Fuzzy Multi-attribute Group Decision Making Method for Wine Evaluation Model

1Haiping Ren and 2Lianwu Yang
1School of Software, Jiangxi University of Science and Technology, Nanchang 330013, 2School of Mathematics and Computer Science, Yichun University, Yichun 336000, P.R. China

Abstract: In recent years, with the fast increase of red wine consumption, the wine evaluation becomes more important for wine enterprises. Establishing the grade of wine needs many qualified members (experts) to evaluate the wine according to several evaluation indexes. Evaluation indexes are mainly quality indexes. Interval numbers are more suitable than numerical numbers to demonstrate these indexes. Then, the wine evaluation model is a fuzzy multiple attribute group decision making model. In this study, we will propose a fuzzy TOPSIS method for the wine evaluation model, in which the evaluation index values are expressed with interval numbers. The coefficient of variation method is used to determine the index weights. Finally, an application example is given to illustrate the validity and practicability of the method.

Keywords: Interval number, multi-attribute group decision making, TOPSIS, wine evaluation

INTRODUCTION

In recent years, more and more people began to drink wine with the development of economy and improvement of people’s life. Wine industry shows blowout type development and wine demand is growing fast. To gain the sustainable development in the fierce market competition, more and more wine enterprises attach great importance to the wine quality evaluation. The wine quality evaluation is important for brand wine enterprises. Establishing the grade of wine needs many qualified members (experts) to evaluate the wine according to several evaluation indexes. Evaluation indexes are mainly quality indexes. Interval numbers are more suitable than numerical numbers to demonstrate these indexes. Then, the wine evaluation model is a fuzzy multiple attribute group decision making model. In this study, we will propose a fuzzy TOPSIS method for the wine evaluation model, in which the evaluation index values are expressed with interval numbers. The coefficient of variation method is used to determine the index weights. Finally, an application example is given to illustrate the validity and practicability of the method.

WINE QUALITY EVALUATION MODEL

Consider a wine quality evaluation problem. Let \( X = \{x_1, x_2, \ldots, x_m\} \) be the set of wine samples (alternatives) and \( O = \{o_1, o_2, \ldots, o_n\} \) be the set of evaluation indexes. \( D = \{D_1, D_2, \ldots, D_s\} \) is the set of wine evaluation experts. Suppose the rating of wine sample \( x_i \) (\( i = 1, 2, \ldots, m \)) on evaluation index \( O_j \) (\( j = 1, 2, \ldots, n \)) given by decision maker \( D_k \) (\( k = 1, 2, \ldots, s \)) is interval number \( \tilde{a}_{ij} = [a_{ij}^L, a_{ij}^U] \). Hence, the wine quality evaluation model is a multi-criteria group decision making problem can be concisely expressed in matrix format as follows:

\[
\tilde{D} = (\tilde{a}_{ij})_{m \times n} = \begin{pmatrix}
\tilde{a}_{11} & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\
\tilde{a}_{21} & \tilde{a}_{22} & \ldots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{m1} & \tilde{a}_{m2} & \ldots & \tilde{a}_{mn}
\end{pmatrix}
\]
where, \( k = 1, 2, \ldots, s \). Suppose that \( w = (w_1, w_2, \ldots, w_n) \) is the indexes weight vector, which satisfies 
\[
\sum_{j=1}^{n} w_j = 1, j = 1, 2, \ldots, n.
\]

For the wine quality evaluation model 
\[
\tilde{D}^k = (\tilde{a}^k)_{n \times n}, \quad k = 1, 2, \ldots, s,
\]

in the following discussion, we will develop a new group decision method for the wine evaluation model.

**FUZZY MULTI-ATTRIBUTE GROUP DECISION MAKING METHOD**

In this section, we will give the calculation steps of the fuzzy decision making method for the wine quality evaluation model as follows:

**Step 1:** For the wine quality evaluation model, collect the evaluation index values of the fuzzy decision matrix 
\[
\tilde{D} = (\tilde{a}_{ij})_{m \times n},
\]

into one decision matrix 
\[
\tilde{D} = (\tilde{a}_{ij})_{m \times n},
\]

where:

\[
\tilde{a}_{ij} = [a^i_{ij}, a^u_{ij}] = \frac{1}{s} (\tilde{a}^1_{ij} + \tilde{a}^2_{ij} + \cdots + \tilde{a}^s_{ij})
\]

**Step 2:** Normalize the decision making matrix: In general, evaluation indexes have two types: benefit indexes and cost indexes. We note \( I_1 \) and \( I_2 \) are the subset of benefit index set and cost index set, respectively.

