# Research Article <br> Periodicity and Circumstances of Occultations of Some Bright Stars 

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#### Abstract

In this paper, we present a study of the phenomenon of lunar occultation of visible stars. In Indian astronomical texts, generally the lunar occultations of some particular visible stars like Regulus ( $\alpha$ Leonis) and Spica ( $\alpha$ Virginis) are considered important. We are presenting the results of the periodicity of this phenomenon. The instants of true conjunctions of the Moon with some of the stars are discussed. Further, these conjunctions are of greater interest when they turn out to be occultations. We have worked out the actual circumstances of some occultations of stars cited by earlier authors as also for modern times.


Keywords: Conjunction, occultation, parallax, regulus, semi-diameter, spica

## INTRODUCTION

The famous mathematical astronomer Meeus (1997), in recent years initiated fresh scientific investigations in the phenomenon of lunar occultations of the stars and the planets. Successive occultations of a star by the Moon (for the earth in general) occur in series, which are separated by periods during which the Moon does not occult the star. Within a series, there is an occultation at each conjunction of the Moon with the star, hence at intervals of 27.3216615 days the Moon's sidereal period. Again in Meeus (1983) provided a deeper study of this phenomenon in his book "Astronomical tables of the Sun, Moon and Planets". Meeus in his presentation, discusses the procedure with relevant data, to compute the occurrence and circumstances of the lunar occultation of a star. But the Moon can occult a star only when it actually covers even as in the case of solar eclipse. A lunar conjunction with a star turns out to be an occultation only when their latitudes are sufficiently close. Since the lunar orbit has a mean inclination of about $5^{\circ} 8^{\prime}$ with the ecliptic, it follows that only those stars whose latitudes, south or north, are less than about $5^{\circ} 8^{\prime}$ are eligible for occultation. However, the Moon's orbital plane is not fixed but oscillates so that the Moon's maximum latitude may go upto $6^{\circ} 21^{\prime}$. Thus, for example the visible stars which are eligible for lunar occultation are Regulus ( $\alpha$ Leonis), Spica ( $\alpha$ Virginis), Aldebaran ( $\alpha$ Tauri) and Anteres ( $\alpha$ Scorpii).

In the present study we have discussed the procedure for predicting the lunar occultation of a star
in particular the phenomenon and its periodicity are worked out for some important visible (to the naked eye) bright stars like Regulus ( $\alpha$ Leonis), Spica ( $\alpha$ Virginis), Aldebaran ( $\alpha$ Tauri) and Anteres ( $\alpha$ Scorpii) etc., we have shown the periods of actual occultations as also of no-occultations for different ranges of the difference in the latitudes of the Moon and the concerned star. Our investigation presented in this study not only corroborates the past records of occult observations as also supporting future investigations by the upcoming researchers in the field. While the modern procedure requires the use of the Besselian elements for the computations of the circumstances of an occultation, we have shown how the predictions are close to the results by using only the celestial longitudes and latitudes of the Moon and the concerned star. We have proposed a new and simple procedure called 'Improved Siddhantic procedure' (Balachandra and Padmaja, 2009) the word 'Siddhantic' means the classical procedure adopted by Indian astronomers.

## MATERIALS AND METHODS

Conditions for lunar occultation of a star: Suppose $\beta$ and $B$ are the latitudes respectively of a star and the Moon, $\Pi$ is the Moon's horizontal parallax and $s$ is the Moon's semi-diameter. The condition for occultation is that the absolute difference in latitudes should be less than the sum of the Moon's semi-diameter and parallax:

$$
\begin{equation*}
\text { i.e., }|B-\beta|<\Pi+s \tag{1}
\end{equation*}
$$

[^0]The minimum values of $\Pi$ and $s$ are respectively about 3223 ". 5 and $878^{\prime \prime} .5$ and the maximum value are 3672 ". 3 and $1000^{\prime \prime} .7$. Accordingly, the minimum and the maximum values of $(\Pi+s)$ are respectively 4102 ". 0 and $4673^{\prime \prime} .0$ i.e., $1^{\circ} 8^{\prime} 22^{\prime \prime}$ and $1^{\circ} 17^{\prime} 53$ ". Thus, we have:

