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Theoretical Analysis of the Influence of the Thermal Diffusivity of Clay Soil on the Thermal Energy Distribution in Clay Soil of Abakaliki, Nigeria

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Abstract: The influence of the thermal diffusivity of clay soil on thermal energy distribution in clay soil was studied using one and two dimensioned heat equation, which was solved, by using separation of variables method. In the analysis, heat was assumed to be propagated along rectangular moldedclaywithlength(L)with the width being considered negligible in the case of one dimension with different temperature ranging from 350 to 1290ºC within zero to one minute chosen where some parameters such as thermal diffusivity In the second case, a steady state heat flow was considered in two dimensions with the assumption that temperature distribution is constant. Different temperature ranging from 350 to 1290 ºC within zero to one minute were chosen with some parameters such as thermal diffusivity, specific heat and mass per unit length of the clay are specified. The variation of the thermal conductivity and diffusivity with temperature was analyzed while that of the energy flux, $u(x,t)$ variation with time for different chosen length were plotted Two dimensional thermal energy distribution viewed at different points was also considered using different values of thermal diffusivity respectively.

Key words: Boundary condition, clay soil, partial differential equation, thermal conductivity, thermal diffusivity, temperature, thermal energy distribution, temperature gradient

INTRODUCTION

The important of soil thermal properties was recognized as early as the 19th century (Forbes, 1849). Thermal properties of soil influence the partition of energy at the ground surface, and are related to the soil temperature and the transfer of heat and water across the ground surface. For these reasons soil physicist, crop scientist, biologist and micrometeorologist study thermal properties. These properties are also important in engineering applications for example; the electric current ratting for buried cables depends on the thermal conductivity of the surrounding soil, as does the efficiency of heat pump system. Thermal conductivity is also an important property of soil that is required in many areas of engineering, agronomy and soil science.

Seed germination seedling emergence and subsequent stand establishment are influenced by the microclimate which is an important roles played by thermal prosperities of soil (Ghuman and Lac, 1985). According to (Eugene, 1994) thermal conductivity is defined as the measure of the ability of substance to transmit heat and its depend on the atomic structure of the material. The most common method of determining thermal conductivity of soil is the transient thermal probe method (Steinanis, 1982). This method consists of the heat source and temperature sensor.

According to (Carslaw and Jeager, 1959) Thermal conductivity is the rate at which heat is dissipated and this can be determined from a theoretical solution of conductive heat flow. According to Kerstin (1949), De Vries (1963), Johansen (1975), Sundberg (1988) and Tarnawski and Wagner (1992) on their empirical relationship developed to estimate thermal conductivity of soil as function of dry density, saturation, moisture content, mineralogy and temperature. Bristow *et al.* (1994) presented the first technique capable of simultaneously measuring on the three types of soil thermal properties. Generally metals are good thermal conductor because heart is relatively free to move through the metal and so can transport energy over a larger distance (James, 1993).

In addition, modeling water and energy movement in soil require knowledge of heat, salt and water interaction. Heat flow in soil provides .a means of temperature adjustment and may be determined by the knowledge of thermal conductivity and temperature gradient (Neborio and Mclnnes, 1993). Since thermal conductivity (k) is analogy to electrical conductivity according to Wiedermann and Franz‘s law, which states that at a given temperature, the ratio of the thermal conductivity to electrical conductivity is the same (Nelkon and Parter, 1997). The factors are: water content of the clay, salt composition of clay, and porosity salinity level and cat ion

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exchange capacity of clay. The essence of this paper is to analyze both thermal conductivity, thermal diffusivity of clay at different temperature and above all to see the influence of thermal diffusivity which is a function of the soil density on thermal energy distribution on clay soil.

MATERIALS AND METHODS

Theoretical methods: Consider the schematic representation of idealized physical setting for heat conduction in a rectangular molded clay soil with boundary conductions.

