

## Research Article

### Lunar Circle Pit Recognition Based on Multi-level De-noising Hough Transform

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**Abstract:** In order to solve the problem of selecting lunar soft landing spot, this study doped out a solution to the problem. Collect images of the lunar surface and analyze them. According to the fact that the speed and accuracy of selecting lunar soft landing spot are highly required in the field of aviation, this study proposed circle pit recognition method based on Multi-level de-noising Hough transform. In this method, do halved sampling process repeatedly on the original images in order to get a set of variable resolution image of pyramid like. Search the circle pit position in the lowest resolution image roughly. And then refine soft landing point in the high resolution images in order to get the lunar circle pit position accurately. Do experiments using the images of the lunar surface which were collected by Chang 'e-3. What's more, selecting the appropriate landing point selection mechanism to select a soft landing point and verify results. After doing all of these, compare this method with five kinds of traditional methods. The experimental results shows the method which is proposed in this study effectively improves the speed and accuracy of Hough transform circle detection, with strong robustness and high efficiency, so as to improve the speed and accuracy of selected lunar soft landing point.

**Keywords:** Circle pit recognition, halved sampling, lunar soft landing point, multi-level de-noising hough transform

## INTRODUCTION

Lunar soft landing point selection, related to the success of lunar soft landing, is an important part of the process of satellite landing. The satellite moves at a high speed, therefore, the higher requirements of the selection of landing speed and accuracy are put forward. The system inside of the satellite analyses accurately identify the circle pit obstacles through collecting images of the lunar surface and choose the appropriate landing site selection mechanism to choose a soft landing point, therefore, the identification of circle pit location become the key problems in the process of a soft landing. Usually, the satellite will take pictures in long distance near the surface of the lunar to analyze for coarse obstacle avoidance, again in near the surface of the lunar to analyze the information to analyze the barriers for accurate obstacle avoidance. Talking account of the surface of the lunar circle pits in the images are a series of the regular circle, the selection of lunar soft landing point becomes the problem of circle location information in image and choosing a soft landing point through the appropriate mechanism.

In recent years, domestic scholars have carried out simulated researches on the selection of the lunar landing point. Su *et al.* (2014) and Zhang *et al.* (2014) propose that solve the problem of accurate positioning

and landing with recognizing technology related to optics, but this complicated method requires a variety of sensor data and therefore, it is difficult to accomplish in the application. Xu *et al.* (2011) use SURF (Speeded up Robust Features) and least squares to calculate the position of the lunar landing point. The method, with high real time, is easy to implement. But its robustness is poor, greatly influenced by the edge of the noise and the detection accuracy and stability is low. Wu *et al.* (2011) adopt method, which is using sequence feature point image matches designated landing navigation method that has been corrected, to achieve obstacle avoidance in the lunar soft landing process. The method has high precision, but with low stability and poor robustness.

With the advantage of strong robustness and not being sensitive to noise (Yang *et al.*, 2013), Hough Transform, also called the HT, is widely used in the detection of circle (Zhou *et al.*, 2013; Zhou *et al.*, 2014). But due to the disadvantage of the high computational cost and its large amount of storage space, traditional Hough Transform method is not able to satisfy the requirement of instantaneity. Scholars have put forward many ways to improve Hough Transform, mainly including randomized Hough Transform and extended Kalman filtering Hough Transform. Experiments show RHT may fail to get the solution because of the contour distortion and the edge

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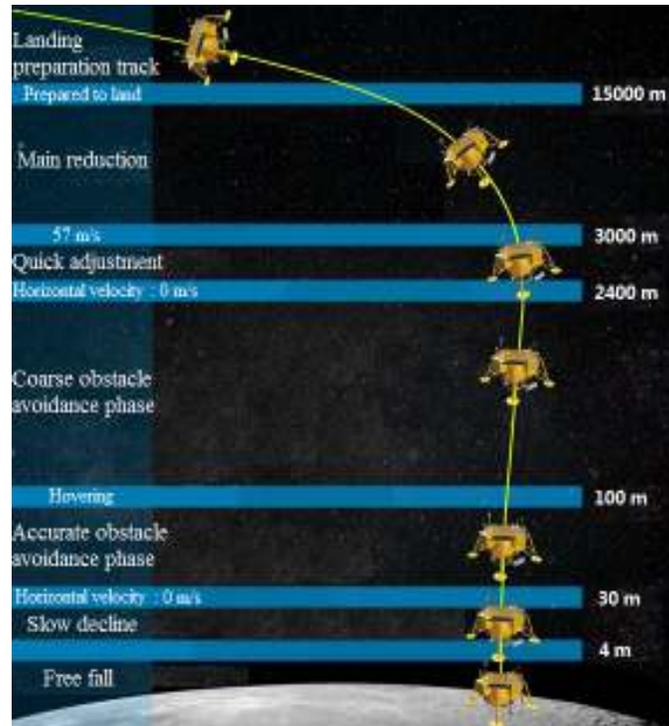