- The lunar occultation of a star of latitude $\beta$ is certain if:
$|B-\beta|<1^{\circ} 8^{\prime} 22^{\prime \prime}$
- Occultation is not possible when $|B-\beta|>1^{\circ} 17^{\prime} 53^{\prime \prime}$
- The phenomenon is doubtful if $1^{\circ} 8^{\prime} 22^{\prime \prime}<|B-\beta|<$ $1^{\circ} 17{ }^{\prime} 53^{\prime \prime}$

Let us denote $(s+\Pi)$ by $S P$. Thus the condition for an occultation becomes:

$$
B-\beta= \pm(S P)
$$

Since, the latitude of a given star $\beta$ is known; we have to determine $B$ for the occurrence of an occultation. If $i$ is the Moon's orbital inclination, then we have:

$$
\begin{equation*}
\tan B=\tan i \sin (M-R) \tag{2}
\end{equation*}
$$

where, $M$ and $R$ are the longitudes of Moon and Moon's ascending node. The above formula can be approximated taking the Moon's orbital inclination as 308 '. 7 as:

$$
\begin{equation*}
B=\tan ^{-1}\left[308^{\prime} .7 \sin (M-R)\right] \tag{3}
\end{equation*}
$$

In the case of lunar occultations of the stars close to the ecliptic, the Moon's latitude has also to be correspondingly small. In that case, the above equation can be approximated as:

$$
\begin{equation*}
B=308 ' .7 \sin (M-R) \tag{4}
\end{equation*}
$$

If $\lambda$ is the stars longitude, in the case of lunar occultation, since the two bodies are in conjunction, the Moon's longitude also becomes $\lambda$. Accordingly, (4) becomes:

$$
\begin{equation*}
\sin (\lambda-R)=\beta \pm(\Pi+s) / 308^{\prime} .7 \tag{5}
\end{equation*}
$$

where, $\beta, s$ and $\Pi$ are expressed in terms of arc minutes. From (5) we get the longitude of the ascending node $R$ given by:

$$
\begin{equation*}
R=\lambda-\sin ^{-1}\left[\beta \pm(\Pi+s) / 308^{\prime} .7\right] \tag{6}
\end{equation*}
$$

Example 1: We shall consider the case of the occultation of Regulus. The ecliptical co-ordinates of the star are respectively, sidereallongitude $\lambda=125^{\circ} 58^{\prime} 21^{\prime \prime}$ and latitude $\beta=0^{\circ} 27^{\prime} 53^{\prime \prime}$. Taking $(\Pi+s)=68^{\prime} 22^{\prime \prime}$, the minimum value, we get:

$$
\begin{aligned}
& R=125^{\circ} 58^{\prime} 21^{\prime}-\sin ^{-1}\left[\left(27^{\prime} .883 \pm 68^{\prime} .3667\right) / 308^{\prime} .7\right] \\
& =125^{\circ} 58^{\prime} 21^{\prime \prime}-18^{\circ} 10^{\prime} 11^{\prime \prime} .78=107^{\circ} 48^{\prime} 9{ }^{\prime \prime} .22 \text { and } \\
& 125^{\circ} 58^{\prime} 21^{\prime \prime}+7^{\circ} 32^{\prime} 8^{\prime \prime} .23=133^{\circ} 30^{\prime} 29^{\prime \prime} .23
\end{aligned}
$$

