Research Journal of Applied Sciences, Engineering and Technology 6(4): 706-711, 2013
DOI:10.19026/rjaset.6.4185
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: September 28, 2012 Accepted: November 23, 2012 Published: June 20, 2013

Research Article

Pro/E-based Kinetic Simulation of Cycloid Steel Ball Planetary Transmission with One-tooth Difference

Chen Houjun, Xu Bo, Cai Xiong and Yang Wenxiang
School of Mechanical Engineering, Nantong University, Nantong, Jiangsu, P.R. China

Abstract: The purpose of this study is to present a kind of novel cycloid steel ball planetary transmission with one-tooth difference and to examine the feasibility of this mechanism using PRO/E-based kinetic simulation. The mechanism's structure and transmission principle are introduced and the geometry of tooth surface for the planet gear is described and analyzed; and further, the rule of determining the directrix of tooth surface is deduced to avoid the interference and self-cross of tooth surface. On PRO/E software platform, the main parts of this mechanism are virtually modeled and assembled and the kinetic simulations on the movement trajectory and the velocity relationship are performed. The obtained results confirm that the cycloid steel ball planetary transmission with one-tooth difference is feasible.

Keywords: Kinetic simulation, pin-cycloid planetary gearing, steel ball, transmission principle

INTRODUCTION

Because of big transmission ratio and high carrying capacity, the pin-cycloid planetary gearing is widely used in the agricultural machinery, the automobile industry and the machine tools, etc. However, there are still some shortages in this mechanism, such as tooth face scuffing, short life of spin arm bearing and low driving accuracy. With the development of technology, RV (Rotate Vector) reducer and the movable-tooth transmission are invented and applied and the drive performances are improved (Qu and An, 1994; Zhou and Chen, 2001). It is rather obvious that as a kind of various-pitch transmission, the inter-medium meshing element (the ball or the rolling column) in the movable-tooth transmission perform the sliding-rolling movement, thus the driving efficiency is affected and the tooth of fixed gear is difficult to be machined. On the other hand, because the inter-medium meshing element is the steel ball, the cycloid steel ball planetary transmission has many excellent transmission performances, such as high meshing stiffness, good force condition, high driving efficiency and free return difference and then it has a good application future in the precision machinery, the indexing mechanism and the frequency reciprocating drive mechanism and related transmission principles, kinematics and dynamic performances have been explored (Zhou, 1996; Wu *et al*., 2007; Li *et al*., 2008; Yang *et al*., 2009). However, in this mechanism, the centre of steel ball locates at the crossing point of directrixes of tooth flanks of the sun and planet gears, which will cause easily that the steel ball is blocked and the machine is stopped when the manufacturing and assembly errors exist.

In this study, a novel cycloid steel ball planetary transmission is introduced and the kinetic simulation is performed to examine the feasibility of this mechanism.

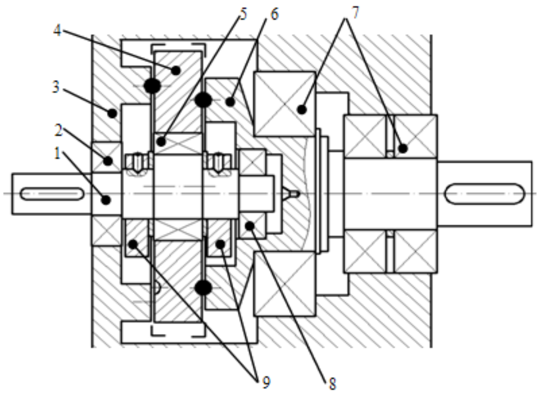
BASIC STRUCTURE AND MATHEMATICAL MODEL OF NOVEL MECHANISM

Basic structure: Compared with conventional cycloid steel ball planetary transmission, this novel mechanism has some competition predominances:

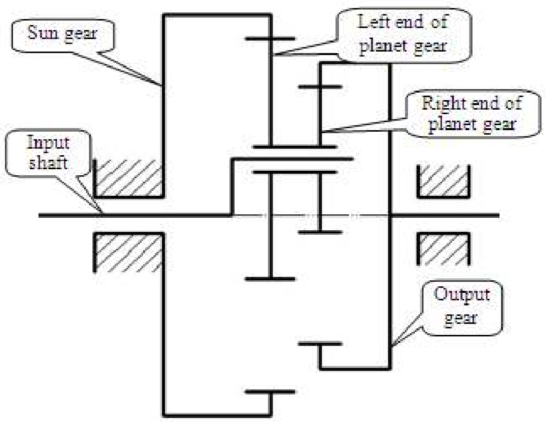
- Compact structure because the holding cage is omitted.
- High carrying capacity because the contact area between the steel ball and the epicycloid groove of planet gear is increased.
- Big reducer ratio because an equi-speed output mechanism is evolved into the second gear pair.

As shown in Fig. 1, one end of the input shaft 1 is mounted in the gearbox by the bearing 2 and another end is installed in the end of the output shaft (gear) 6 by the bearing 8, which ensures that a steady double support for the input shaft is obtained; the planet gear 4 is placed on an eccentric axle journal of input shaft by the bearing 5 and both the left and right ends of planet gear are machined into the self-closed epicycloid grooves with the wave number z_1 and z_3, respectively; the output shaft is installed in the gearbox by the

Corresponding Author: Chen Houjun, School of Mechanical Engineering, Nantong University, Nantong 226019, China



(a) Design drawing



(b) Scheme of mechanism

Fig. 1: Cycloid steel ball planetary transmission with one-tooth difference

bearing 7 and z_4 spherical sustaining sockets, used to place the steel balls, are arranged circumferentially at the left end of output shaft. These steel balls are in mesh with the epicycloid groove at the right end of planet gear and $z_4 - z_3 = 1$. At the left inside of the gearbox, there is z_2 sustaining sockets used to arrange the steel balls which are used as the teeth of the sun gear and are in mesh with the epicycloid groove on the left end of planet gear and $z_2 - z_1 = 1$. In order to enhance the dynamic balance of mechanism, the balance weight 9 is arranged at two sides of the planet gear.

According to the theory of enveloping surfaces, the flank of epicycloid groove is the tube surface (normal circular-arc surface (Chen *et al*., 2006), whose normal section is a circular arc. In order to reduce the sensitivity of errors, the groove flanks of planet gear and the flanks of sustaining sockets are mismatched to be in mesh with the steel balls. That is to say, at the design state, both the radius of normal section of epicycloid groove and the radius of generatrix of sustaining socket are little bigger than that of the steel ball, which will generate two meshing pairs, i.e., the first pair between the steel ball and the epicycloid groove and the second pair between the steel ball and the sustaining socket. As a result, the epicycloid groove consists of two different tube surfaces, the radii of whose normal sections are same. These two surfaces are offset and are linked by the fillet surface at the root. Similarly, the flank of sustaining socket is a torus generated by an offsetting circular arc generatrix rotating about the vertical line of flat end which passes the center of steel ball, thus it avoids the face-to-face contact with the steel ball.

Transmission principle: In the studies course, the input shaft rotates with the rotate speed n_1. Due to the limitation of z_2 steel balls arranged in the sustaining sockets at the left inside of the gearbox, the planet gear works with the rotate speed n_4. At the same time, because the epicycloid groove at the right end of planet gear are in mesh with the steel balls arranged at the end of output shaft, the output shaft are drived with the rotate speed n_6.