$u(x,0) = f(x)$ (Initial temperature distribution)

In this analysis, heat equation, which is an important partial differential equation that describes the distribution of heat or variation in temperature, is given by

$$\frac{\partial u}{\partial t} = c^2 \nabla^2 u, c^2 = k/\sigma\rho$$

Where u is temperature which is a function x, y and z that is a direction over time, k is thermal conductivity of the molded clay, c^2 is the thermal diffusivity, sis the specific heat capacity of the clay, r is mass per unit length of the mould clay $\nabla^2 u$ is the Laplacian of u and with respect to Cartesian coordinate x, y and z

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad (1)$$

From Fig. 1, it was assumed that temperature is oriented along x-axis that is $y = 0$, $z = 0$, so that heat flow in x-direction only which is called one dimensional heat equal given by

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (2)$$

The equation was solved by separation of variable method and boundary condition was imposed with reference to Fig. 1.

(a) The molded clay extended from $x = 0$ to $x = l$
(b) The temperature at the ends of the molded clay is maintained at zero.
(c) The initial temperature distribution along the molded clay is defined by $f(x)$ at $t = 0$ such that $u(x,0) = f(x)$.

Now consider our initial Eq. 2.

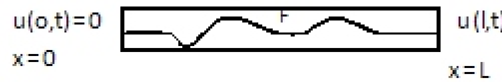

Fig. 1: A model of molded clay with a negligible width and infinite length, L

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (3)$$

$$u(0,t) = 0, u(l, t = 0) \forall t > 0 \quad (4)$$

$$u(0,t) = f(x) \quad (5)$$

The solution of $u(x,t)$ Eq. 3 satisfies the condition given (4) and (5).Using separation of variables method

$$u(x,t) = F(x)T(t) \quad (6)$$

Where F is a function of x only and T is a function of t only. With the correct separation constant, we obtain

$$F^{11} + \rho^2 F = 0 \quad (7)$$

and

$$T^2 + c^2 \rho^2 T = 0 \quad (8)$$

Eq. 7 has a solution of the form

$$f(x) = A\cos px + B \sin px \quad (9)$$

Where A and B are both constants

These constants are eliminated using the appropriate boundary conditions.

The solution becomes

$$f(x) = \sin\frac{n\pi}{l} \quad (10)$$

From Eq. 9

$$T(t) = Be^{-c^2 \rho^2 t} \quad (11)$$

Where B is a constant and

Thus $$T_n(t) = B_n e - \left(\frac{n\pi}{l}\right)^2 \quad (12)$$

Putting (11) and (12) in (6),

$$u_n(x,t) = B_n \sin\frac{n\pi x}{l} 1^{-\lambda_n^2 t} \quad (13)$$

$$\lambda_n = \frac{nc\pi}{l} \text{ and } c^2 = \frac{k}{\sigma\rho}$$

Then,

$$u(x,0)\sum_{n=0}^{\infty} B_n \sin\frac{n\pi x}{l} = f(x) \quad (14)$$

Eq. (13) satisfies the boundary conditions specified in Eq. (4). The series solution of these eigenfunctions is written below

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{l} e^{-\lambda_n t} \quad (15)$$

Where

$$B_n = \frac{2}{l}\int_0^l f(x)\, {}^{n\pi x}\!/\!_l\, dx \quad (16)$$

and n = 1,2,………

When the heat flow is assumed to steady, the energy flow becomes a factor of temperature gradient and the general solution of the problem is now

$$u(x,y) = \sum_{n=1}^{\infty} A_n^* \sin\frac{n\pi x}{l} \sin\frac{n\pi y}{l} \quad (17)$$

Where is Fourier coefficient given as

$$A_n^* = \frac{2}{\sin\left({}^{n\pi b}\!/\!_l\right)}\int_0^1 f(x) \sin\frac{n\pi x}{l} dx \quad (18)$$

Now, if the f(x) = $u_{o,}$ it implies that the temperature gradient along y-axis is zero. Then,

$$A_n^* = \frac{2}{l}\int_0^1 u_0 \sin\frac{n\pi x}{l} dx = \left[\frac{2u_0}{l}\frac{a}{n\pi}\cos\frac{n\pi x}{l}\right]_0^l \quad (19)$$