Since, arcsine function yields two values, corresponding to $18^{\circ} 10^{\prime} 1^{\prime \prime} .78$ and $7^{\circ} 32^{\prime} 88^{\prime \prime} .23$, we get two more value viz., $161^{\circ} 49^{\prime} 58^{\prime \prime} .22$ and $172^{\circ} 27^{\prime} 51^{\prime \prime} .77$. Subtracting these two values from $\lambda$ we get two more values for the position of moon's ascending node:

```
125*58'21"-161*49'58". 22 \equiv 324*8'22".78" (adding
360)
125``58'21"+172*}27'51".77 \equiv298026'12".77
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Thus, totally we have four values for the sidereal position of the Moon's ascending node. Arranged in the descending order (since the node moves backward) these are: $324^{\circ} 8^{\prime} 22^{\prime \prime} .78,298^{\circ} 26^{\prime} 12^{\prime \prime} .77,133^{\circ} 30^{\prime} 29^{\prime \prime} .23$, $107^{\circ} 48^{\prime} 9^{\prime \prime} .22$. This means in sidereal period of the moon's ascending node about 18.6 years there are two series of continuous occultation's of star Regulus viz.:

- Moon's nodes motion from $324^{\circ} 8^{\prime} 22^{\prime \prime} .78$ star Markab ( $\alpha$ Pegasi) to $298^{\circ} 26^{\prime} 12^{\prime \prime} .77$ star $\beta$ Delphini i.e., over an interval of $25^{\circ} 42^{\prime} 10^{\prime \prime} .01$
- $133^{\circ} 30^{\prime} 29^{\prime \prime} .23$ star $\varepsilon$ Hydrae to $107^{\circ} 48^{\prime} 9^{\prime \prime} .22$ star 41 Arietis i.e., over an interval of $25^{\circ} 42^{\prime} 10^{\prime \prime} .01$

Since, nodes sidereal period is about 18.6 years, the time taken by the node to cover $25^{\circ} 42^{\prime} 10^{\prime \prime} .01$ is about 1.32798 years i.e., about 1 year, 4 months. For example the latest series of Regulus occultation was from 2007 January 7 upto 2008 May 12. The next series is from 2016 December 18 up to 2018 April 24.

Remark: It is estimated that:

- A star whose latitude is less than $3^{\circ} 56^{\prime}$ has two series of lunar occultation during the sidereal period of the Moon's node.
- For a star whose latitude lies between $3^{\circ} 56^{\prime}$ to $6^{\circ} 21^{\prime}$ has only one series of lunar occultation.
- Star whose latitude is greater than about $6^{\circ} 21^{\prime}$ is never occulted by the Moon.

Example 2: Lunar occultation of star Antares ( $\alpha$ Scorpii) its sidereal longitude $\lambda=225^{\circ} 54^{\prime} 19^{\prime \prime}$ and latitude $\beta=4^{\circ} 34^{\prime} 09^{\prime \prime}$. Since in this case star's latitude $\beta$ lies between $3^{\circ} 56^{\prime}$ and $6^{\circ} 21^{\prime}$ there is only one series of lunar occultation. This series is between moon's nodes successive sidereal longitudes $185^{\circ} .1448$ star Spica ( $\alpha$ Virginis) and $86^{\circ} .66572$ star Pollux ( $\alpha$ Germinorum) i.e., over an interval of $98^{\circ} .47908$. The time taken by the moon's mode to cover this interval is about 5.0880 years i.e., 5 years, one month. For example, the last series of the star occultation was from 2005 January 8 to 2010 February 8 . The next series will be from 2023 August 25 to 2028 August 27.

Possibility of a lunar conjunction becoming an occultation: A conjunction of a star with the Moon (in longitude) takes place in about every 27.32 days, the Moon's sidereal period. But every such conjunction need not be an occultation just as every new Moon day does not result in a solar eclipse. Even as there are ecliptic limits for a lunar or solar eclipse, there are lower and upper occultation limits for a star. As mentioned earlier, if the Moon's angular semi diameter and horizontal parallax are denoted respectively by $s$ and $\Pi$ we know that the minimum and maximum values of $(s+\Pi)$ are respectively $1^{\circ} .1394$ and $1^{\circ} .2989$. Let DLAT be the numerical difference between the latitudes of the Moon and the star i.e., if $B$ and $\beta$ are respectively the latitudes, positive or negative, of the Moon and the star than DLAT $=|B-\beta|$.

