

## Research Article

# Combined Effects of Gain and Two-photon Absorption on Dark-bright Vector Soliton Propagation and Interaction

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**Abstract:** Interaction as well as dynamics of the dark-bright vector solitons induced by incorporation of gain and two-photon absorption are investigated by the numerical method in a birefringent fiber. The numerical results show that the gain and the two-photon absorption may lead to the destructive effects on the soliton propagation and interaction. Their combined effects play a crucial role in the dynamics and depend strictly on the sign and magnitude of their coefficients, so the proper incorporation may reduce effectively their destructive effects. The nonlinear gain combined with the filter can be used to suppress effectively the disadvantaged effects and the stabilization mechanism is demonstrated.

**Keywords:** Combined effect, dark-bright vector soliton, gain, two-photon absorption

## INTRODUCTION

Optical solitons in fibers are formed by a balance of group dispersion and Kerr non-linearity, numerical simulation and experiments have demonstrated solitons can propagate an extended distance without distortion, so they maybe become the ideal message carrier in long distance communication (Hasegawa and Kodama, 1995).

There are many perturbations in a practical soliton system, which lead to the fluctuation in amplitude or pulse-width of soliton and deform the soliton shape. Their effects cannot be neglected on the soliton system characteristics, such as degrading the stability of coherently amplified solitons, increasing the time jitters in arrival and reducing the signal-noise-ratio of communication system. Specially, an optical fiber exhibits birefringence because of the asymmetry of the refractive index profile of the core. If the birefringence is randomly varying during the soliton propagation, the traveling pulses will experience waveform distortion; a dispersive wave radiation will appear in the system and result in the decrease of transmission capacity (Kivshar and Davies, 1998; Wang *et al.*, 2005).

So-called coupled dark-bright vector soliton, where a bright optical solitary wave exists in a system with defocusing nonlinearity because it is trapped within a co-propagating dark soliton, has some interesting and distinguishing dynamics, which are different from those of the bright soliton and the dark soliton. Nonlinear effects and perturbations (i.e., high-order effects and the

periodically inhomogeneous birefringence) may modify the effects of the short-ranged interactions between solitons (Li and Wang, 2007a; Li *et al.*, 2005). For example, the relative phase of the two bright components strongly affects the details of the interaction and dark-bright vector solitons may repel or attract each other. This short range behavior is opposite to that of the scalar bright solitons and their phase-dependent interaction can significantly affect their oscillations (Liu *et al.*, 1995a, b).

Two-Photon Absorption (TPA) is a high-order nonlinear optical process. Specially, the imaginary part of the third-order nonlinear susceptibility is related to the extent of the two-photon absorption in a given molecule. If there are the gain and the two-photon absorption in the birefringence fiber during the soliton propagation, the traveling pulses will experience waveform distortion and frequency shift; a dispersive wave radiation may appear in the system and result in the decrease of transmission capacity. The effect of the two-photon absorption on the soliton is not neglected and may be compensated for by the frequency-dependent optical gain in an optical fiber (Li and Wang, 2007b).

Former limiting factors for optical communication capacity, such as fiber local loss and fiber dispersion, have been overcome with the development of optical amplifiers and special fibers. Now perturbations caused by the high-order nonlinear optical process become leading limiting factors with the communication capacity increasing and may be the final limiting factor. Today

the research region has become the focus on optical fiber communication. Optical fibers are birefringent in reality and pulses travel along two orthogonal polarizations of the fiber at slightly different speeds. A technique called polarization-division multiplexing has been proposed and this technique doubles the transmission rate compared to the launching of pulses along the same polarizations. In fact, theory and experiments show the single-wavelength bit-rate capacity of an ultra-long distance soliton transmission system can be doubled by the polarization-division multiplexing of orthogonal polarization solitons and the capacity can be furtherly increased by using a combination of polarization and other multiplexing (such as time-division multiplexing and wavelength-division multiplexing) (Maxum *et al.*, 2008).

Recent numerical and analytical theoretical studies demonstrated both stable and unstable evolution of the vector soliton in the birefringent fiber and show that the high-order nonlinear optical process plays a crucial role in the physical features of the vector solitons, such as the modulation instability and transmission capacity. In the study, the interaction and dynamics of the dark-bright vector solitons induced by incorporation of the gain and the two-photon absorption are investigated.

