# Research Article An Empirical Relationship between Modulus of Elasticity, Modulus of Rupture and Compressive Strength of M60 Concrete Containing Metakaolin

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**Abstract:** This study presents the relationship between modulus of elasticity and, modulus of rupture relationship with compressive strength of M60 concrete incorporating Metakaolin. Comparing the experimentally obtained result with the mechanical properties calculated using the recommended relationship from the various design codes, one finds substantially variation in the data. A new empirical relationship between elastic modulus, modulus of rupture and compressive strength for Metakolin based M60 concrete is proposed.

Keywords: Compressive strength, high performance concrete, metakaolin, modulus of elasticity, modulus of rupture

## INTRODUCTION

The static modulus of elasticity, modulus of rupture and compressive strength are important properties of concrete. These are the basic parameters for computing deflection in reinforced concrete structures. Various countries have been established their design codes based on this empirical relationship between static modulus of elasticity, modulus of rupture and compressive strength of plain concrete at 28 days of curing.

The Indian code of practice (IS: 456) recommends the empirical relation between the static modulus of elasticity and cube compressive strength of concrete as:

$$Ec = 5000 fc \tag{1}$$

The ACI code (ACI-318) (Beeby and Narayanan, 1995b) defines the relationship between elastic modulus of concrete and cylinder compressive strength for calculating deflection as:

$$Ec = 4734\sqrt{f'c} \tag{2}$$

The New Zealand Code (NZS-3101) defines elastic modulus for normal strength concrete as:

$$Ec = 4734 \left( \sqrt{f'c} + 6900 \right) \tag{3}$$

The Euro-code (Beeby and Narayanan, 1995a) recommends the following equation for static modulus of elasticity of concrete from its cube compressive strength of concrete as:

$$Ec = 9500(\sqrt{f'c} + 8)0.33 \tag{4}$$

The British Code of practice (BS-8110) recommends the following expression for static modulus of elasticity with cube compressive strength of concrete as:

$$Ec = 20000 + 0.2fc \tag{5}$$

where,

 $E_c$ : The static modulus of elasticity at 28 days in Mpa

 $f_c$ : Cube compressive strength at 28 days in Mpa

f'<sub>c</sub> : Cylinder compressive strength at 28 days in Mpa

Also, the Indian code of practice (IS 456) recommends the empirical relation between the static modulus of rupture and cube compressive strength of concrete as:

$$fr = 0.7\sqrt{fc} \tag{6}$$

The ACI Code (ACI-318), (Beeby and Narayanan, 1995b) defines the flexural tensile or modulus of rupture of concrete as:

$$fr = 0.62\sqrt{f'c} \tag{7}$$

The New Zealand Code (NZS-3101) defines flexural tensile or modulus of rupture for normal strength concrete as:

$$fr = 0.60\sqrt{f'c} \tag{8}$$

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The Euro-Code (EC-02) (Beeby and Narayanan, 1995a) recommends the relationship between flexural tensile or modulus of rupture of concrete and cube compressive strength of concrete as:

$$fr = 0.3 (fc) 0.67 \tag{9}$$

The Canadian Code of Practice (CSA) (Beeby and Narayanan, 1995c) defines the flexural tensile or modulus of rupture of concrete from its cylinder compressive strength of concrete as:

$$fr = 0.60\sqrt{f'c} \tag{10}$$

where,

- fr : Flexural tensile or modulus of rupture of concrete in Mpa
- $f_c$ : Cube compressive strength at 28 days in Mpa
- f'<sub>c</sub> : Cylinder compressive strength at 28 days in Mpa

To determine the modulus of elasticity of concrete precisely is very important for structures that require strict control of the deformability. International codes propose a wide variety of formulae which establish a relationship between modulus of elasticity and compressive strength. Most of these codes are valid up to 50 MPa, the usual limit for normal concretes. Logan *et al.* (2009) studied the short term mechanical properties of concrete. The elastic deformation characteristics of concrete with thirteen types of common aggregates are investigated to estimate the modulus of concrete and a modification of the BS code approach is suggested.

Alendar (2001) reported that, the applicability of the BS approach to different aggregates appreciating the enormous influence on concrete properties in particularly in elastic modulus.

Fawzi *et al.* (2013) reported the mechanical property of partial replacement of metakaolin in lightweight porcelinate aggregate concrete improves the strength properties and modulus of elasticity.