Remark: According to the Sūryasiddhānta, (Ebenezer Burgess, 1989) the apparent diameter of the Moon comes to $32^{\prime} 12^{\prime \prime}$. In modern astronomy the minimum and maximum values of the Moon's angular diameter are about $29^{\prime} .183,31^{\prime} .32$ and $33^{\prime} .3566$ respectively. In the siddhāntic texts the usual practice of determining the true angular diameter of the Moon is as follows:

$$
d_{m}=\frac{\text { Mean diameter } \times \text { Moon's true daily motion }}{\text { Moon's mean daily motion }}
$$

Since the Moon's mean daily motion $=790$ '. 57 , we get:

$$
d_{m}=\frac{\text { Moon's true daily motion }}{24.55186335} \text { in kalās }
$$

The above result divided by three yields Moon's true diameter in añgulas:

$$
d_{m}=\frac{\text { Moon's true daily motion }}{73.65559006} \text { in angulas }
$$

Gaṇeśha Daivajña in his Grahalagavm (Balachandra and Uma, 2006) gives:

$$
d_{m}=\frac{\text { Moon's true daily motion }}{74} \text { in aj̈gulas }
$$

Now, we have the following conditions for a lunar conjunction with a star to be an occultation:

- If DLAT is less than lower limit $1^{\circ} .1394$ then there is the occultation of the star.
- If DLAT is greater than the upper limit $1^{\circ} .2989$ there will be no occultation.
- If DLAT lies between the two limits $1^{\circ} .1394$ and $1^{\circ} .2989$ there may or may not be an occultation, in that case the possibility of occultation or otherwise is decided actually by determining the Moon's semi diameter $s$ and the horizontal parallax $\Pi$ at the instant of true conjunction of the Moon and the star. The occultation takes place if $(s+\Pi)$ is greater than DLAT.

Example 3: The lunar conjunctions of star Spica during the year 2013 every month the lunar conjunction of Spica turned out to be an occultation. On the other hand during the subsequent year 2014 the first conjunction on January 23 was a case of probable occultation. While none of the rest during the year was an occultation.

We shall now examine the probable case of occultation on January 23, 2014. In this case the sum $(s+\Pi)=1^{\circ} .20023$ and DLAT $=1^{\circ} .2893$. Since $(s+\Pi)$ $<$ DLAT there is no occultation.

During the year 1993 between January and September none of the conjunctions was an occultation. But the case of conjunction on 1993 October 15/16 was a case of probable occultation. On actual computation of $s, \Pi$ and DLAT we find that $(s+\Pi)=1^{\circ} .30075$ and DLAT $=1^{\circ} .26284$. Since $(s+\Pi)>$ DLAT occultation took place on that day.

Remark: The series of non occultation during the said year 1993 turned out to be "freak" situation of having an occultation on October $15 / 16$ while there was no occultation in the preceding months. This was followed by non occultation in the subsequent two months.

## RESULTS AND DISCUSSION

Variation of moon's latitude in an occultation cycle: Considering an occultation series of star Regulus during 2016-2018 we have listed the instants of conjunctions week day, Moon's latitude and DLAT. In the last column the occurrence or otherwise is mentioned as "No" and "Occn" (Table 1).

Table 1: Lunar conjunction/occultation of Regulus during 2016-2018

| Date | Day and time | Moon's latitude | DLAT | Occultation |
| :---: | :---: | :---: | :---: | :---: |
| 2016 Sept. 28 | Wed. $3^{\text {h }} 19^{\mathrm{m}}$ | $1^{\circ} 9^{\prime}(\mathrm{S})$ | $1^{\circ} .62$ | No |