Fig. 1: The soft landing process of Chang 'e-3

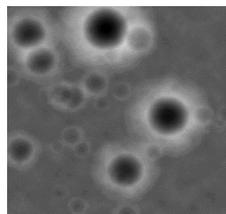


Fig. 2: (2.4 km) away from the surface of the lunar

noise jamming, while extended Kalman filtering Hough Transform is susceptible to the influence of the initial guesses and it is only suitable for straight line detection. In recent years, various kinds of intelligent optimization methods provide a new opportunity to improve Hough Transform. As a result, this paper will list a method which is to identify circle based on multi-level de-noising Hough transform, in order to solve the problem of the selection of lunar soft landing point. It converts the problem of the selection of lunar soft landing point into the problem of identifying the location of the circle in the image information. Then, by repeatedly halving the information of the original image, the variable resolution image of a group of pyramid and the information of circle pit's position roughly in the lowest resolution image will be carried out. Next, refine the soft landing point in the high resolution image and get the accurate position information of circle pit on the surface of the lunar. The simulation results and actual research verify the method and show that the validity of the method is good and the robustness is strong.

Therefore, this study proposes a lunar circle pit recognition based on Multi-level De-noising Hough Transform, mainly researches on rough obstacle avoidance phase stage and accurate obstacle avoidance phase stage of Chang'e-3 lunar satellite. That is, the stage away from the surface of the lunar 2400 m and the stage away from the surface of the lunar 30 m, accessing to the lunar surface digital elevation map by camera, using Multi-Level De-noising Hough Transform to identify the lunar circle pit and selecting the appropriate landing point to achieve automatic obstacle avoidance in the soft landing process.

**Problem description:** Due to the landing environment of the soft landing process of Chang'e-3 is universe, so the satellite takes photos of the lunar surface by using the camera installed on the satellite, collect images and then determine the landing site. Among them, the satellite soft landing process is divided into six stages, as shown in Fig. 1.

**Coarse obstacle avoidance phase:** Coarse obstacle avoidance phase, ranging from 2.4 km to 100 m away from the surface of the lunar, which mainly require that avoid large craters, achieve hover over the designed landing site 100 m and initially identified the falling location of the lunar. Chang 'e-3, 2.4 km away from the surface of the lunar, takes photos towards the surface of the lunar in the range of 2300×2300, which is just below itself. A digital elevation map, whose resolution

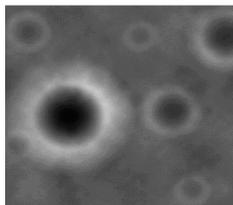


Fig. 3: One hundred meter away from the surface of the lunar

is 2300×2300, is obtained as Fig. 2 shows. It shows that the vertical projection of the Chang 'e-3 on lunar surface is located at the center position of intended landing area.

**Accurate obstacle avoidance phase:** Accurate obstacle avoidance phase, ranging from 100 to 30 km away from the surface of the lunar, which requires that Chang 'e-3 can hover over the surface of the lunar 100 m, take photos of 100 m range within the region near the landing site and obtained a three-dimensional digital elevation map. Analyze the three-dimensional digital elevation map, avoid large craters and then determine the best landing site, to achieve the level of 30 m above the landing site at the direction of the velocity of 0 m/sec. The three-dimensional digital map shown in Fig. 3 is shot by hovering at the level of 100 m away from the surface of the lunar, with a resolution of 1000×1000.