This then becomes

$$A_n^* = \frac{2u_0}{n\pi}\left[(-1)^n - 1\right] = \begin{cases} {}^{4u_0}\!/\!_{n\pi} \\ 0 \end{cases} \quad (20)$$

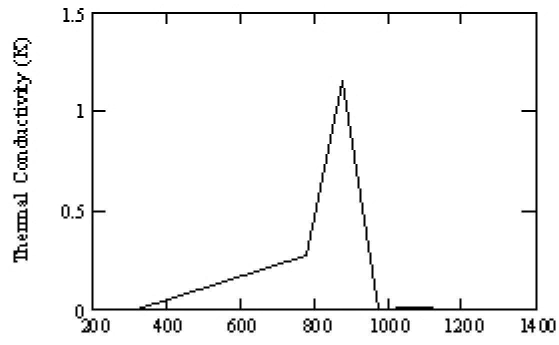

Fig. 2: Graph of thermal conductivity against temperature

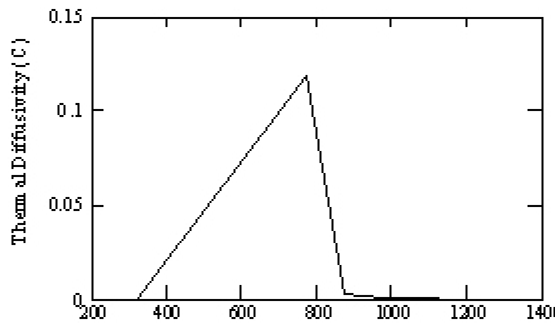

Fig. 3: Graph of thermal diffusivity against temperature, $\rho = 2.25$ kgm^{-3}, $\sigma = 880$ Jkg^{-1}k^{-1} and L = 0.18 m

O for n even and ${}^{4u_0}\!/\!_{n\pi}$ for n odd

Therefore the required solution is

$$u(x,y) = \sum_{nodd} \frac{au_0}{n\pi} e^{\frac{n\pi x}{l}} \sin\frac{n\pi y}{l} \quad (21)$$

RESULTS AND DISCUSSION

Figure 1 depicts the modeled molded block of finite length. Figure 2 shows the graph of the thermal conductivity as a function of temperature while that of Fig 3 represents the graph of the thermal diffusivity against the temperature. These two graphs depict peak at temperature of 800k. Figure 4a, b to 9 shows the thermal energy flow with time for different lengths when the soil density is low that result from the computed thermal diffusivity, c is 0.1178 m^{2}s^{-1} based on the relation $c^2 = {}^{k}\!/\!_{\sigma\rho}$. These three graphs exhibited exponential decrease of the energy flow over the time. Figure 6 and 7 depict the graph of the thermal energy flow when the

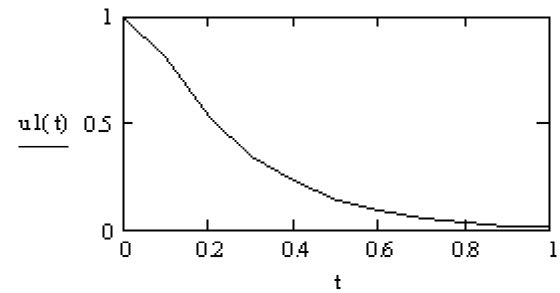

Fig. 4a: Graph of u(x,t)against time, $c = 0.1178\ m^2s^{-1}$, x = 0.1m and L = 0.18m

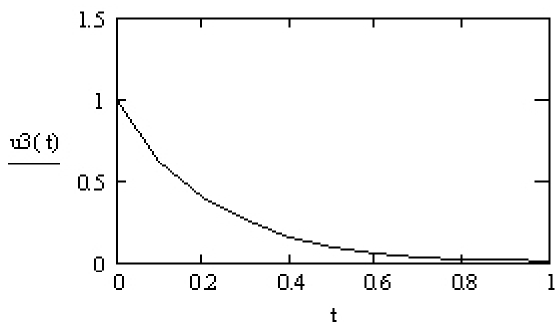

Fig. 4b: Graph of u(x,t) against time, $c = 0.1178\ m^2s^{-1}$, x = 0.116m and L = 0.18m