Gutierrez and Canovas (1995a to e) established the modulus of elasticity-compressive strength curve for low and high strength concretes i.e., 120 MPa. In addition to that the effect of aggregate is also studied and reported that the model code formula provides good correlation with experiment up to compressive strength of 120 MPa.

All the above empirical relationship is only for plain concretes. Therefore, these experiments focused on establishing an empirical relationship between static modulus of elasticity and modulus of rupture based on the compressive strength of concrete containing Metakaolin.

Properties	Observed values	Specification limit
Brightness (%) (ISO)	76.80%	75-80%
Bulk density (gm/L)	297	270-370
Oil absorption (gm/100 gm)	60.11%	45-65%
Water demand (gm/100 gm)	N/A	N/A
Particle size analysis:		
(a) D (50) -50% particle below	N/A	N/A
(b) % below 10 µm	N/A	N/A
(c) % below 2 µm	N/A	N/A
Moisture (Ex. work) (%)	0.36%	0.5% max
Specific gravity	N/A	N/A
pH (10% Aq.slury)	6.11	5.0-8.0
Viscosity concentration (% solid for 500 cps.)	N/A	N/A
Residue (on 400#) (%)	0.36%	1.0% max
Flow point (mL/100 gm)	N/A	N/A

Table 2: Concrete mix design details

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	Partial replacement	t i						
	of cement by	Cement	Metakaolin		C.A. in			
Mix	metakaolin, (%)	in kg/m <sup>3</sup>	in kg/m <sup>3</sup>	F.A. in kg/m <sup>3</sup>	kg/m <sup>3</sup>	W/C ratio	f <sub>c</sub> in Mpa	f <sub>c</sub> ' in Mpa
Mix-I	0.0	504.21	00.00	683.20	1108.13	0.28	62.44	49.95
Mix-II	8.0	463.87	40.33	683.20	1108.13	0.28	53.63	42.90
Mix-III	10.0	453.79	50.42	683.20	1108.13	0.28	68.77	55.01
Mix-IV	12.0	443.70	60.50	683.20	1108.13	0.28	57.17	45.73

Table 3: Com	parison of c	odal provisi	ons for static	modulus of e	elasticity Eq	n in N/mm <sup>2</sup>
rable 5. Com	pullison of c	ouur provisi	ons for static	mouulus or c	Justicity, L	2 III I V/IIIIII

	As per measured		As ACI:318	As per New Zealand	As per Euro code,	As per
Mixes	value, Ec	As per IS: 456 code	code	code, NZS: 3101	EC: 02	BS: 8110
Mix-I	27405.07	39509.49	33458.36	30364.67	36269.05	20012.48
Mix-II	25399.01	36616.25	31008.24	28646.38	34749.75	20010.72
Mix-III	28761.55	41463.83	35113.38	31524.44	37285.70	20013.75
Mix-IV	26221.93	37805.42	32015.28	29352.62	35376.19	20011.43

Mix-I: Plain concrete; Mix-II: Plain concrete + 8% metakaolin by partial replacement of cement; Mix-III: Plain concrete + 10% metakaolin by partial replacement of cement; Mix-IV: Plain concrete + 12% metakaolin by partial replacement of cement

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Fig. 1: Shows the experimental setup

## METHODOLOGY

**Research significance:** This study provides information on the relationship between experimentally obtained modulus of elasticity, modulus of rupture and compressive strength of concrete containing metakaolin at 28 days. From the experimental results, the comparison of mechanical properties of concrete are derived from the codes of various countries. An attempt is made to form an empirical modulus of rupture using compressive strength of concrete containing Metakaolin up to 12%.

**Objectives:** For localized materials and conditions the effect of Metakaolin on the relationships between static modulus of elasticity, modulus of rupture and compressive strength of concrete has not been clearly established:

- To study the design codes of various countries for understanding the static modulus of elasticity, modulus of rupture and compressive strength relationships and compared with the relationships to concrete containing Metakaolin.
- To propose new relationships linking static elastic modulus, modulus of rupture and compressive strength of Metakaolin based concreted by experimentally obtained results.

#### **Experimental program:**

**Material properties:** The materials consisted of 53grade Ordinary Portland Cement, Natural River Sand, Crushed Granite Coarse Aggregate of maximum size 12.5 mm, Ordinary portable water for mixing and curing and a Super Plasticizing admixture. High Reactive Metakaolin used in this investigation was procured from 20 Microns Private Limited.

