well defined and
derivable became an ambitious target as well as a very useful tool for the researcher and engineer. The aim followed during the derivation of the entropy velocity profile as suggested by Chiu (1987) assume the role of well describing tool of the local energy budget as well as a direct instrument for the evaluation of the average and maximum cross velocity.

The maximum cross velocity plays a primary part into the entropy description of the velocity profile through the relation (Chiu, 1987):

$$\frac{\xi - \xi_0}{\xi_{\text{max}} - \xi_0} = \int_0^\mu F(u) du$$

where $u$ is the local velocity measured in the flow field along a vertical line, $\xi$ is a dimensionless variable depending on the reference system employed for the local representation of the flow field, $\xi_0$ and $\xi_{\text{max}}$ are the value of the dimensionless variable at which correspond the minimum ($u = 0$) and the maximum ($U = U_{\text{max}}$) of the velocity respectively while $p(u)$ is the density probability function. Assuming a probability distribution law as:

$$F(u) = e^{a_1 + a_2 u}$$

with $a_1$ and $a_2$ distribution parameters, and going through simple math and integrating the Eq. (1) the velocity distribution became:

$$\frac{u}{U_{\text{max}}} = \frac{1}{M} \ln \left[ 1 + (e^M - 1) \frac{\xi - \xi_0}{\xi_{\text{max}} - \xi_0} \right]$$

in which $M$ is the dimensionless entropy parameter needed together to the maximum velocity, $U_{\text{max}}$, for the analytical problem closure. The distribution parameters can be obtained by alternative methods allowing three equations into the three unknowns $M$, $U_{\text{max}}$, and $a_i$:

$$\frac{\bar{U}}{U_{\text{max}}} = e^M (e^M - 1)^{-1} - \frac{1}{M}$$
$$U_{\text{max}} e^{a_1} = M (e^M - 1)^{-1}$$
$$\bar{U} e^{a_1} = (M e^M e^{a_1} + 1)(e^M - 1)^{-2}$$

where, $\bar{U}$ is the mean cross velocity.

Particular interest is related to the Eq. (4). The $M$ parameter can be derived by the use of the ratio $\bar{U} / U_{\text{max}}$. Recent studies have shown the possibility to describe the relation between mean and maximum velocities through the linear equation (Xia, 1997; Chiu, 1995):

$$\bar{U} = b U_{\text{max}}$$

where the coefficient $b$ depend on the flow regime. This means that the Eq. (4) can be reprocessed as:

$$b = e^M (e^M - 1)^{-1} - \frac{1}{M}$$

and the solution of the Eq. (8) is related to the evaluation of the $b$ parameter thus on the peculiarity of the flow field.

In the physical space. On the vertical axis Relation between Mean and Maximum Velocity hereinafter, on which the maximum velocity $u_{\text{max}}$ occurs, $\xi$ may be expressed (Chiu and Lin, 1983) as a function of $y$ such as

$$\xi = \frac{y}{D-y} \exp \left( 1 - \frac{y}{D-y} \right) = \frac{y}{D} \exp \left( 1 - \frac{y}{D} \right)$$

in which $y =$vertical distance from the bed.

Relation between mean and maximum velocity: The relation between the mean velocity, $u_m$, and the maximum velocity, $U_{\text{max}}$, can be expressed in Chiu’s form (Chiu and Said, 1995) as:

$$u_m = \Phi(M) U_{\text{max}}$$

in which

$$\Phi(M) = \frac{u_m}{U_{\text{max}}} = e^M (e^M - 1)^{-1} - \frac{1}{M}$$

and $M$ = entropy parameter

Experimental setup: The experiments were carried out using a rectangular laboratory flume of the University of Tehran. The flume was 6 m long, 0.5 m wide and 0.6 m depth. In order to calculation of entropy parameters and velocity profile, 12 velocity profiles were measured in depth of channel so that each profile consist 20 point. Table 1 showed specification of each 12 velocity profiles.

**RESULTS AND DISCUSSION**

**Entropy parameter:** By using the pairs of $u_m$ and $U_{\text{max}}$ collected at the flume laboratory the best-fit mean velocity
TABLE 1: Specification 12 velocity profiles

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>$U_{\text{AVE}}$(cm/s)</th>
<th>$U_{\text{MAX}}$(cm/s)</th>
<th>H/D</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>52.54</td>
<td>50.0</td>
<td>0.21</td>
<td>0.56</td>
</tr>
<tr>
<td>9</td>
<td>46.04</td>
<td>44.0</td>
<td>0.20</td>
<td>0.46</td>
</tr>
<tr>
<td>10</td>
<td>42.19</td>
<td>40.0</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>11</td>
<td>38.04</td>
<td>36.0</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>12</td>
<td>35.06</td>
<td>33.0</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>13</td>
<td>30.56</td>
<td>29.1</td>
<td>0.31</td>
<td>0.27</td>
</tr>
<tr>
<td>14</td>
<td>30.08</td>
<td>28.6</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>15</td>
<td>28.31</td>
<td>26.6</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>16</td>
<td>27.26</td>
<td>25.0</td>
<td>0.31</td>
<td>0.19</td>
</tr>
<tr>
<td>17</td>
<td>25.60</td>
<td>23.5</td>
<td>0.41</td>
<td>0.18</td>
</tr>
<tr>
<td>18</td>
<td>24.06</td>
<td>22.0</td>
<td>0.21</td>
<td>0.16</td>
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<td>21.0</td>
<td>0.30</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>21.56</td>
<td>20.0</td>
<td>0.31</td>
<td>0.14</td>
</tr>
</tbody>
</table>