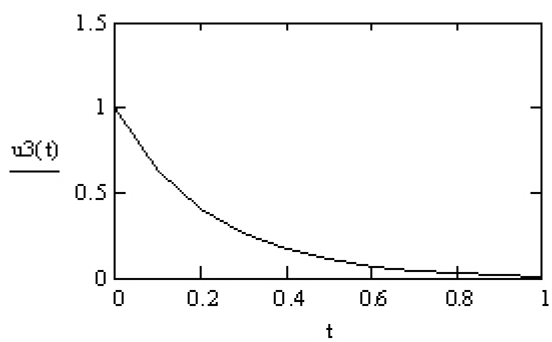

Fig. 5: Graph of u(x,t) against time, $c = 0.1178\ m^2s^{-1}$, x = 0.132m and L = 0.18m

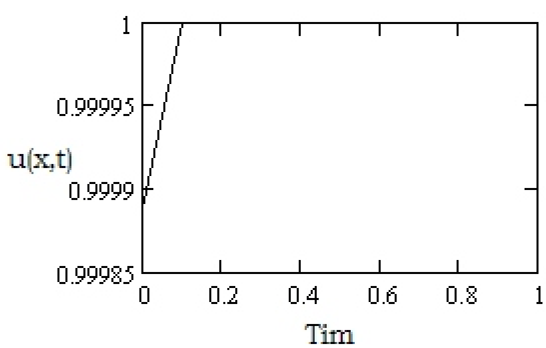

Fig. 6: Graph of u(x,t) against time, $c = 1.08 \times 10^{-3}\ m^2s^{-1}$, x = 0.1m and L = 0.18m

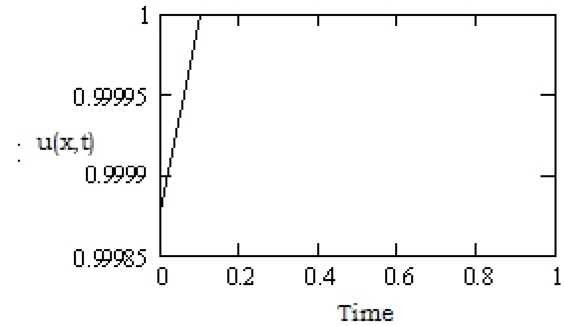

Fig. 7: Graph of u(x,t) against time, $c = 1.08 \times 10^{-3}\ m^2s^{-1}$, x = 0.116m and L = 0.18m

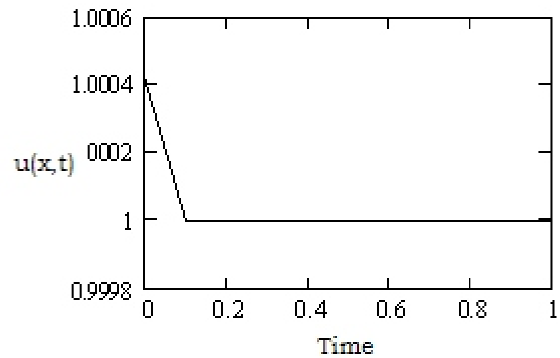

Fig. 8: Graph of u(x,t) against time, $c = 1.08 \times 10^{-3}\ m^2s^{-1}$, x = 0.132m and L = 0.18m

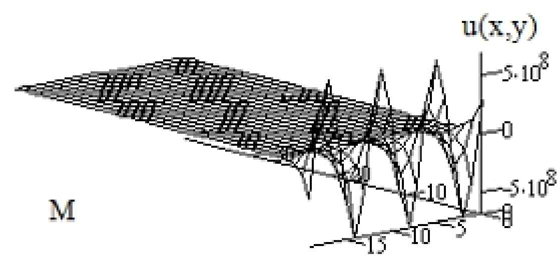

Fig. 9: Thermal energy distribution, u(x,y) when the thermal diffusivity = $0.1178\ m^2s^{-1}$, with length= 0.112m. 140º