The image Chang'e-3 satellite camera obtained, after grayscale processing, binary processing and edge detection, obtains an image point set, which is  $I = \{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}, (a_i, b_i), i = 1, \dots, n$ , stands for horizontal and vertical coordinates of the points in the image and  $n$  stands for the number of points. According to the image-forming principle in the camera of the circle pit on the moon's surface,  $r$  is the radius of the circle pit. Therefore, fitting the circle on the basis of the image point set  $I$  to get the parameters of the circle, which is  $(a, b, r)$  and then get the location information of the circle. The standard equation of the circle is:

$$(x-a)^2 + (y-b)^2 = r^2 \quad (1)$$

In where,

- $a, b$  = The center coordinates of the circle
- $r$  = Radius of the circle

**At present, there are four kinds of methods of hough transform to identify the image:** RANSAC (Bolles and Fischler, 1981; Xie and Jun, 2010) (random sample consensus), the Least Squares Method (Qiao *et al.*, 2009), the Traditional Hough Transform (Illingworth and Kittler, 1988) and Random Hough Transform (Chen and Qi, 1998; Shu and Qi, 2003). Among them, compared with other methods, the robustness of RANSAC has improved, but it is easily

affected by marginal noise. The Least Squares Method is fast and is simple to implement, but the detection curvature has high deviation. Traditional Hough Transform has strong robustness and is not sensitive to the noise, but because of its high computation and large occupied storage space, it can't satisfy the requirement of real-time. Random Hough Transform in dealing with complex image by randomly sampling is easy to cause a large number of invalid accumulations. Therefore, this paper will list a method which is based on multi-level de-noising Hough Transform about detecting lunar circle pit.

### HOUGH TRANSFORM BASED ON MULTILEVEL DE-NOISING ALGORITHM

**Multi-level transform algorithm:** Multi-level Hough Transform is similar to a process of human eyes, which from a rough look to scan. First of all, do dimension transform with the original image. And then, do halved sampling by using the algorithm in this text in order to get a lower resolution image. Next, the new image could be processed by do halved sampling. Repeating this cycle over and over again, by that it will get a set of variable resolution of the images. The so-called halved sampling is something that, for binary text image, in order not to affect the tilt angle detection of the low resolution image, a variable resolution pyramid should be formed on the basis of retaining the black spots. Methods for this are showed as follows (Liu *et al.*, 2010):

$$f_l(i, j) = f_{l-1}(2i, 2j) \cap f_{l-1}(2i+1, 2j) \cap f_{l-1}(2i, 2j+1) \cap f_{l-1}(2i+1, 2j+1) \quad (2)$$

In where,  $l$  stands for the number of layers of the image and  $f_l(i, j)$  stands for the pixel value of the row  $i$ , column  $j$  of the  $l^{\text{th}}$  floor in the pyramid. Formula (2) means that pixel of the image in each layers are determined by points in corresponding area of upper image. Only when all points in the corresponding area are white, the point is white. In Fig. 1, the number one is used to represent white dot, which can also be called background and the number zero are used to represent black dot, so that black dots can be kept effectively.

Hough transform is first performed on the lower-resolution binary image to get the circle parameters of the image roughly. And then refine the parameters at a higher-resolution image until obtain a satisfactory precision. As can be seen from the above procedure, lunar circle pit recognition is mainly performed on lower-resolution images. Due to the small size of this kind of image, the search scope of angles can be set very large and the step can also be set rather fine. Oppositely, when refine the angle at the higher-resolution image after getting the rough position information of the unidentified parameters, the search

scope of angles is limited. Therefore, in this algorithm, although the overall amount of computation is not big, it can obtain high precision. What's more, appropriate half sampling process also helps to improve the anti-noise ability of the algorithm.

**De-noising algorithm:** Because of the influence of all kinds of noise in image space, the circle pit location information identified by the use of the Hough transform often has duplicate values in the same place, with interference coordinate information. In this study, a kind of de-noising method based on the highest acquaintance degree is put forward, that is, to meet the location information of the formula (3), only keep the most appropriate one of them, among them, the selection of reference threshold de-noising, which is  $k_1, k_2, k_3$  should be set according to the accuracy requirement:

$$\begin{cases} |a_i - a_j| < k_1 \\ |b_i - b_j| < k_2 \ (i \neq j) \\ |r_i - r_j| < k_3 \end{cases} \quad (3)$$

In where,  $a_i, a_j$  mean the abscissa of the center of a circle that is initially identified,  $b_i, b_j$  mean the ordinate of the center of a circle that is initially identified,  $r_i, r_j$  mean the radius of the circle that is initially identified and parameters including  $k_1, k_2, k_3$  is reference threshold de-noising.