| Nov. 21 | Mon. $15^{\mathrm{h}} 4^{\mathrm{m}}$ | $0^{\circ} 48^{\prime}$ (S) | $1^{\circ} .26$ | No |
| Dec. 18 | Sun. $23^{\text {h }} 23^{\mathrm{m}}$ | $0^{\circ} 31{ }^{\prime}$ (S) | $0^{\circ} .99$ | Occn |
| 2017 Feb. 11 | Sat. $19{ }^{\text {h }} 29^{\text {m }}$ | $0^{\circ} 18^{\prime}$ (S) | $0^{\circ} .77$ | Occn |
| Apr. 7 | Fri. $9^{\mathrm{h}} 55^{\text {m }}$ | $0^{\circ} 14{ }^{\prime}$ (S) | $0^{\circ} .70$ | Occn |
| Jun. 28 | Wed. $6^{\text {h }} 14^{\text {m }}$ | $0^{\circ} 26^{\prime}$ (N) | $0^{\circ} .04$ | Occn |
| Aug. 22 | Tue. $2^{\mathrm{h}} 3^{\mathrm{m}}$ | $0^{\circ} 32^{\prime}$ (N) | $0^{\circ} .07$ | Occn |
| Oct. 15 | Sun. $16^{\mathrm{h}} 55^{\mathrm{m}}$ | $0^{\circ} 40^{\prime}$ (N) | $0^{\circ} .20$ | Occn |
| Dec. 9 | Sat. $4^{\text {h }} 33^{\text {m }}$ | $1^{\circ} 10^{\prime}(\mathrm{N})$ | $0^{\circ} .70$ | Occn |
| 2018 Feb. 2 | Fri. $0{ }^{\text {h }} 51^{\mathrm{m}}$ | $1^{\circ} 23^{\prime}(\mathrm{N})$ | $0^{\circ} .92$ | Occn |
| Apr. 25 | Wed. $2^{\text {h }} 19^{\text {m }}$ | $1^{\circ} 38^{\prime}(\mathrm{N})$ | $1^{\circ} .17$ | Occn |
| May. 22 | Tue. $7^{\mathrm{h}} 40^{\mathrm{m}}$ | $1^{\circ} 53{ }^{\prime}$ (N) | $1^{\circ} .42$ | No |
| Jul. 15 | Sun. $23^{\text {h }} 9^{\text {m }}$ | $2^{\circ} 10^{\prime}(\mathrm{N})$ | $1^{\circ} .70$ | No |

Table 2: Length of series and interval between successive series

| Star | Magnitude | Latitude | Interval between |
| :--- | :--- | :--- | :--- | :--- |
| successive beginnings |  |  |  |



Fig. 1: Lunar occultation of star Regulus

Figure 1 gives the image of lunar occultation of star Regulus which will happen on 2016 Dec. 18, clearly it is seen that star Regulus (highlighted) is occulting moon; this image is taken from Stellarium software.

## Periodicity of occultation cycles of stars:

For stars latitude $<\mathbf{3}^{\circ} \mathbf{5 6}$ ': In Table 2, we have compared the lengths of each series of occultations, the interval between two successive beginnings of the series for four stars whose latitudes numerically are less than $3^{\circ} 566^{\prime}$. From Table 2, we observe that:

- As the latitude numerically increases from $0^{\circ}$ to $3^{\circ} 56^{\prime}$ the length of each series goes on increasing with a minimum of about 1.4 years for Regulus and a maximum of about 2.02 years for $\delta$ Sagittarii.
- On the other hand, the interval between two successive series decreases from a maximum of about 9.854 years for Regulus to a minimum of 4.614 years for $\delta$ Sagittarii.

For stars latitude between $3^{\circ} 56^{\prime}$ to $6^{\circ} 21^{\prime}$ : As pointed out earlier the stars in this range of latitude, have only one series of occultation in the node's period of about 18.6 years. In Table 3, we have compared the series length for different stars. From Table 3, we note that as the latitude of a star increases numerically from $4^{\circ} 03^{\prime} 02^{\prime \prime}$ for Alcyone ( $\eta$ Tauri) to a maximum of $5^{\circ} 28^{\prime} 04^{\prime \prime}$ for Aldebaran the series length decreases from 5.838 to 3.529 years.

Circumstance of lunar occultation of a star: Even as in the case of an eclipse an occultation will have its circumstances namely, the beginning (First contact), the

Table 3: Length of series

| Star | Magnitude | Latitude | Series length |
| :--- | :--- | :--- | :--- |
| Alcyone, $\eta$ tauri | 2.96 | $4^{\circ} 03^{\prime} 02^{\prime \prime}$ | 5.8390 |
| Antares, $\alpha$ scorpii | 1.20 | $-4^{\circ} 34^{\prime} 09^{\prime \prime}$ | 5.0880 |
| Aldebaran | 1.06 | $-5^{\circ} 28^{\prime} 04 \prime$ | 3.5290 |

Table 4: Historical references to lunar occultations of stars

| Table 4: Historical references to lunar occultations of stars |  |  |