### THEORETICAL FORMULA AND NUMERICAL RESULTS

In a real transmission system, the envelop of the dark-bright vector soliton becomes inhomogeneous because of the gain and the two-photon absorption in a birefringent fiber and can be described by the coupled nonlinear Schrödinger equation below (Li and Wang, 2007b; Li *et al.*, 2005; Liu *et al.*, 1995a, b):

$$\begin{aligned}
 j \frac{\partial u_1}{\partial Z} + j \eta \frac{\partial u_1}{\partial \tau} - \frac{1}{2} \frac{\partial^2 u_1}{\partial \tau^2} + (|u_1|^2 + \frac{2}{3} |u_2|^2) u_1 &= j \gamma u_1 + j \alpha |u_1|^2 u_1, \\
 j \frac{\partial u_2}{\partial Z} - j \eta \frac{\partial u_2}{\partial \tau} - \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + (|u_2|^2 + \frac{2}{3} |u_1|^2) u_2 &= j \gamma u_2 + j \alpha |u_2|^2 u_2,
 \end{aligned}
 \tag{1}$$

where,  $u_i$  ( $i = 1, 2$ ) is each normalized elliptically polarized component along two orthogonal directions, respectively.  $\eta$  is the group-velocity delay caused by the birefringence in the fiber.  $Z = z/z_d$ ,  $z_d = \tau_0^2/|\bar{d}|$  and  $\tau = t/\tau_0$ .  $z$  and  $t$  are actual distance coordinates and time.  $\tau_0$ ,  $z_d$  and  $\bar{d}$  are the pulse-width, the dispersion length and the path-average dispersion, respectively.  $\gamma$  and  $\alpha$  are the coefficients of the gain and the two-photon absorption, respectively.

From the evolution Eq. (1), we can see the motion equations for the bright soliton are the standard nonlinear Schrödinger equations when there is no the gain and the two-photon absorption. Namely, the special case coincides with the optical bright soliton under the framework of Eq. (1) without the perturbation (gain and the two-photon absorption). The result means that the soliton propagation and interaction are modulated by the gain and the two-photon absorption. Although Eq. (1) has a Hamiltonian structure, it is not exactly integrable because of the perturbations (gain and the two-photon absorption, and  $z$  dependent coefficient). Such a behavior of the normalized wavepacket functions  $u_i$  ( $i = 1, 2$ ) may be obtained as their response averaged over the perturbations (gain and the two-photon absorption), which result in path-averaged soliton. However, taking a simple average of the perturbations (gain and the two-photon absorption) fails to provide the proper response because of the correlations with variations of  $u_i$  ( $i = 1, 2$ ) and the perturbations. So the effects of the perturbations (gain

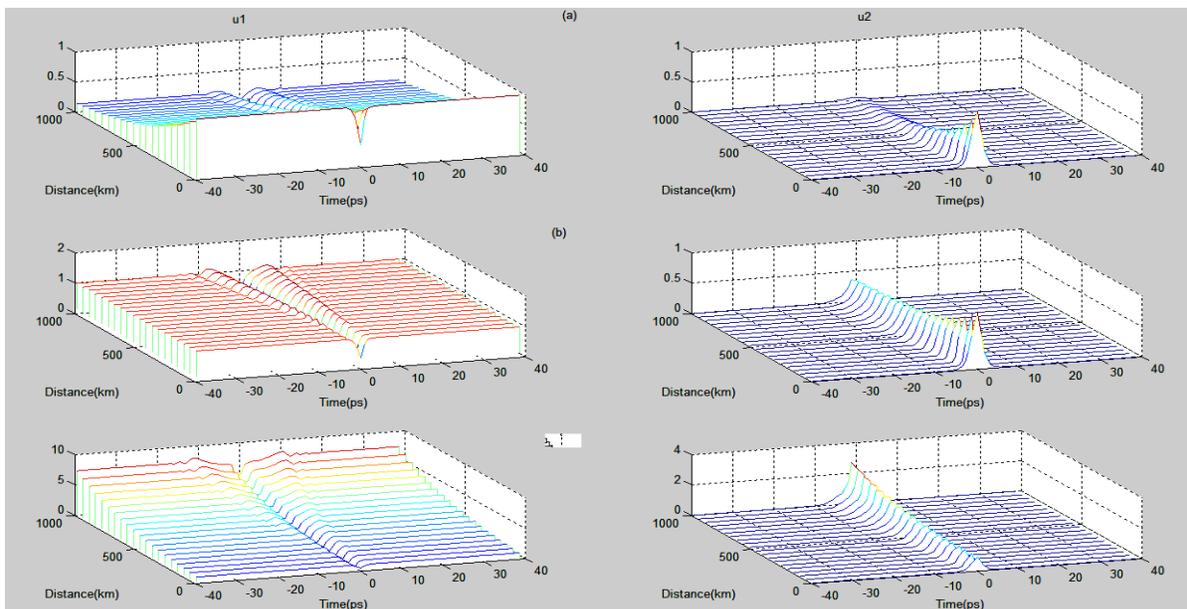


Fig. 1: The normalized vector soliton intensity versus the propagation distance, (a)  $\gamma = -0.2$  and  $\alpha = 0$ , (b)  $\gamma = 0$  and  $\alpha = 0$ , (c)  $\gamma = 0.2$  and  $\alpha = 0$