**Classic hough transform:** The general procedures of the Hough transform algorithm used to detect the circle are as follows.

**Step 1:** Establish the right angle coordinate system. At the upper left corner of the image is the origin, the upper edge of the image is regarded as y axis and the left edge is regarded as x axis. According to the formula (1), an arbitrary circle in plane can be determined by the coordinate  $(a, b, r)$ , among them, the coordinate  $(a, b)$  determines the center of the circle in plane and  $r$  determines the radius of the circle.

**Step 2:** As for one of the black dots  $(x_i, y_i)$  in the image space, choose a certain step as radius sequence, choose a certain step around the black dot as angle sequence and then determine the value of a series of  $(a', b', r')$  corresponding to them. The values will be mapped to a set of accumulator in Hough space P  $(a', b', r')$  and then conduct the voting process that is to say, accumulate in the Hough space.

**Step 3:** Repeat Step 2 until all points in the image space are mapped to the Hough space.

**Step 4:** After the polls had closed, the value of  $(a', b', r')$  corresponding to the local maximum of P  $(a', b',$

$r')$  in the Hough space is the parameters of several circles that are supposed to identify.

Therefore, the problem of identifying the position information of circles in the image can be transformed into the problem of searching in the parameter space to make the accumulator reach the optional solution of  $(a_1, b_1, r_1)$ . Now adjusting the searching object of the Hough transform several times. Then conduct variable resolution recognition of pyramid like, segmenting from low resolution to high resolution.

**The steps of multi-level de-noising hough transform:**

**Step 1:** Do half processing on the sampling image of the lunar surface to reduce the resolution of the image gradually and get a set of variable resolution image of pyramid like.

**Step 2:** Using the classical Hough transform to do Hough recognition on the lowest resolution image and the rough position information of the circle pit on the surface of the lunar will be obtained. After that, remove the interferential circle location information points with de-noising algorithm. In this method, the characteristics of classical Hough transform, which is with strong robustness and insensitive to noise, is well reserved.

**Step 3:** After getting the rough position information of the circle pit on the lunar surface in Step 2, add an appropriate mini-zone range to each center position. Then identify the circle pit position information again in the higher resolution image and remove the interference position information points of the circle pits with de-noising algorithm. In this case, the recognition speed of Hough transform is accelerated as the interval range of the searching point is smaller.

**Step 4:** Do Step 3 repeatedly until getting the most refined recognition results of the position information of the circle pits.

**Step 5:** Select the best lunar landing site: It is known that the satellite is located above the center of the image.  $O$  is set as the starting position of the satellite and  $D$  is set as the optimal landing point. It can be seen from the select conditions of the optimal landing point that the optimal landing point  $D$  should be as close as possible to the starting position  $O$ , away from the center of each circle pit  $O_i$  ( $i = 1, 2, \dots, n$ ) and maintain a certain distance with the circle pit edge. That is, need to meet:

$$\begin{cases} \min |D - O| \\ \max \sum_{i=1}^6 |D - O_i| \\ |D - O_i| - R_i > L \\ i = 1, 2, 3, 4, 5, 6 \end{cases} \quad (4)$$

In where,  $L$  is the shortest distance between the optimal landing point and the circle pit edge, aimed at avoiding the peak at the circle pit edge.

### ANALYSIS OF IMAGE RECOGNITION RESULTS

In order to verify the effectiveness of the algorithm proposed in this study, this section compared the algorithm with five traditional methods (The least squares method, HT, RHT, PSOHT and RANSAC) in running speed, which had verified the real-time property and effectiveness of the algorithm proposed in this study. By validating the validity of the algorithm in other environment, proves that the algorithm has strong robustness. In the first experiments, the lunar surface image obtained by Chang'e-3 was processed. As the noise is much, use de-noising algorithm to do multi-level de-noising experiments and compared with five kinds of traditional methods. In the second experiment, Hough transform algorithm without de-noising and the algorithm in this study are used to process the lunar surface image separately to verify that the algorithm proposed in this study maintains the original characteristics of traditional Hough transform, which is having strong robustness and insensitive to noise and it's adapted to the image recognition under a variety of environments.