The normalization method is to preserve the property that the range of a normalized interval number \( \tilde{r}^k_{ij} \) belongs to the closed interval \([0, 1]\). We transform the fuzzy decision matrix 
\[
\tilde{D} = (\tilde{a}_{ij})_{m \times n},
\]

into the normalized fuzzy decision matrix 
\[
\tilde{R} = (\tilde{r}_{ij})_{m \times n},
\]

where 
\[
\tilde{r}_{ij} = [r^l_{ij}, r^u_{ij}] \text{ obtained by the following formulas (Xu, 2004)}:
\]

\[
\begin{align*}
\tilde{r}^l_{ij} &= \frac{1}{s} \left( \sum_{i=1}^{n} a^i_{ij} \right)^2, \quad i \in M, j \in I_1 \\
\tilde{r}^u_{ij} &= \frac{1}{s} \left( \sum_{i=1}^{n} a^u_{ij} \right)^2, \quad i \in M, j \in I_1
\end{align*}
\]

and,

\[
\begin{align*}
\tilde{r}^l_{ij} &= \left( \frac{1}{s} \right) \sum_{i=1}^{n} \left( \frac{1}{a^i_{ij}} \right)^2, \quad i \in M, j \in I_2 \\
\tilde{r}^u_{ij} &= \left( \frac{1}{s} \right) \sum_{i=1}^{n} \left( \frac{1}{a^u_{ij}} \right)^2
\end{align*}
\]

where, \( M = \{1, 2, \ldots, m\} \)

**Step 3:** Determine the positive and negative ideal solution:

The Positive Ideal Solution (PIS) is defined as 
\[
x^+ = (x^+_1, x^+_2, \ldots, x^+_n), \quad \text{where, } x^+_j = [1, 1]
\]

And the negative ideal solution (NIS) is defined as 
\[
x^- = (x^-_1, x^-_2, \ldots, x^-_n), \quad \text{where, } x^-_j = [0, 0]
\]

**Step 4:** Calculating the index weights as follows:

- Defuse \( \tilde{R} = (\tilde{r}_{ij})_{m \times n} \) into a crisp number decision matrix 
  \( G = (g_{ij})_{m \times n} \) by the expectation method given as follows (Hu and Zhang, 2010):
  \[
g_{ij} = \frac{1}{2} (r^l_{ij} + r^u_{ij})
\]

- The indexes weights are calculated by coefficient of variation method as follows (Men and Liang, 2005):
  \[
  w_j = \frac{\delta_j}{\sum_{j=1}^{n} \delta_j}, \quad j = 1, 2, \ldots, n
  \]
  \[
  \delta_j = \frac{s_j}{\bar{x}_j}, \quad \bar{x}_j = \frac{1}{m} \sum_{i=1}^{m} x_{ij} \quad \text{and}
  \]
  \[
  s_j = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (x_{ij} - \bar{x}_j)^2}
  \]

Obviously, \( w_j \geq 0, \sum_{j=1}^{n} w_j = 1, j = 1, 2, \ldots, n \)

**Step 5:** Calculate the distance measures of each alternative \( x_i \) with the PIS and NIS, as follows:

\[
\begin{align*}
d(x_i, x^+) &= \sum_{j=1}^{n} w_j d^2(\tilde{r}_{ij}, r^l_{ij}), \\
d(x_i, x^-) &= \sum_{j=1}^{n} w_j d^2(\tilde{r}_{ij}, r^u_{ij})
\end{align*}
\]

where, \( d(\cdot, \cdot) \) is the distance measure defined as follows:

Let \( \tilde{a} = [a^l, a^u] \) and \( \tilde{b} = [b^l, b^u] \) are two interval numbers, then the distance measure between them is defined as (Zhang and Fan, 2008):

\[
d(\tilde{a}, \tilde{b}) = \sqrt{(a^l - b^l)^2 + (a^u - b^u)^2}
\]