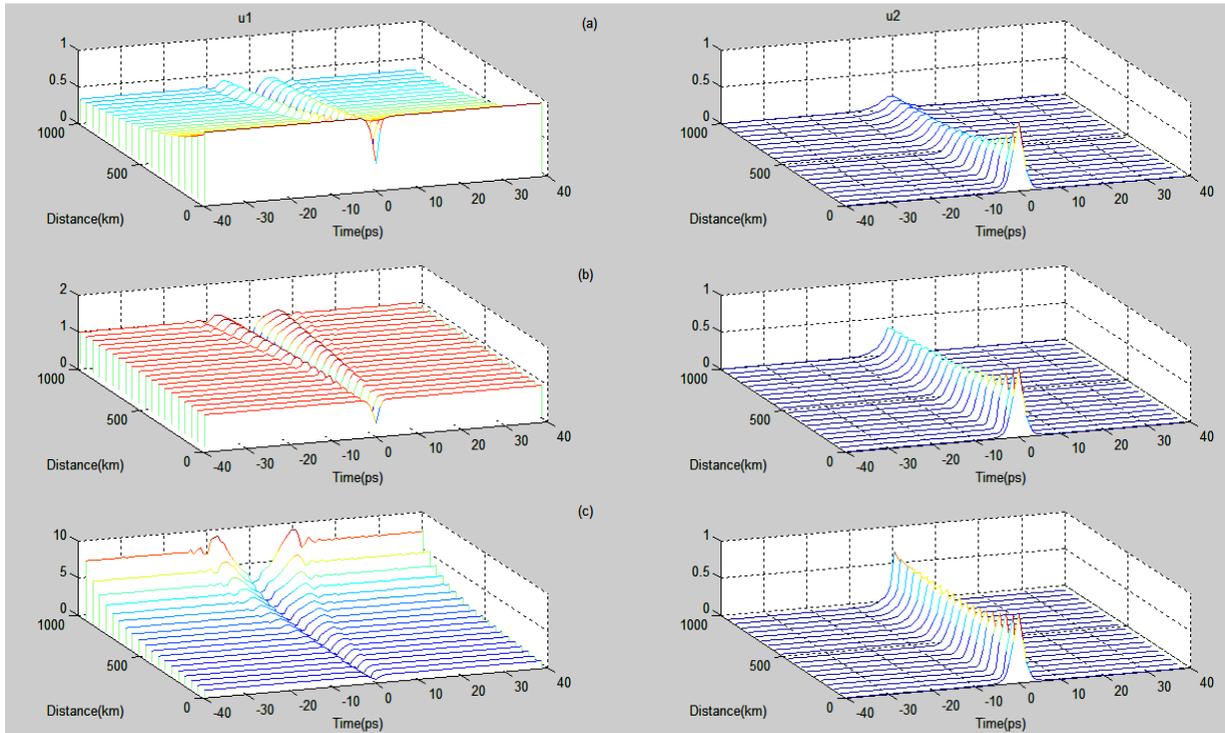


Fig. 2: The normalized vector soliton intensity versus the propagation distance, (a)  $\gamma = 0$  and  $\alpha = -0.2$ , (b)  $\gamma = 0$  and  $\alpha = 0$ , (c)  $\gamma = 0$  and  $\alpha = 0.2$