**Set the algorithm's initial conditions:** The reference threshold de-noising parameters including  $k_1$ ,  $k_2$ ,  $k_3$  are set to 4 and the shortest distance between the optimal landing point and circle pit edge is set to 180.

**The grayscale and binary processing on the image:** Firstly, based on the proper mathematic tools, the high-resolution image is calculated by graying and binarization processing. Secondly, after halving processing, the previous image, presented in Fig. 2 and 3, is changed into a series of pyramidal variable-resolution images. Finally, the lowest-resolution image, presented in Fig. 4 and 5, is carried out with the resolution of  $200 \times 200$ .

**Using sobel operator for edge detection:** Select the Sobel operator to do edge detection for Fig. 4 and 5, as shown in Fig. 6 and 7.

**Analysis of image recognition results:** Using the multi-level de-noising Hough transform algorithm this paper introduces for image processing on the lunar surface, then get the results of the algorithm. The circle pit recognition results on the lunar surface in Fig. 8 and 9 shows that after doing multi-level de-noising recognition on images that from low resolution to high



Fig. 4: (2.4 km) away from the surface of the lunar



Fig. 5: One hundred meter away from the surface of the lunar

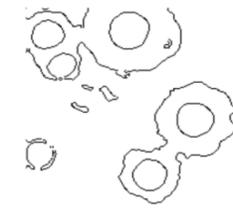


Fig. 6: (2.4 km) away from the surface of the lunar



Fig. 7: One hundred meter away from the surface of the lunar

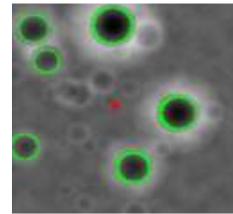


Fig. 8: (2.4 km) away from the surface of the lunar

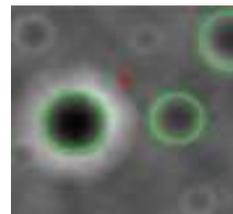


Fig. 9: One hundred meter away from the surface of the lunar

Table 1: The identification results of circle pit location on the surface of the lunar and the soft landing point location

The circle pit position which are 2.4 km away from the surface of the lunar				The circle pit position which are 100 m away from the surface of the lunar			
No.	Abscissa	Ordinate	Radius	No.	Abscissa	Ordinate	Radius
1	1576	153	138	1	540	280	135
2	299	230	155	2	525	725	110
3	1802	1227	161	3	165	955	130
4	1208	1668	180	/	/	/	/
5	259	1018	190	/	/	/	/
6	598	311	184	/	/	/	/
Landing position	1027	1035	/	Landing position	345	500	/

Table 2: The root mean square error of different algorithms

Parameter	Multi-level de-noising HT	TLSM	RHT	PSOHT	RANSAC	HT
a	0.0132	0.1102	0.1273	0.1021	0.0977	0.0434
b	0.0089	0.1739	0.1856	0.0182	0.2031	0.0132
c	0.0118	0.0915	0.2372	0.0961	0.1023	0.0891
Running time/sec	1.5321	16.2918	0.9023	13.4511	48.3724	8342.8317

Table 3: The lunar soft landing points identified by different algorithm, at the height of 2.4 km away from the lunar surface

Parameter	Multi-level de-noising HT	TLSM	RHT	PSO HT	RANSAC	HT
x/m	1027	1029	1031	1021	1025	1028
y/m	1035	1037	1034	1031	1033	1036

resolution, the running time of this algorithm is significantly reduced compared to traditional Hough Transform algorithm and the effectiveness and real-time of results are greatly improved.