$R^2=0.9991$

Fig. 1: Relation between $U_{\text{AVE}}$ and $U_{\text{MAX}}$

Fig. 2: Relation between (h/D) ratio and $U_{\text{MAX}}$

was calculated wherein the correlation coefficient, $R^2$, is also reported (Fig. 1). Also Fig. 2 shows the relation of $U_{\text{max}}$ and h/D. Since $\Phi$ is a function of M, the mean value of M at a channel section obtained from the mean value of $\Phi$ also tends to stay constant. The extensiveness of the laboratory analyzed and the consistency of the results show that the tendency for the mean values of $\Phi$ and M to remain constant at a channel section has regularity, (time) invariance, and universality required of a natural law (Chiu, 1996). Statistically, the fact that the mean values of $\Phi$ and M are constant at a channel section implies that the probability density function $f(U/U_{\text{max}})$. Therefore, $\Phi$ (M) can be assumed constant. Finally according to calculate $\Phi$ (M) and h/D, the parameter M was obtained equal to 18.94.

Comparison Chiu, logarithmic and Exponential method to estimation of velocity profile: 5 profiles was calculated in width of flume in three depths of water equal to 10, 15 and 20 cm. in order to evaluating of power estimation of velocity profile each three Chiu, logarithmic and exponential method, RMSE indicator was used. Table 2 shows the RMSE value of each three methods at three depths and five sections of flume width. Results show that the Chiu method has the best power estimation in velocity profile, so that it has the least value of RMSE between the two other methods. Also the Chiu method could estimate a velocity profile near the wall side of flume better than logarithmic and exponential method.

Figure 3 shows the estimated velocity profile with three methods at depth of 20 cm. According to observed profile, a velocity profile which was estimated by Chiu method has the best accuracy at five section of flume width. The Chiu method has the best power of estimation at the center part of channel width.

Table 2: RMSE value of three methods five sections of flume width

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Chiu</th>
<th>Logarithmic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.21</td>
<td>1.37</td>
<td>1.49</td>
</tr>
<tr>
<td>15</td>
<td>0.85</td>
<td>1.24</td>
<td>1.37</td>
</tr>
<tr>
<td>20</td>
<td>0.67</td>
<td>1.18</td>
<td>1.28</td>
</tr>
<tr>
<td>10</td>
<td>1.37</td>
<td>1.18</td>
<td>1.32</td>
</tr>
<tr>
<td>15</td>
<td>1.29</td>
<td>1.29</td>
<td>1.48</td>
</tr>
<tr>
<td>20</td>
<td>1.15</td>
<td>4.96</td>
<td>5.89</td>
</tr>
<tr>
<td>10</td>
<td>4.87</td>
<td>4.61</td>
<td>4.9</td>
</tr>
<tr>
<td>15</td>
<td>3.09</td>
<td>3.25</td>
<td>3.01</td>
</tr>
<tr>
<td>20</td>
<td>4.97</td>
<td>4.85</td>
<td>5.57</td>
</tr>
</tbody>
</table>

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CONCLUSION

The velocity profile derived by Chiu through an entropy-probabilistic approach has been scaled downward obtaining a local velocity distribution varying along the cross section. The scaling approach make a good estimation of the real velocity profile giving the possibility to derive more information concerning the flow field assessment as well as the energetic processes in it active. Moreover the possibility to scale the velocity law led to derive the transverse bottom shear stress distribution dealing a comparison in between those derived by the global scale and the local one. This comparison gives rise to further consideration concerning the value of the general entropy profile. A general result is the possibility to employ the entropy profiles both global scale and local in the numerical modelization of the open channel flow limiting the number of the parameter to the average and maximum cross velocity. The simple method developed for reconstructing the velocity profiles at width of channel, which is based on the assumption that Chiu’s velocity distribution can be applied locally, is found capable of estimating with a reasonable accuracy the shape of the observed velocity profiles. Finally, It explains the regularities that exist in various flow patterns observable at a channel section, which can be summarized and represented by the following constants: M, $\Phi$, h/D and energy and momentum coefficients. These constants are useful in hydraulic engineering.
Fig. 3: Velocity profile at the depth of 20 cm in width of flume

REFERENCES