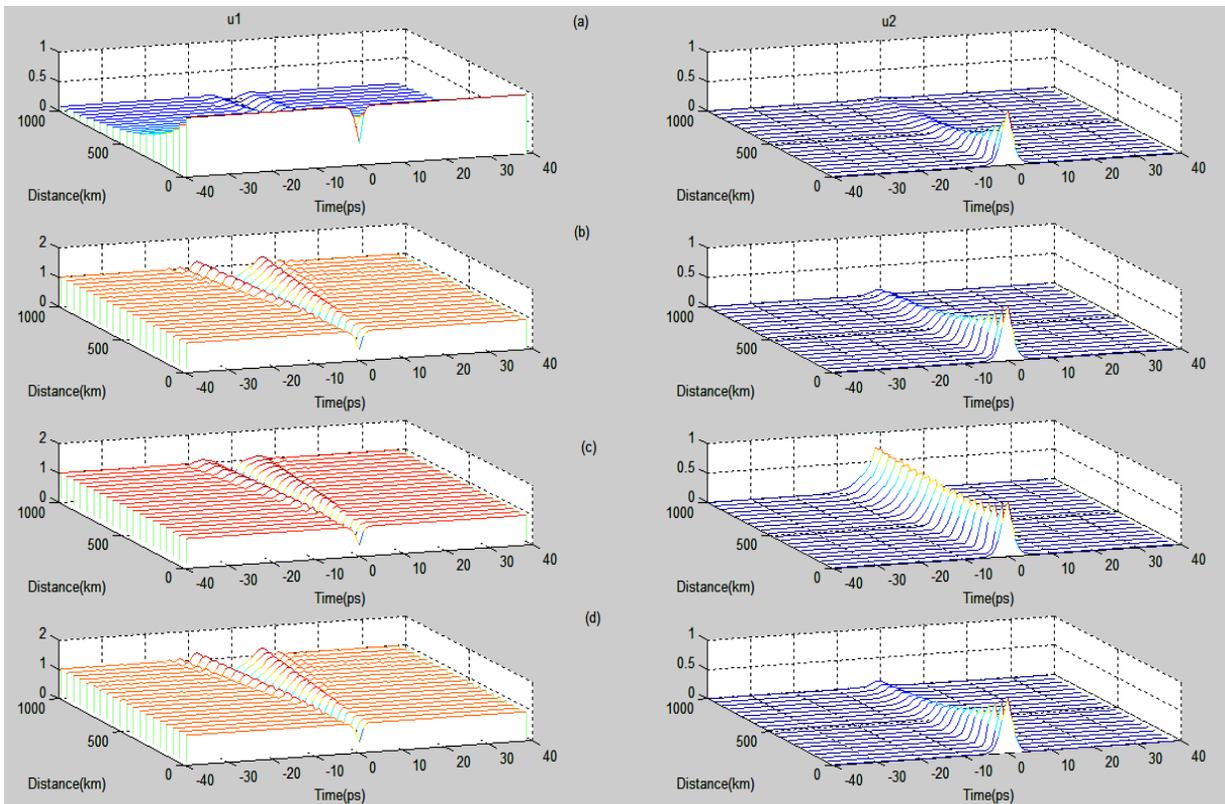


Fig. 3: The normalized four vector soliton intensity versus the propagation distance, (a)  $\gamma = -0.2$  and  $\alpha = -0.2$ , (b)  $\gamma = -0.2$  and  $\alpha = 0.2$ , (c)  $\gamma = 0.2$  and  $\alpha = -0.2$ , (d)  $\gamma = 0.2$  and  $\alpha = 0.2$

and the two-photon absorption) are investigated by the direct numerical simulation to give proper response.

Effects of the gain and the two-photon absorption on the dark-bright vector soliton propagation are investigated by using the split-step Fourier algorithm. The initial soliton pulse-width of  $\tau_0 = 10$  ps and the group-velocity delay of  $\eta = 0.3$  are used in the letter. The average dispersion of the fiber is  $\bar{d} = -1.00$  ps<sup>2</sup>/km and the dispersion length is about 100 km corresponding to the average dispersion for the soliton of pulse-width  $\tau_0 = 10$  ps.

Figure 1 and 2 are the normalized one dark or bright vector soliton intensity versus the propagation distance with the different gain or two-photon absorption. Figure 3 shows the normalized one dark or bright vector soliton intensity versus the propagation distance with their combined effects. The coefficients of the gain and the two-photon absorption are selected as  $\gamma = \pm 0.2$  and  $\alpha = \pm 0.2$ . The initially input vector soliton pulses are:

$$u_1(\tau, z = 0) = \tanh(\tau), u_2(\tau, z = 0) = \sec h(\tau) \quad (2)$$

From figures above, we can see that the dynamics of the dark-bright vector solitons result from the gain and the two-photon absorption for the soliton propagation, respectively. The gain and the two-photon absorption may lead to the destructive effects. There is only the gain in the birefringent fiber, the depth

(amplitude) of the dark (bright) soliton will change during propagation and the effects depend strictly on the sign and magnitude of the gain coefficient. For example, the dark and bright solitons will experience waveform attenuation (amplification) if the gain coefficient is negative (positive). There is only the two-photon absorption, the dark and bright solitons will experience waveform distortion during propagation and the effects also depend strictly on the sign and magnitude of the coefficient. When there are simultaneously the gain and the two-photon absorption in the birefringent fiber, they may enhance or reduce their combined effects. For example, when there are simultaneously the gain and the two-photon absorption of the same sign coefficients, their combined effects will enhance partly the perturbation-induced effects. So their combined effects play a crucial role in the dynamics and depend strictly on the sign and magnitude of their coefficients. These features show that the proper design for the sign of coefficients for the gain and the two-photon absorption may reduce effectively perturbation-induced effects on vector soliton propagation. Further, we find that the two-photon absorption has larger destruction on soliton propagation than that of the gain if their coefficients have the same sign and magnitude.