Thus, we have:
Table 1: Evaluation values of different experts

<table>
<thead>
<tr>
<th>Sample</th>
<th>Index</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$a_1$</td>
<td>[10, 11]</td>
<td>[11, 12]</td>
<td>[9, 10]</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>[18, 20]</td>
<td>[23, 24]</td>
<td>[24, 25]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>[24, 26]</td>
<td>[33, 35]</td>
<td>[34, 38]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>[8, 10]</td>
<td>[7, 9]</td>
<td>[8, 10]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>$a_1$</td>
<td>[5, 8]</td>
<td>[8, 10]</td>
<td>[9, 12]</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>[18, 21]</td>
<td>[22, 25]</td>
<td>[14, 18]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>[21, 24]</td>
<td>[25, 28]</td>
<td>[19, 23]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>[7, 10]</td>
<td>[8, 10]</td>
<td>[7, 9]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td>$a_1$</td>
<td>[8, 10]</td>
<td>[8, 10]</td>
<td>[9, 11]</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>[17, 23]</td>
<td>[23, 25]</td>
<td>[25, 27]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>[33, 37]</td>
<td>[36, 38]</td>
<td>[37, 39]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>[9, 10]</td>
<td>[8, 10]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_4$</td>
<td>$a_1$</td>
<td>[11, 12]</td>
<td>[10, 12]</td>
<td>[9, 11]</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>[21, 23]</td>
<td>[22, 25]</td>
<td>[23, 25]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>[33, 35]</td>
<td>[38, 40]</td>
<td>[40, 43]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>[8, 9]</td>
<td>[9, 10]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 1: According to the Eq. (1) and (2), calculate the fuzzy decision matrix $\tilde{D} = (\tilde{a}_{ij})_{n \times m}$:

$$d(\tilde{r}_i, \tilde{r}_j) = \sqrt{(1 - r^{d}_{ij})^2 + (1 - r^{o}_{ij})^2}$$

Step 2: The normal decision matrix $\tilde{R} = (\tilde{r}_{i,j})_{n \times m}$ is calculated as:

$$\tilde{R} = \begin{bmatrix}
0.2336 & 0.2796 \\
0.5061 & 0.5845 \\
0.7086 & 0.8387 \\
0.1791 & 0.2457 \\
0.1747 & 0.2398 \\
0.4542 & 0.5802 \\
0.7407 & 0.8819 \\
0.1747 & 0.2321
\end{bmatrix}$$

Step 3: The PIS and NIS are respectively given as:

$$x^* = (x_1^*, x_2^*, x_3^*, x_4^*) = ([1,1],[1,1],[1,1],[1,1])$$

$$x^- = (x_1^-, x_2^-, x_3^-, x_4^-) = ([0,0],[0,0],[0,0],[0,0])$$

Step 4: Calculate the index weight vector:

- Calculate the crisp number decision matrix $(g_{ij})_{n \times m}$:

$$G = \begin{bmatrix}
0.2566 & 0.2693 & 0.2072 & 0.2323 \\
0.5453 & 0.6076 & 0.5172 & 0.4962 \\
0.7736 & 0.7200 & 0.8113 & 0.8168 \\
0.2124 & 0.2638 & 0.2034 & 0.1964
\end{bmatrix}$$

- Then the weight vector can be obtained by coefficient of variation method

$$w = (0.2432, 0.2078, 0.2764, 0.2726)$$

Step 5: Calculate the distance measures:

$$d(x_i, x^*) = 1.0782, d(x_j, x^-) = 0.6635, d(x_k, x^*) = 0.3276, d(x_l, x^-) = 1.1102$$

and:

$$d(x_i, x^*) = 0.3439, d(x_j, x^-) = 0.7672, d(x_k, x^*) = 1.1162, d(x_l, x^-) = 0.3120$$

Then we have $d(x^*) = 0.7225, d(x^-) = 0.6988$.
Step 6: The relative closeness coefficient of each wine sample obtained as follows:

\[ C_1 = 0.2418, C_2 = 0.5362, C_3 = 0.7731 \]

and,

\[ C_4 = 0.2194 \]

Step 7: Rank the alternatives: It is easy to see \( C_3 > C_2 \) \( C_1 > C_4 \), thus the wine quality evaluation result is:

\( x_3 > x_2 > x_1 > x_4 \)

The wine sample \( x_3 \) is the best wine.

CONCLUSION

This study is focus on wine quality evaluation problem, which is a multi-attribute group decision making problem. Interval numbers are used to demonstrate the evaluation values given by experts. For the determination of indexes weights, we use coefficient of variance method, which is an objective method. Coefficient of variance method can use of number information itself reflects the index weight, thus overcomes the artificial and uncertainty of subjective weight. A group decision method for the wine evaluation model is put forward based on the concept of TOPSIS. An application example about wine quality evaluation is given to illustrate the validity and practicability of the method. The proposed method can also be extended to other aspect, such as investment project selection, employee performance evaluation.

ACKNOWLEDGMENT

This study is partially supported by Natural Science Foundation of Jiangxi Province of China (No. 20132BAB211015); Natural Science Foundation of Jxust (No. NSFJ2014-G38).

REFERENCES