Based on Multi-level De-noising Hough transform algorithm, the results of the lunar circle pit identification, with the green sign in the picture, is presented in Table 1. And the soft landing point (marked in red), which is selected by using above appropriate mechanisms (The landing point should located outside the lunar circle pit and the sum of distance between the landing point and each lunar circle center must be the largest and the distance between landing point and the images' center must be the smallest). According to the data of Table 1, the identification results of Multi-level De-noising Hough Transform algorithm is more accurate and the value is refined. What's more, it can identify faster and the characteristics of traditional Hough Transform, which is with strong robustness and non-sensitive to noise, are well reserved. The difference of the average running time and root mean-square error in 300 times independent testing are presented in Table 2, among the least squares method, RHT, PSOHT, RANSAC, HT and Multi-level De-noising HT algorithm. According to the data of Table 2, the detection precision of multi-level de-noising Hough transform algorithm is superior to others in running speed.

Table 3 compares the algorithm proposed in this study with other algorithms in the selection of the lunar soft landing location. Table 2 and 3 show the algorithm proposed in this study has accurate detection accuracy and high detection speed, which are conform to the accurate and real-time requirements on the determination of soft landing point for satellite.

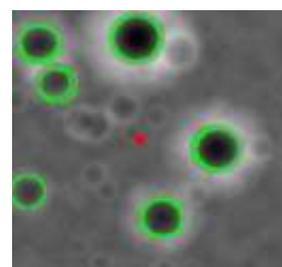


Fig. 10: No de-noising hough transform

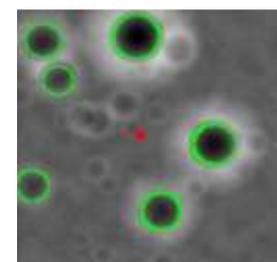


Fig. 11: Multi-level de-noising hough transform

**De-noising performance analysis:** In order to test the algorithm in this study of the multi-level de-noising performance, select the digital elevation map shot at 2400 m away from the lunar surface, shown in Fig. 2. Use the non-de-noising Hough transform for image's circle pit recognition and the recognition results are shown in Fig. 10. Then, use the multi-level de-noising Hough transform proposed in this study to identify the circle pit and the result of recognition is shown in Fig. 11. It can be seen from Fig. 10 that, the non-de-noising Hough transform algorithm had identified 8 circles in total and two of them are repeated. Thus it can be seen non-de-noising Hough transform algorithm is influenced by noise greatly and the robustness is poor.

In contrast, it can be seen from Fig. 11 that, this study's algorithm can still identified 6 large circle pit correctly even in the case of much noise, maintaining its strong robustness and the insensitive to noise characteristics. What's more, its de-noising effect is obvious, the effectiveness and accuracy is high and the real-time performance is strong. Its recognition of circle with noise is obviously superior to other algorithm.

## CONCLUSION

In order to improve the detection speed and precision on the circle pit during the coarse obstacle avoidance phase stage and accurate obstacle avoidance phase in the process of lunar soft landing and then choose the appropriate landing site selection mechanism (The landing point should located outside the lunar circle pit and the sum of distance between the landing point and each lunar circle center must be the largest and the distance between landing point and the images' center must be smallest) to choose a soft landing point. This article first transforms the selection problem of lunar soft landing point for identifying the location information of the circle in the image and then puts forward a kind of circle pit detection method that was based on multi-level de-noising Hough transform. The method repeatedly does halved sampling on the images collected originally to get a set of variable-resolution images pyramid, roughly searches the location information of the lunar circle pit on the lowest resolution image and then refines the soft landing points on the high resolution images. Finally it can get the exact location information of the lunar circle pit and choose the location on the lunar for the satellites' soft landing.

Finally, make an simulation experiment of the real images which have been taken by the Chang 'e -3 satellite on the lunar by using the algorithm in this study and compare it with the 5 methods like Least Square Method, traditional Hough transform, randomized Hough transform, the PSO Hough transform and RANSAC. The experimental results show that the algorithm in this study effectively improves the precision and speed of Hough transform to solve the problems of the circle detection, thus effectively improves the detection speed and precision on the circle pit during the coarse obstacle avoidance phase stage and accurate obstacle avoidance phase in the process of lunar soft landing. By comparing the multi-level de-noising Hough transform with the traditional Hough transform on the identification results of the lunar circle pit to analyze the de-noising performance of the algorithm and then reach a conclusion that comparing with the no de-noising Hough Transform, the effectiveness and the robustness

of the algorithm in the study are greatly improved and make the satellites determine the soft landing point quicker and more accurate, which is of great significance for the development of the aviation industry and the study on the soft landing of the satellites.

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