Figure 4 and 5 demonstrate the normalized intensity of four dark-bright vector solitons in the same polarizing direction versus the propagation distance

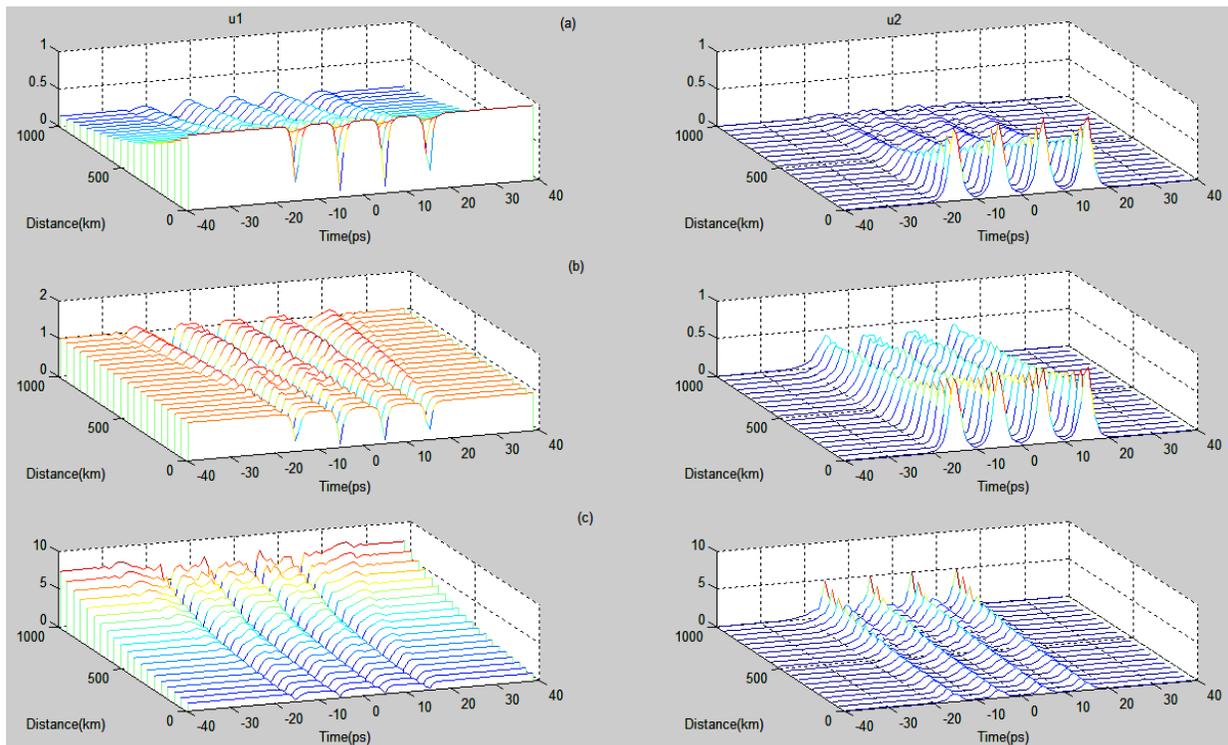


Fig. 4: The normalized four vector soliton intensity versus the propagation distance, (a)  $\gamma = -0.2$  and  $\alpha = 0$ , (b)  $\gamma = 0$  and  $\alpha = 0$ , (c)  $\gamma = 0.2$  and  $\alpha = 0$