According to the kinematics of the planetary gear train, the following relationships of speed ratio can be obtained:

First stage:

$$I_{34}^{1} = \frac{n_3 - n_1}{n_4 - n_1} = \frac{z_1}{z_2} = \frac{z_1}{z_1 + 1} \tag{1}$$

Second stage:

$$I_{46}^{1} = \frac{n_4 - n_1}{n_6 - n_1} = \frac{z_4}{z_3} = \frac{z_3 + 1}{z_3} \tag{2}$$

where,
n_3 = The rotate speed of sun gear

To the cycloid steel ball planetary transmission with one-tooth difference presented in this study, the sun gear is fixed at the gearbox, i.e., $n_3 = 0$, thus the total speed ratio can be deduced from Eq. (1) and (2) as:

$$I_{61} = \frac{n_1}{n_6} = \frac{z_1(z_3 + 1)}{z_1 - z_3} \tag{3}$$

It can be found that when $z_1 - z_3 = \pm 1$, the total speed ratio is maximal. In order to obtain the compact structure, there are two design schemes for the tooth number distribution:

- z_1, $z_2 = z_1 + 1$, $z_3 = z_1 - 1$, $z_4 = z_1$, $I_{61} = (z_1)^2$
- z_1, $z_2 = z_1 + 1$, $z_3 = z_1 + 1$, $z_4 = z_1 + 2$, $I_{61} = -z_1(z_1+2)$

Because the wave number of epicycloid groove is one less than the number of steel balls in mesh with the epicycloid groove and two sides of an epicycloid groove

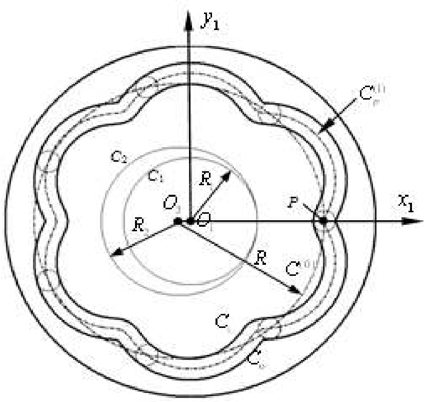


Fig. 2: Generation of epicycloid groove

can become alternately the driving side, all of steel balls can participate simultaneously in the conjugate operation and the contact ratio is two times more than that of conventional pin-cycloid planetary gearing.

Mathematical model of epicycloid groove: Relative to the planet gear, the locus of the center of steel ball is a cyloid in geometry. In order to avoid the sharp point at the wave crest or the wave trough of epicycloid groove, some elementary conditions need to be firstly determined at the design state, which can assure that the locus of the center of steel ball is a curtate epicycloid in geometry.

As shown in Fig. 2, C_1 and C_2 are the pitch circles of the planet and sun gears and their radii are R_1 and R_2, respectively and the centre distance is $A = R_2 - R_1$ ($R_2>R_1$). The radius of circle $C^{(0)}$, where the center P of steel ball fixed at the pitch circle C_2 locates, is $R = R_2/K$ (K is the radius variation ratio of epicycloid). When the pitch circle C_2 performs a pure rolling motion on the pitch circle C_1, the locus $C_p^{(1)}$ of the center of steel ball is just the curtate epicycloid. In the coordinate system $\{O_1;\ x_1y_1\}$ attached to the pitch circle C_1, the radius-vector function of $C_p^{(1)}$ can be described as:

$$\begin{cases} \mathbf{R}_p^{(1)} = -A\mathbf{e}(\varphi^{(1)}) + R\mathbf{e}(\varphi^{(1)} - \varphi^{(2)}) \\ \qquad = -A\mathbf{e}(\varphi^{(1)}) + R\mathbf{e}(J\varphi^{(1)}) \\ I = \dfrac{R_1}{R_2} = \dfrac{\varphi^{(2)}}{\varphi^{(1)}},\ J = 1 - I \end{cases} \tag{4}$$

where,

I	: The speed ratio
$\varphi^{(1)}$ & $\varphi^{(2)}$	: The rotation angle parameters of the planet and sun gears, respectively
$e(\varphi^{(1)})$ & $e(J\varphi^{(1)})$	: The circular vector functions and their concrete expressions are:

$$\begin{cases} \mathbf{e}(\varphi^{(1)}) = \mathbf{i}\cos\varphi^{(1)} + \mathbf{j}\sin\varphi^{(1)} \\ \mathbf{e}(J\varphi^{(1)}) = \mathbf{i}\cos(J\varphi^{(1)}) + \mathbf{j}\sin(J\varphi^{(1)}) \end{cases}$$