| :--- | :--- | :--- |
| Star | Date | Reference |
| $\beta$ scorpii | 259 B.C. Dec. 21 | Timocharis, Alexandria |
| $\alpha$ virginis | 294 B.C. March 9 | Timocharis, Alexandria |
| Pleiades | 283 B.C. Jan. 29 | Timocharis, Alexandria |
| Spica | 283 B.C. Nov. 9 | Timocharis, Alexandria |
| Pleiades | 92 Nov. 29 | Agrippa, Bithynia |
| $\alpha$ virginis | 98 Jan. 11 | Menelaus, Rome |
| $\beta$ scorpii | 98 Jan. 14 | Menelaus, Rome |
| Regulus | 139 Feb. 23 | Ptolemy, Alexandria |
| Aldebaran | 1497 March 9 | De Revolutionibus |
| Spica | 1883 Jan. 29 | Cited by Venkatesh |

second contact, the third contact and the end (Fourth contact). In between the second and the third contacts the concerned star will be totally hidden by the moon.

Example 1: The lunar occultation of Spica on 2006 April 13 (Balachandra et al., 2008):

The instant of conjunction $22^{\mathrm{h}} 47^{\mathrm{m}}$ IST, Moon's latitude, $B=-1^{\circ} 45^{\prime} .8$
Star's latitude, $\beta=-2^{\circ} 2^{\prime} .7$, DLAT $=16^{\prime} .9$, Moon's horizontal parallax, $\Pi=55^{\prime} .008$, Moon's angular semi diameter $s=14$ '. 99
I contact $=20^{\mathrm{h}} 36^{\mathrm{m}}$, II contact $=20^{\mathrm{h}} 37^{\mathrm{m}}$,
Middle $=22^{\mathrm{h}} 50^{\mathrm{m}}$, III contact $=25^{\mathrm{h}} 2^{\mathrm{m}}$
IV contact $=25^{\mathrm{h}} 3^{\mathrm{m}}$
Some historical references to occultations of bright stars: In the famous text of Ptolemy's Almagest and Copernicus' De-Revolutionibus we get some interesting references to the occultations of some bright stars observed by the two great astronomers or their predecessors we provide some of these references in Table 4. The observations of occultations between the years -282 and 98 are from Pedersen (1974) A Survey of the Almagest.

Improved siddantic procedures for occultations: The Indian astronomical texts have discussed in detail the phenomenon of conjunctions of the Sun, the Moon and the planets, between any two of them as also with some bright stars like Aldebaran, Regulus and Spica. The conjunction of two heavenly bodies is said to take place when their celestial longitudes are equal. Depending on the bodies involved, their conjunctions are classified as follows in Sūryasiddhānta.
If the conjunction is:

- Between two planets it is called yuddha (war or encounter).
- Between a planet and the Moon then it is called samāgama. In modern parlance we call it lunar occultation.
- Between a planet and the Sun it is called astamana or astañgata (heliacal setting). The transits of the interior planets Mercury and Venus are special cases of this category.

In the case of Mercury or Venus an inferior conjunction can result in a transit. In addition, these two planets can have a superior conjunction with the Sun in which case these bodies will be covered by the Sun.

For superior planets viz., Mars, Jupiter and Saturn the conjunction results in a planet coming behind the Sun as observed from the Earth. This is also a case of heliacal setting.

Note: The classical Indian texts have given great importance for the lunar conjunction of a fixed star. As pointed out earlier lunar conjunction of a star or a planet can result in an occultation, even as in an eclipse, if the difference between the latitudes of the two bodies is within the prescribed limits.

In modern astronomy circumstances of an occultation of a star or a planet is worked out by using the Besselian elements. This procedure, though yields accurate results, is rather cumbersome. We have evolved a simple but effective procedure called Improved Siddhantic Procedure (ISP) (Balachandraand Padmaja, 2009) within the framework of Indian classical texts. We explain the ISP by working out historically important cases of lunar occultation.