Figure 1 shows the experimental setup, Table 1 Shows the properties of Metakaolin, Table 2 Shows the Concrete mix design details and Table 3 Shows the comparison of codal provisions for static modulus of elasticity, Ec in N/mm<sup>2</sup>.

## **RESULTS AND DISCUSSION**

**Static modulus of elasticity:** A comparison of static modulus of elasticity obtained experimentally and that obtained from the empirical expressions given by the various design codes for both plain concrete and Metakaolin concrete is presented in Fig. 2 and 3.

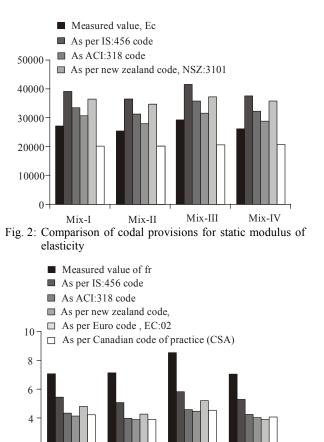


Fig. 3: Comparison of Mixal provisions for flexural tensile strength concrete

The figures show the modulus of elasticity predicted by IS: 456-2000 and EC:02 are higher than those predicted by ACI:318 and NZS:3101. The figures also show that the experimentally obtained modulus of elasticity is higher than the corresponding modulus of elasticity calculated from the BS: 8110 codes.

Figure 2 shows the Modulus of Elasticity predicted by IS: 456-2000 (Beeby and Narayanan, 1995d) and EC-02-1995 (Beeby and Narayanan, 1995a) are higher than compare to other code prediction. From the Fig. 2 it is concluded that the measured value of Modulus of Elasticity (MOE) is lower side compare with IS: 456-2000, EC:02 code, ACI:318 and NZS:3101 code predictions and higher side compare to BS:8110. As the compressive strength of concrete varies, the measured and predicted values of concrete also varying.

Table 2 shows the details of cube and cylinder specimen of M60 concrete. From the table it is concluded that the incorporation of Metakaolin increases upto 10% and further addition the values are decreasing.

From the literature<sup>12</sup>, the isolated effect of Metakaolin increases the strength properties. Also the static modulus of elasticity increases with increasing of compressive strength upto 15%.

Based on the regression analysis of the experimentally obtained tests results, the proposed correlations of the modulus of elasticity and compressive strength of cube and cylinder for plain and Metakaolin based concrete are given below. For cube compressive strength:

$$Ec = C_1 \sqrt{fc} \tag{11}$$

For cylinder compressive strength:

$$Ec = C_2 \sqrt{f'c} \tag{12}$$

where,

- $E_c$  : The static modulus of elasticity at 28 days in Mpa
- $f_c$  : Cube compressive strength at 28 days in Mpa

f'<sub>c</sub> : Cylinder compressive strength at 28 days in Mpa

C<sub>1</sub>, C<sub>2</sub>: Constants given in the Table 4

**Modulus of rupture:** A comparison of static modulus of rupture obtained experimentally and that obtained from the empirical expressions given by the various design codes for both plain concrete and Metakaolin concrete is presented in Table 5.

From the Table 5, the flexural tensile strength of experimental values of concrete is in higher side compare to other code provisions. Table 6a and b are showing the details of empirical relationships between flexural tensile strength vs. cube compressive strength and flexural tensile strength vs. cylindrical compressive strength, respectively.

Table 4: (a) Constants for empirical relationship b	etween static modulus of elasticity and compressive strength
C <sub>1</sub> for cube compressive stren	oth

Mixes	As per measured value, Ec	As per IS: 456 code	As per ACI: 318 code	As per New Zealand code, NZS: 3101	As per Euro code, EC: 02	As per BS: 8110		
Mix-I	3469.00	5001.20	4235.23	3843.62	4591.01	2533.22		
Mix-II	3063.81	5002.21	4236.09	3913.43	4747.23	2733.70		
Mix-III	3452.76	5001.66	4235.63	3802.70	4497.67	2414.20		
Mix-IV	3103.18	5000.71	4234.82	3882.62	4679.39	2647.01		

Table 4: (b) Constants for empirical relationship between static modulus of elasticity and compressive strength