Due to the limiting condition of one-tooth difference, there are following relationships among the parameters:

$$I = \frac{z_1}{z_1+1},\ J = \frac{1}{z_1+1},\ R_1 = z_1A,\ R_2 = (z_1+1)A \tag{5}$$

According to the differential geometry, the curvature $k^{(1)}$ of $C_p^{(1)}$ can be calculated by the formula:

$$\begin{cases} k^{(1)} = \dfrac{\left(\dot{\mathbf{R}}_p^{(1)} \times \ddot{\mathbf{R}}_p^{(1)}\right)\cdot\mathbf{k}}{\left|\dot{\mathbf{R}}_p^{(1)}\right|^3} = \dfrac{A^2 + J^3R^2 - (1+J)JRA\cos\left(I\varphi^{(1)}\right)}{\left[J^2R^2 + A^2 - 2JAR\cos\left(I\varphi^{(1)}\right)\right]^{\frac{3}{2}}} \\ \dot{\mathbf{R}}_p^{(1)} = \dfrac{\mathrm{d}\mathbf{R}_p^{(1)}}{\mathrm{d}\varphi^{(1)}} = -A\mathbf{e}_1(\varphi^{(1)}) + JR\mathbf{e}_1(J\varphi^{(1)}) \\ \ddot{\mathbf{R}}_p^{(1)} = \dfrac{\mathrm{d}^2\mathbf{R}_p^{(1)}}{\mathrm{d}\varphi^{(1)^2}} = A\mathbf{e}(\varphi^{(1)}) - J^2R\mathbf{e}(J\varphi^{(1)}) \end{cases} \tag{6}$$

where, k is the unit vector vertical to the coordinate system $\{O_1;\ x_1y_1\}$ and constructs a right-hand coordinate system with the vectors i and j, which are the unit vectors along x_1- and y_1-axes, respectively.

In the engineering, the reciprocal of curvature, i.e., the radius of curvature is usually used to describe the bending degree of curve. To the curtate epicycloid, the minimum radius of curvature occurs at the wave trough. When $\varphi^{(1)} = 0$, the minimum radius of curvature $\rho_{\min}^{(1)}$ of $C_p^{(1)}$ occurs and its expression is:

$$\rho_{\min}^{(1)} = \frac{J\left(R - R_2\right)^2}{JR - A} \tag{7}$$

As mentioned previously, the flank of epicycloid groove is the tube surface whose directrix is the locus $C_p^{(1)}$ of the center of steel ball. According to the generating theory of tube surface, the flank of epicycloid groove can be generated by the sweeping of circular-arc tooth profile along the directrix with single-DOF (Degree of Freedom) and this tooth profile always lies on the normal plane to the directrix. Thus, the radius-vector function for the flank of epicycloid groove can be described as:

$$\mathbf{R} = \mathbf{R}_p^{(1)} + r\left(\mathbf{e}_2\cos\theta + \mathbf{e}_3\sin\theta\right) \tag{8}$$

where,

r	: The radius of normal section of epicycloid groove (is also the radius of steel ball)
θ	: The angle parameter used to define the circular-arc tooth profile in the moving frame of the directrix $C_p^{(1)}$
e_2, e_3	: The unit vectors along the coordinate axes of moving frame and their concrete expressions are:

$$\begin{cases} \mathbf{e}_3 = \mathbf{k} \\ \mathbf{e}_2 = \dfrac{A\mathbf{e}(\varphi^{(1)}) - JR\mathbf{e}(J\varphi^{(1)})}{\sqrt{A^2 + (JR)^2 - 2JAR\cos(I\varphi^{(1)})}} \end{cases}$$