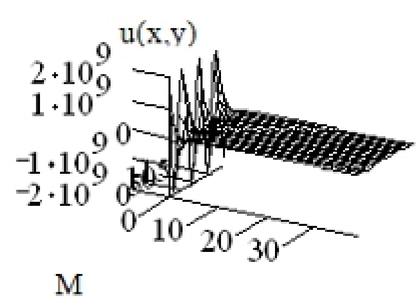

Fig. 10: Thermal energy distribution, u(x,y) when the thermal diffusivity = $0.1178\ m^2s^{-1}$, with length= 0.112m viewed at 180º

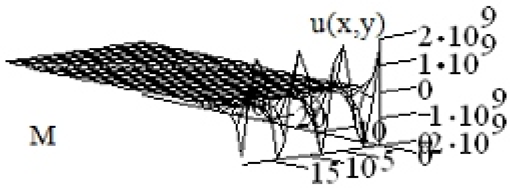

Fig. 11: Thermal energy distribution, u(x, y) when the thermal diffusivity = 0.1178, with length =0.118m. 135º

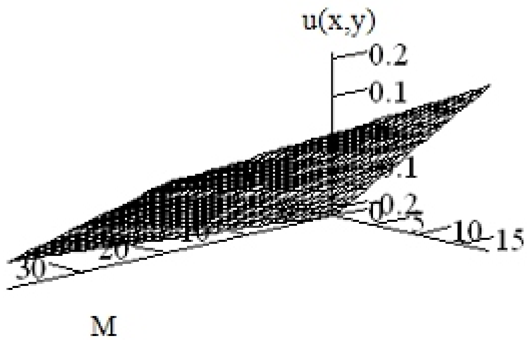

Fig. 12: Thermal energy distribution, u(x, y) when the thermal diffusivity = 0.00108, with length =0.118m. 60º

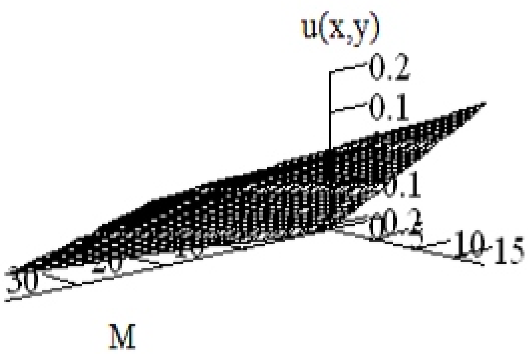

Fig. 13: Thermal energy distribution, u(x, y) when the thermal diffusivity = 0.00108, with length =0.118m.viewed at 45º

density of the clay is high such that the computed thermal diffusivity c is $1.08x10^3$ m^2s^{-1} with length 0.1 and 0.116 m. These two graphs showed the same pattern, while that of Fig. 8 where the length used is 0.132 m depicted a different pattern. The inverse relation observed from the expression showing relation of the thermal diffusivity and density indicates that the porous nature of the soil which is known generally to have effect on the density plays a part in the thermal diffusivity the and energy flow in the clay. (Nweke and Ugwu, 2007). The graphs of the thermal energy distribution are plotted in two dimensions using different values of thermal diffusivity(c) considering the lengths 0.112 and 0.118 m with summation of n = 1,n = 3 and n = 5 taken. Figure 9-13 shows the pattern of the energy distribution profile viewed at various angles when c = 0.00108 and 0.1178 respectively.

CONCLUSION

The concept of heat equation was used in the analysis of the influence of the thermal diffusivity of clay soil on the energy distribution profile. In both one-dimensional and two-dimensional approaches, it was found that thermal diffusivity of the molded clay model affected the energy distribution profile in both cases. This could be as a result of the influence of the density of soil, which is highly affected by the porous nature of the soil.

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