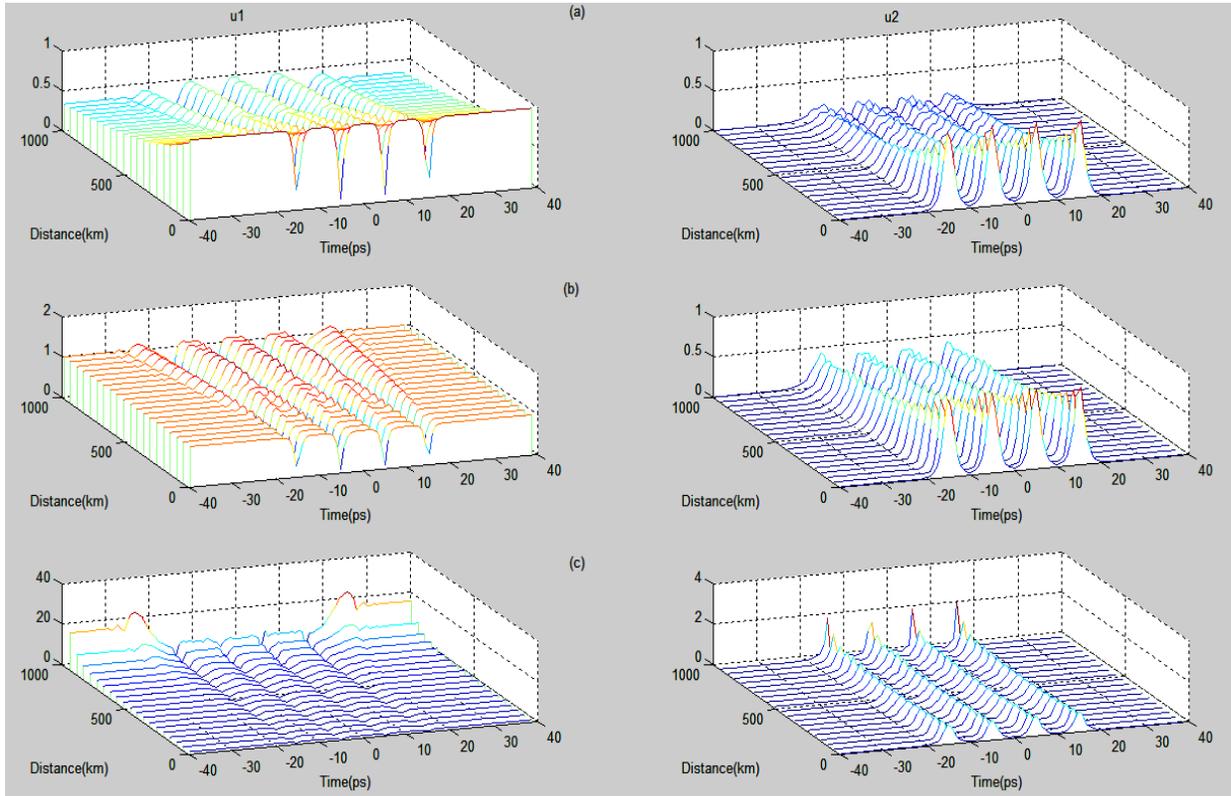


Fig. 5: The normalized four vector soliton intensity versus the propagation distance, (a)  $\gamma = 0$  and  $\alpha = -0.2$ , (b)  $\gamma = 0$  and  $\alpha = 0$ , (c)  $\gamma = 0$  and  $\alpha = 0.2$

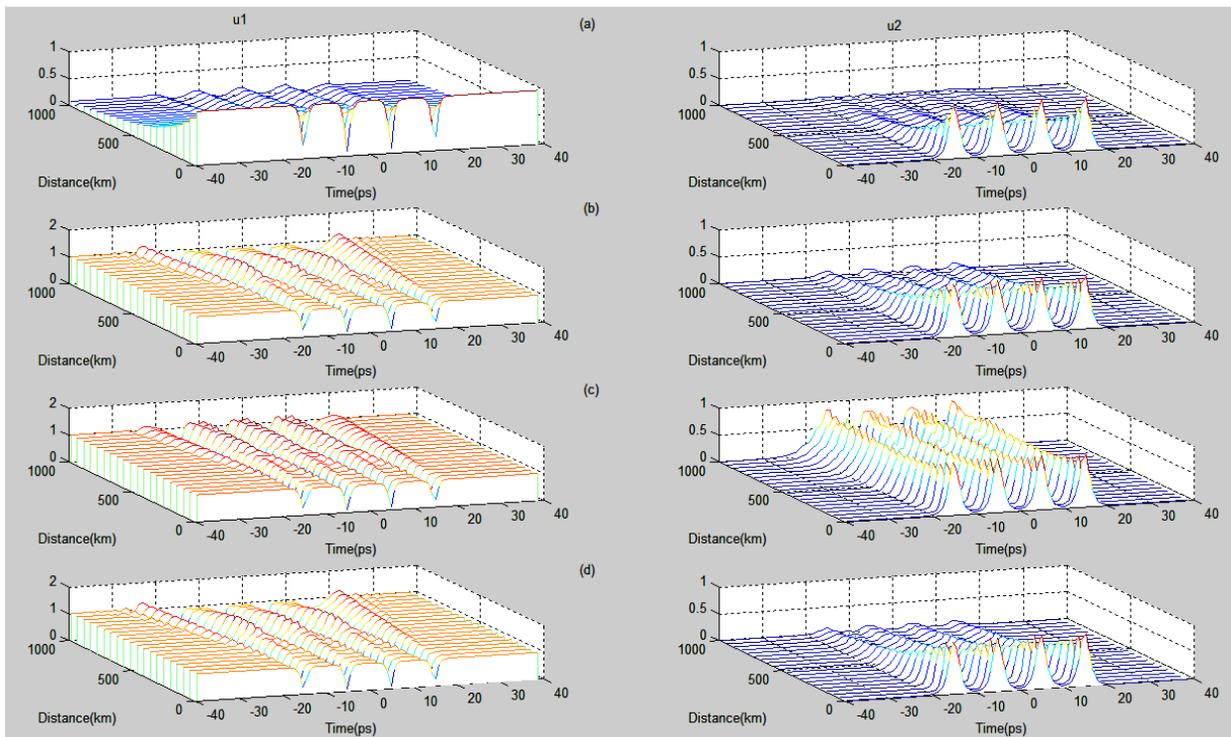


Fig. 6: The normalized four vector soliton intensity versus the propagation distance, (a)  $\gamma = -0.2$  and  $\alpha = -0.2$ , (b)  $\gamma = -0.2$  and  $\alpha = 0.2$ , (c)  $\gamma = 0.2$  and  $\alpha = -0.2$ , (d)  $\gamma = 0.2$  and  $\alpha = 0.2$