The intersecting lines between the epicycloid groove and the end plane of planet gear are the inner and outer equidistant curves C_i, C_o of the directrix $C_p^{(1)}$ and the offset distance just is r. According to the defination of the equidistant curve, the radius-vector functions of C_i and C_o can be described as:

$$\mathbf{R}_{i,o} = -A\mathbf{e}(\varphi^{(1)}) + R\mathbf{e}(J\varphi^{(1)}) \pm \frac{K\mathbf{e}(\varphi^{(1)}) - \mathbf{e}(J\varphi^{(1)})}{\left|K\mathbf{e}(\varphi^{(1)}) - \mathbf{e}(J\varphi^{(1)})\right|} r \quad (9)$$

In the notation "±",
+ : Used to calculate the inner equidistant curve C_i
- : For the outer equidistant curve C_o

To the inner and outer equidistant curves, the location where their minimum radii of curvature appear is consistent with that of the directrix $C_p^{(1)}$. Thus, the minimum radii of curvature of the inner and outer equidistant curves C_i, C_o can be described as:

$$\rho = \rho_{\min}^{(1)} \pm r \quad (10)$$

where,
+ : For the inner equidistant curve
- : For the outer equidistant curve

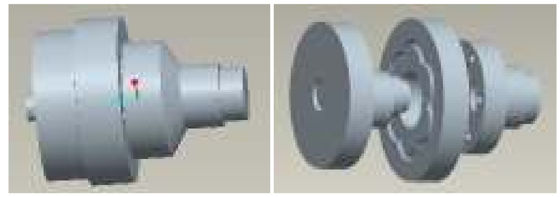
It can be found that the radius of curvature of the outer equidistant curve is less than that of the inner equidistant curve generated by a same directrix, thus the tooth-surface interference analysis should be performed at the outer equidistant curve.

In order to avoid the flank interference between the steel ball and the epicycloid groove, the radius of steel ball must satisfy the condition:

$$r \le \rho_{\min}^{(1)} = \frac{J\left(R - R_2\right)^2}{JR - A} \quad (11)$$

At the gear design state, the transmission ratio, the teeth number distribution and the intensity distribution are firstly determined by the working condition. To the cycloid steel ball planetary transmission, when the centre distance, the speed ratio and the radius of steel ball are known, the condition that the radius of circle $C^{(0)}$ should satisfy for avoiding the flank interference can be deduced from Eq. (11) as:

$$R \ge R_2 - \frac{1}{2}\left(r - \sqrt{r^2 + 4rz_1R_2}\right) \quad (12)$$



(a) Assembling solid (b) Explosive sketch

Fig. 3: Model of cycloid steel ball planetary transmission

Furthermore, the condition that the radius variation ratio K of epicycloid should satisfy can be deduced as:

$$K \le \frac{2R_2}{2R_2 - r + \sqrt{r^2 + 4rz_1R_2}} \quad (13)$$

MODELING AND SIMULATION OF NOVEL MECHANISM

In order to examine the feasibility of novel cycloid steel ball planetary transmission with one-tooth difference, main parts of this mechanism are virtually modeled and assembled by PRO/E software and the kinetic simulation is performed in this section.

Modeling and assembling of novel mechanism: In the modeling process, main parts, whose motion can affirm the transmission principle, are modeled, however other assistant parts, such as the sealing devices, the bearings and the fasteners, etc., are ignored.

To novel cycloid steel ball planetary transmission, main parts consist of the input shaft, the planet gear, the sun gear and the output shaft. According to the design parameters presented in Table 1, the modeling and assembling results are shown in Fig. 3.