Example 2: Timocharis (320-260 B.C.) has recorded his observation of the occultation of Spica (Citra $\bar{a}$ ) on 293 March 9 at Alexandria about 4 seasonal h before midnight. The instant of lunar conjunction of Spica: about 27 h (IST), true tropical longitude of the Moon and the star $=171^{\circ} .36$, Moon's latitude, $B=-93$ ', Star's latitude $=-123^{\prime} .25$, DLAT $=30^{\prime} .25$, Moon's horizontal parallax, $\pi=55$ '.1438, Moon's angular semi-diameter, $s=15^{\prime} .0257$. The circumstances of the occultation are as follows:

External Ingress $=25^{\mathrm{h}} 1^{\mathrm{m}} .2$, Middle $=26^{\mathrm{h}} 56^{\mathrm{m}}$ and External egress $=28^{\mathrm{h}} 50^{\mathrm{m}} .7$

Remark: We have middle of the occultation at $26^{\mathrm{h}} 56^{\mathrm{m}}$ (IST) this corrected for $\Delta \mathrm{T} \approx 4 \mathrm{~h}$ we get $22^{\mathrm{h}} 26^{\mathrm{m}}$ (IST). Now, for Alexandria (longitude: 595' (E)) the civil time comes to $21^{\mathrm{h}} 26^{\mathrm{m}}$ (LMT). Timocharis has given the time of his observation as 20 seasonal h .

Example 3: Astronomer Agrippa (c. $1^{\text {st }}$ cent. C.E.) has recorded his observation of the lunar occultation in Bithynia of the Pleiades which took place on the night of between $29^{\text {th }}$ and $30^{\text {th }}$ of November, 92 C.E. The brightest star in that group corresponds to Alcyone. The instant of the lunar conjunction with the star is about $22^{\mathrm{h}} 19^{\mathrm{m}}$ (Indian Standard Time (IST)), true tropical
longitudes of the star and the Moon $=33^{\circ} .4736$, instant of conjunction $=22^{\mathrm{h}} 19^{\mathrm{m}}$ (IST).

Star's latitude, $\beta=243$ '.0334, Moon's latitude, $B=294 '$ and difference in latitude, DLAT $=50 ' .967$, Moon's parallax, $\Pi=54$ '. 4349 , Moon's angular semi diameter, $s=14$ '.8326. Now, by ISP we have the following circumstances of the occultation of Alcyon:

External Ingress $=20^{\mathrm{h}} 37^{\mathrm{m}} \cdot 6$, Middle $=22^{\mathrm{h}} 20^{\mathrm{m}} \cdot 9$ and External egress $=24^{\mathrm{h}} 4^{\mathrm{m}} .2$

## Remark:

- The beginning and the end of occultation are respectively at $20^{\mathrm{h}} 37^{\mathrm{m}} .6$ and $24^{\mathrm{h}} 20^{\mathrm{m}} 4.2^{\mathrm{s}}$. Locally at Alexandria, Timocharis mentions his time of observation as about four seasonal hours before midnight. This time is close to the beginning of the occultation.
- Kim (2012) refers to the ancient text śata-patha Brāhmaṇa which mentions the stars Alcyon (Kṛttikās) a group consisting of the conjunctionstar (yogatārā) Krttikā (identified as Alcyon) and six less bright stars comprising the Pleiades group.

Example 4: Menelaus (70-140 C.E.) records his observation of Spica on 98, January, 11 at Rome. By ISP we get the following. The instant of lunar conjunction of Spica: 10 h (IST), true tropical longitude of the Moon and the star $=177^{\circ} 23^{\prime} 21^{\prime \prime}$, Moon's latitude, $B=1^{\circ} 22^{\prime}(\mathrm{S})$, DLAT $=1^{\circ} 8^{\prime}$, Moon's horizontal parallax, $\pi=57^{\prime} .14993$, Moon's angular semi-diameter, $s=15 ' .572293$. The circumstances of the occultation are as follows:

External Ingress $=7^{\mathrm{h}} 49^{\mathrm{m}} 31^{\mathrm{s}}$, Middle $=9^{\mathrm{h}} 51^{\mathrm{m}} 54^{\mathrm{s}}$ and External egress $=11^{\mathrm{h}} 54^{\mathrm{m}} 16^{\mathrm{s}}$

## Remark:

- We have middle of the occultation at $9^{\mathrm{h}} 51^{\mathrm{m}} 54^{\mathrm{s}}$. This corrected for $\Delta \mathrm{T} \approx 2^{\mathrm{h}} 24^{\mathrm{m}} 37 \mathrm{~s} .61$ we get $7^{\mathrm{h}} 27^{\mathrm{m}} 16^{\mathrm{s}} .31$ (IST). Now for Rome (longitude $=12^{\circ} 27^{\prime} \mathrm{E}$ ) the civil time comes to be $2^{\mathrm{h}} 47^{\mathrm{m}} 4^{\mathrm{s}} .31$ (LMT). Menelaus has given the time of his observation as 4 seasonal $h$ (LMT) after midnight.