with the different gain and two-photon absorption. Figure 6 is the normalized intensity of four dark-bright vector solitons versus the propagation distance with their combined effects. The initially input four dark-bright vector soliton pulses are:

$$\begin{aligned} u_1(\tau, z=0) &= \tanh(\tau - 3\Delta/2) \times \tanh(\tau - \Delta/2) \times \tanh(\tau + \Delta/2) \times \tanh(\tau + 3\Delta/2), \\ u_2(\tau, z=0) &= \operatorname{sech}h(\tau - 3\Delta/2) + \operatorname{sech}h(\tau - \Delta/2) + \operatorname{sech}h(\tau + \Delta/2) + \operatorname{sech}h(\tau + 3\Delta/2), \end{aligned} \quad (3)$$

where,  $\Delta$  is the separation between two neighboring vector solitons in the same polarizing direction and  $\Delta = 10$  is given in these figures. The coefficients of the gain and the two-photon absorption are selected as  $\gamma = \pm 0.2$  and  $\alpha = \pm 0.2$ .

When initial separation between neighboring solitons is larger than five times of the Full Width at Half Maximum (FWHM) of the soliton in a path-average dispersion transmission system, the interaction can be suppressed effectively (Hasegawa and Kodama, 1995). From Fig. 4 to 6, we can see that the gain and the two-photon absorption enhance the interaction between two neighboring solitons even if the separation ( $\Delta = 10$ ) is about the 5.68 times of FWHM of the soliton) is larger than five times of the soliton FWHM. The gain and two-photon absorption may lead to the weighty effects on interaction of the dark-bright vector solitons. The interaction distance, which is defined as the

distance where the timing shifts of the neighboring solitons exceed a half of their FWHM (Matsumoto *et al.*, 1997; Li and Wang, 2007c) depends strictly on the sign and magnitude of their coefficients and the separation between solitons in the same polarizing direction. Also their combined effects play a crucial role in the interaction and depend strictly on the sign and magnitude of their coefficients.

Results in Fig. 1 to 6 remind that dark-bright vector solitons result from the gain, the two-photon absorption and their combined effect in the optical birefringent fiber. The perturbations induced by the gain and the two-photon absorption result in the avalanche of the side band, lead to the submergence of the dark vector soliton and the enhance the vector soliton interaction. Meantime, the perturbations cause the time relative displacement between elliptically polarized components, then the time relative displacement leads to the disintegration of the bright vector soliton and enhances the interaction between the neighboring vector solitons. The proper design for the sign and magnitude of coefficients of the gain and the two-photon absorption may reduce perturbation-induced destructive effects. The effective propagation distance and the interaction distance are determined by the two-photon absorption because the two-photon absorption has larger destruction than that of the gain on the vector soliton propagation and interaction.