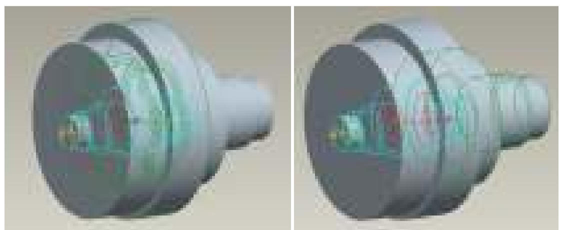
Definition of gear pair: Because the novel mechanism is a two-stage planetary transmission, two gear pairs should be firstly established at the assembling model before the kinetic simulation is performed.

Entering into the "Application/Mechanism" function of PRO/E software, the virtual gear pair can be established. For the first-stage transmission, it is the meshing relationship between the planet and sun gears by the inter-medium meshing element-the steel ball. Selecting the axis line of the sun gear as the first motion axis and it is set up as the "Ground" mode because the sun gear is fixed at the gearbox. According to the design parameters in the Table 1, the value "70" is considered to be the pitch diameter of sun gear. Selecting the axis line of planet gear as the second motion axis and the value "60" is considered to be the pitch diameter at the left end of planet gear.

For the second-stage transmission, it is the meshing relationship between the planet gear and the output shaft (gear) by the inter-medium meshing element-the

Table 1: Main design parameters of novel cycloid steel ball planetary transmission with one-tooth difference

Items	Value	Items	Value
Centre distance	5 mm	Radius of steel ball	5 mm
Wave number z_1	6	Wave number z_3	5
Teeth number z_2 of sun gear	7	Teeth number z_4 of output gear	6
Radius of pitch circle of sun gear	35 mm	Radius of pitch circle of output gear	30 mm
Radius variation ratio of epicycloid groove at the driving side	0.5	Radius variation ratio of epicycloid groove at the output side	0.55
Total speed ratio	36		



(a) First pair (b) Second pair

Fig. 4: Definition of gear pairs

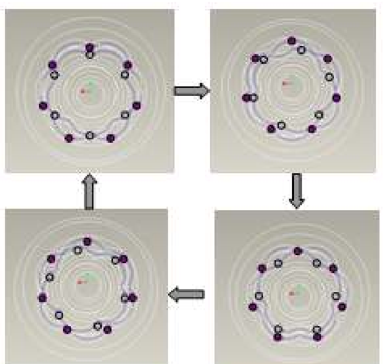


Fig. 5: Movement loci of steel balls

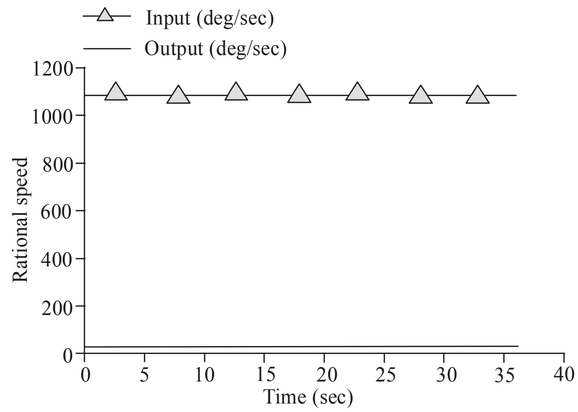
steel ball. Selecting the axis line of planet gear as the first motion axis and its pitch diameter at the right end is considered to be the value "50". Selecting the axis line of output shaft (gear) as the second motion axis and its pitch diameter is considered to be the value "60".

After two virtual gear pairs are established, two pairs of gear symbols appear in the assembling model of mechanism, as shown in Fig. 4.