- Kim (2012) referring again to śata-patha Brāhmaṇa points out that if the references to the two neighbouring stars Aldebaran and $\lambda$ Orionis symbolizes the shift of the vernal equinoxes, then the event to be dated approximately as 4000 B.C.E.

Example 5: Copernicus records in his De Revolutionibus (Swerdlow and Neugebauer, 1984) his observation of the occultation of Aldebaran on 1497 March 9 at Bologna one hour before midnight. The
instant of lunar conjunction of Aldebaran $=26^{\mathrm{h}} 15^{\mathrm{m}}$ (IST), true topical longitude of the Moon and the star $=62^{\circ} .833$, Moon's latitude, $B=-288^{\prime}$, Star's latitude, $\beta=-310^{\prime}$, DLAT $=-22^{\prime}$, Moon's horizontal parallax, $\pi=56$ '.852, Moon's angular semi-diameter, $s=15^{\prime} .49$. The following are the circumstances of the occultation obtained by ISP:

External Ingress $=24^{\mathrm{h}} 7^{\mathrm{m}} .8$, Middle $=26^{\mathrm{h}} 15^{\mathrm{m}}$ and External egress $=28^{\mathrm{h}} 22^{\mathrm{m}} .2$

Remark: We have middle of the occultation at $26^{\mathrm{h}} 15^{\mathrm{m}}$ (IST). This when corrected for $\Delta \mathrm{T} \approx 3^{\mathrm{m}} 49^{\mathrm{s}}$ we get $26^{\mathrm{h}} 11^{\mathrm{m}} 11^{\mathrm{s}}$ (IST). Now, for Balogna (longitude $11^{\circ} 21^{\prime} \mathrm{E}$ ) the civil time comes to $21^{\mathrm{h}} 26^{\mathrm{m}} 11^{\mathrm{s}}$ (LMT). The beginning and the end of occultation in Balogna time are respectively $19^{\mathrm{h}} 18^{\mathrm{m}} .8$ and $23^{\mathrm{h}} 33^{\mathrm{m}} .2$. The time of observation mentioned by Copernicus is 1 h before the local midnight 23 h (LMT). This time occurs indeed during the occurrence of the occultation.

## CONCLUSION

In the preceding section we have discussed the condition for a lunar conjunction of some bright stars to become occultations we have noted that in the case of stars whose celestial latitudes are less than about $3^{\circ} 56^{\prime}$ there will be two series of occultations in a cycle of about 18.6 years, the sidereal period of the Moon's node. Those stars whose latitudes, south or north, lie between $3^{\circ} 56^{\prime}$ and $6^{\circ} 21^{\prime}$ will have only one cycle of occultation in a period of almost 18.6 years. On the other hand, if a star's latitude exceeds about $6^{\circ} 21^{\prime}$ it's conjunction with the moon can never be an occultation.

We have also presented apparently freak possibilities in which amidst a series of occultations there arise one or two cases of no occultation. So also, there can arise a case of occurrence of occultation amidst a series of no occultations.

Further, lunar conjunction of a star is sure case of occultation if DLAT, the difference between the latitudes of the two bodies is less than the lower limit $1^{\circ} .20023$ similarly, when DLAT is greater than upper limit 1.30075 the lunar conjunction of the star will not be an occultation. The interesting situation is when DLAT lies between the two limits this situation is designated as 'probable' case of occultation. We have given two examples of stars lunar conjunction -one case of 'probable' occultation turns out to be an actual occultation and in the other case an actual occultation and in the other case the 'probable' situation yields no occultation.

Finally, in above section we have given a list of historically important references to the lunar occultation of some bright stars recorded by Ptolemy, Copernicus and others.

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