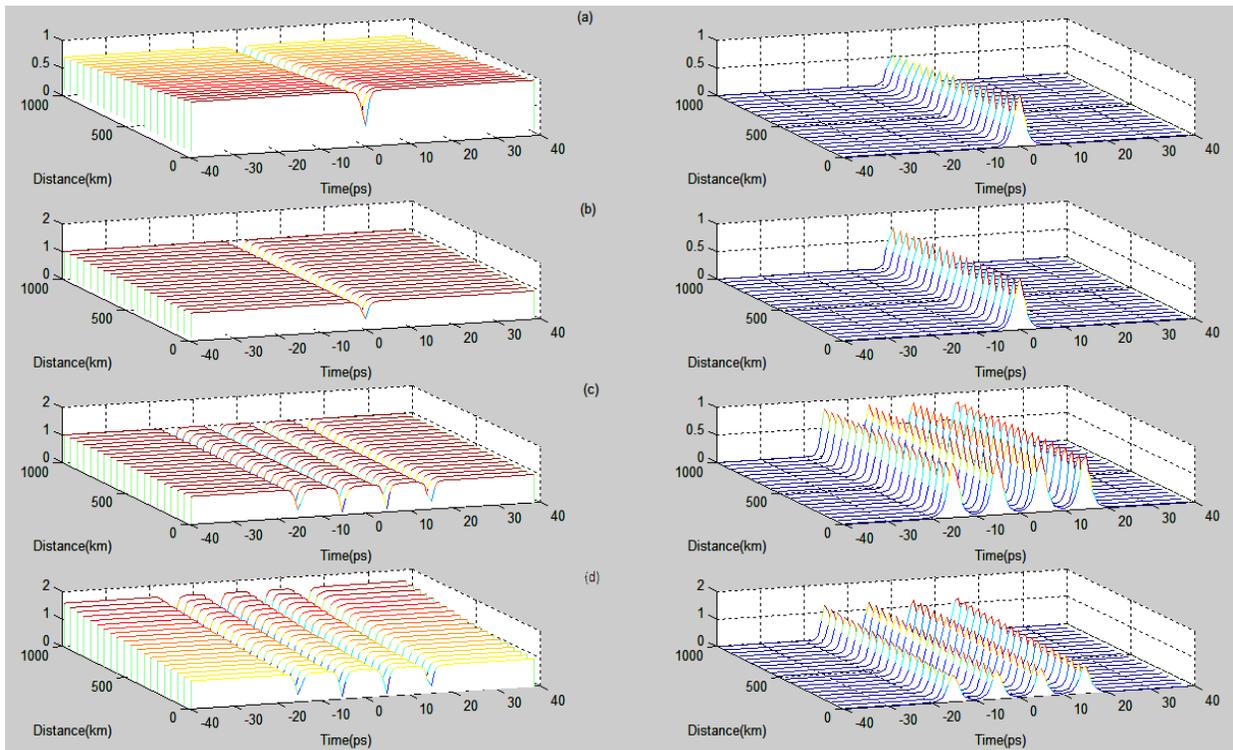


Fig. 7: The normalized one or four vector soliton intensity versus the propagation distance with the control (a)  $\gamma = -0.2$  and  $\alpha = -0.2$ , (b)  $\gamma = 0.2$  and  $\alpha = 0.2$ , (c)  $\gamma = -0.2$  and  $\alpha = -0.2$ , (d)  $\gamma = 0.2$  and  $\alpha = 0.2$

## CONTROL FOR THE COMBINED EFFECTS

When there are the gain and the two-photon absorption in the birefringence fiber during the optical soliton propagation, the traveling pulses will experience wave form distortion and frequency shift if the mismatch between the gain and the two-photon absorption is serious, a dispersive wave radiation may appear in the system and result in the decrease of transmission capacity. The combined effects on the soliton are not neglected and may be compensated for and controlled.

In order to stabilize the vector soliton propagation, some controllable methods can be used to suppress the disadvantaged effects of the mismatch between the gain and the two-photon absorption. When propagating in the birefringent fiber exhibiting the gain and the two-photon absorption with the control of the nonlinear gain and the filter, the vector soliton pulses can be described by the modified nonlinear Schrödinger equation below:

$$\begin{aligned} j\frac{\partial u_1}{\partial Z} + j\eta\frac{\partial u_1}{\partial \tau} - \frac{1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + (|u_1|^2 + \frac{2}{3}|u_2|^2)u_1 &= j\gamma u_1 + j\alpha|u_1|^2 u_1 + j\delta|u_1|^4 u_1 + j\beta\frac{\partial^2 u_1}{\partial \tau^2}, \\ j\frac{\partial u_2}{\partial Z} - j\eta\frac{\partial u_2}{\partial \tau} - \frac{1}{2}\frac{\partial^2 u_2}{\partial \tau^2} + (|u_2|^2 + \frac{2}{3}|u_1|^2)u_2 &= j\gamma u_2 + j\alpha|u_2|^2 u_2 + j\delta|u_2|^4 u_2 + j\beta\frac{\partial^2 u_2}{\partial \tau^2}, \end{aligned} \quad (4)$$

where,  $\delta$  and  $\beta$  are the coefficients of the nonlinear gain saturation and filter, respectively. These parameters are selected on the condition below (Matsumoto *et al.*, 1997; Li and Wang, 2007c):

$$\delta < 0, \quad \beta < 2\alpha, 15\gamma - 5(\beta - 2\alpha) + 8\delta = 0 \quad (5)$$

Figure 7 shows the normalized intensity of one and three dispersion path-average dark or bright vector solitons in the same polarizing direction versus the propagation distance with the controllable nonlinear gain and the filter. The initially input soliton pulses are the same as those in Fig. 1 to 6. We can see the linear gain, the nonlinear gain and the nonlinear gain saturation combining with filter can be used to suppress effectively these effects in the vector soliton system and the soliton shape is recovered and the interaction is suppressed. The nonlinear gain can suppress the linear wave growth due to the excess gain in the dark-bright vector soliton system and the narrow-band filter can suppress the relative time displacement caused by the mismatch between the gain and the two-photon absorption. Combined with the narrow-band filter, the nonlinear gain with high-order terms perfectly amplifies the soliton with the large amplitudes while the linear wave with the small amplitudes is not amplified and the relative time displacement is suppressed.

## CONCLUSION

The coupled dark-bright vector solitons are considered in a birefringent fiber with the gain and the

two-photon absorption and their sole and combined effects on the vector soliton propagation and interaction are investigated by the numerical method. The numerical results show that the gain and the two-photon absorption may lead to the destructive effects on the dark-bright vector soliton propagation and interaction; their combined effects play a crucial role in the dynamics and depend strictly on the sign and magnitude of their coefficients. The features mean that the proper design for the sign and magnitude of their coefficients may reduce effectively perturbation-induced destructive effects. The nonlinear gain combined with the filter can be used to suppress effectively the some disadvantaged effects and the stabilization mechanism is demonstrated.

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