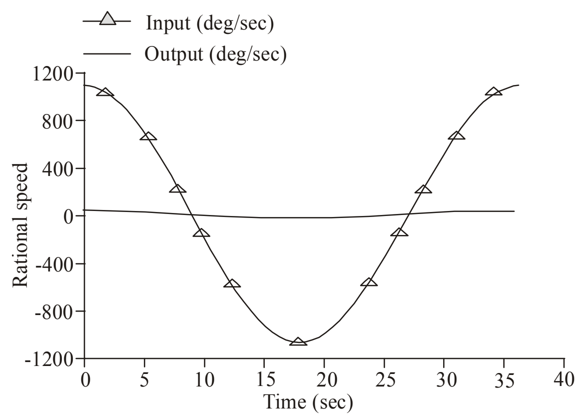
KINETIC SIMULATION AND DISCUSSION

In order to make the assembling model perform the motion, a virtual servo motor should be added to drive the input shaft and the velocity is set up to satisfy the requirement of analyzing the motion performance of the mechanism.

Figure 5 presents four instantaneous loci of steel balls in the running process of novel mechanism. Thereinto, the hatched circles represent the steel balls used as the teeth of sun gear and the hollow circles denote the steel balls used as the teeth of output gear. We can find that all solid parts are transparently treated and the steel balls always move along respective directrix of epicycloid grooves at two ends of planet gear.



(a) Constant-speed input



(b) Variable-speed input

Fig. 6: Velocity curves of the input and output shafts

Figure 6 presents the velocity curves of the input and output shafts in this mechanism. Thereinto, at the case of constant-speed input, when the input speed is 1080°/sec, the resulting output speed is 30°/sec; at the case of variable-speed input, when the cosine function 1080*cos (360*t/36) deg/sec is chosen as the input speed, the output speed is 30*cos (360*t/36) deg/sec, where *t* is the time. It can be concluded that whether the input speed is constant or not, the output and input speeds always satisfy the relationship of speed ratio.

The above results show that the transmission principle of novel cycloid steel ball planetary gearing with one-tooth difference is right and it is feasible to obtain this mechanism in the engineering.

CONCLUSION

Based on the basic structure and the transmission principle of novel cycloid steel ball planetary gearing with one-tooth difference, the directrix-based mathematic model of tooth surface of planet gear is deduced and the rule to determine the radius variation ratio of the directrix is explored, which ensures that the tooth surface is free of the interference and the self-cross. On PRO/E software platform, the kinetic simulations are performed and the obtained results affirmed the feasibility of this mechanism.

It provides a theoretical basis for the design and the kinetic analysis of cycloid steel ball planetary gearing with one-tooth difference.

ACKNOWLEDGMENT

This study is financed by National Natural Science Foundation of China (Grant No. 51105210) and Nantong University (Grants No. 201010 and No. 10ZY006).

REFERENCES

Chen, H.J., Z.Y. Duan, H.J. Wu and J. Liu, 2006. Study on the general principle of normal circular-arc gear transmission. Mech. Mach. Theory, 41(12): 1424-1442.

Li, X.P., B.K. Chen and S.Y. Wang, 2008. Study on the floating plate output mechanism of a new cycloidal planetary transmission. Mod. Manufact. Eng., 2: 96- 99.

Qu, J.F. and Z.J. An, 1994. Research on zero clearance cycloid ball transmission. Chin. J. Mech. Eng., 7(1): 17-23.

Wu, Q.B., Z. Ran and X.F. Xin, 2007. Design and numerical control machining on cycloidal slot of double-cycloidal steel ball reducer. J. Machine Des., 24(10): 68-70.

Yang, Z.M., Z.J. An and P. Zhang, 2009. Research on undercutting in cycloid ball planetary transmission based on space meshing theory. T. Chinese Soc. Agric. Mach., 40(10): 216-222.

Zhou, J.J., 1996. Planetary ball cycloid transmission. J. Hangzhou Inst. Electron. Eng., 16(2): 35-44.

Zhou, J.J. and Z.C. Chen, 2001. Study and performance test of full complement cycloidial ball reducer with ceramic balls as its gears teeth. Chin. J. Aeronaut., 14(4): 245-255.