C<sub>2</sub> for cylinder compressive strength

		A	A ACI 210		а Г I	A DC
2.0	As per measured	As per	As per ACI: 318	As per New Zealand	As per Euro code,	As per BS:
Mixes	value, Ec	IS: 456 code	code	code, NZS: 3101	EC: 02	8110 code
Mix-I	3881.72	5596.24	4739.14	4300.94	5128.76	2834.62
Mix-II	3424.43	5590.26	4734.08	4373.49	5305.30	3055.07
Mix-III	3856.46	5595.65	4738.64	4254.31	5031.80	2700.91
Mix-IV	3468.50	5592.51	4735.98	4342.10	5233.16	2960.27

Table 5: Comparison of codal provisions for flexural tensile strength concrete, fr in N/mm<sup>2</sup>

		As per IS: 456		As per New Zealand	As per Euro	As per Canadian code
Mixes	Measured value, fr	code	As ACI: 318 code	code, NZS: 3101	code, EC: 02	of practice (CSA)
Mix-I	7.14	5.531	4.377	4.236	4.787	4.236
Mix-II	7.12	5.126	4.061	3.930	4.323	3.930
Mix-III	8.44	5.804	4.594	4.446	5.107	4.446
Mix-IV	7.02	5.292	4.192	4.057	3.885	4.057

Table 6: (a) Constants for empirical relationship between flexural tensile strength and compressive strength

	C <sub>1</sub> for cube compressive strength							
Mixes	As per measured value	As per IS: 456 code	As per ACI: 318 code	As per New Zealand code, NZS: 3101	As per Euro code, EC: 02	As per Canadian code of practice (CSA)		
Mix-I	0.903	0.700	0.554	0.536	0.605	0.536		
Mix-II	0.972	0.700	0.554	0.536	0.590	0.536		
Mix-III	1.018	0.700	0.554	0.536	0.616	0.536		
Mix-IV	0.928	0.678	0.537	0.519	0.571	0.519		

Table 6: (b) Constants for empirical relationship between flexural tensile strength and compressive strength

C<sub>2</sub> for cylinder compressive strength

	As per	As per IS: 456	As per ACI:	As per New Zealand	As per Euro	As per Canadian code
Mixes	measured value	code	318 code	code, NZS: 3101	code, EC: 02	of practice (CSA)
Mix-I	1.011	0.783	0.619	0.600	0.678	0.600
Mix-II	1.087	0.782	0.620	0.600	0.660	0.600
Mix-III	1.138	0.783	0.619	0.600	0.689	0.600
Mix-IV	1.038	0.758	0.600	0.581	0.639	0.581

The values of  $C_1$  and  $C_2$  are high for Mix-III due to higher percentage incorporation of Metakaolin in plain concrete and low for Mix-I i.e., plain concrete.

Based on the regression analysis of the experimentally obtained tests results, the proposed correlations between flexural tensile strength and compressive strength of cube and cylinder for plain and Metakaolin based concrete are given below.

For cube compressive strength:

$$fr = C_1 \sqrt{fc} \tag{13}$$

For cylinder compressive strength:

$$fr = C_2 \sqrt{f'c} \tag{14}$$

where,

- f<sub>r</sub> : The flexural tensile strength in Mpa
- $f_c$  : Cube compressive strength at 28 days in Mpa
- f'<sub>c</sub> : Cylinder compressive strength at 28 days in Mpa

 $C_1, C_2$ : Constants given in the Table 6.

## CONCLUSION

This study of the experimentally obtained elastic modulus, modulus of rupture of plain concrete containing Metakaolin and the corresponding codal provisions of select countries, led to the following conclusions:

- The experimental measured values of static modulus of elasticity of Metakaolin concrete are lower side compare to IS: 456-2000 Code, (Beeby and Narayanan, 1995d) ACI: 318 code and NZS: 3101 code provisions. The prediction of BS: 8110 code values are in higher side compare with measured values.
- IS: 456-2000 and EC: 02 predict higher modulus of elasticity than BS: 8110, ACI: 318 and NZS: 3101.
- The experimental flexural tensile strength was higher than the other for all the percentage of addition of Metakaolin in plain concrete.
- The new empirical relations for elastic modulus, modulus of rupture and compressive strength of concrete containing different dosage of Metakaolin in plain concrete are proposed.

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