

## Research Article

### A Fractal Model for Capillary Pressure of Porous Media

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**Abstract:** Capillary pressure is a basic parameter in the study of the behavior of porous media containing two or more immiscible fluid phases. In this study, the capillary pressure of porous media is predicted based on fractal property of pore in porous media. The formula of calculating the capillary pressure of porous media is given. The capillary pressure of porous media is expressed as a function of porosity, fractal dimension of pore and saturation. Based on the parametric effect analysis, we conclude that the capillary pressure of porous media is negatively correlated with the porosity and saturation. Besides, it is shown that the capillary pressure of unsaturated porous media decreases with the increase of saturation. No additional empirical constant is introduced. This model contains less empirical constants than the conventional correlations. The model predictions are compared with the existing experimental data and good agreement between the model predictions and experimental data is found. The validity of the present fractal model is thus verified.

**Keywords:** Capillary pressure, fractal, porosity, porous media, saturation

#### INTRODUCTION

In recent years, the capillary pressure of porous media have captured great attentions from engineers and scientists in various areas including filtration, fuel cells, functional clothing, thermal insulation, paper products and medical science. Many parameters such as capillary pressure and the porosity are very important for design and manufacture of fiber-reinforced composites with high quality (Meng and Hu, 2009). The capillary pressure of porous media is critically important in many applications. So far, a number of experimental techniques and theoretical methods have been developed to predict the capillary pressure of porous media.

Ahn *et al.* (1991) measured capillary pressure difference of thermosetting matrices in woven fabric reinforcements. They found that the capillary pressure difference decreases with the increase of porosity and at low porosity the capillary pressure difference is quite sensitive to porosity for saturated porous media. Gauvin *et al.* (1996) suggested that capillary pressure difference has great effect on the permeability measurements. Accordingly the relative permeability is also affected by capillary pressure difference. Masalmeh *et al.* (2003) studied the effect of wet ability heterogeneity on the capillary pressure difference and relative permeability. They found that capillary pressure difference effect has to be taken into account in relative permeability. But no

quantitative relation between relative permeability and capillary pressure difference was reported.

Since the limitations of experiment conditions and various postulated mechanisms, a comprehensive theory and unified model still lacking. The literature on the interrelationship between capillary pressure and permeability has been few.

Until now many researchers are arguing the capillary pressure of porous media. There are many empirical correlations and models for fabric structures and mechanics in the literature, with each applicable to somewhat narrow range of experimental conditions. And they have own disadvantages because of the limitations of experiment conditions and various postulated mechanisms. So, comprehensive theories and unified models are desirable. In this study, we derive a model for capillary pressure of porous media based on fractal theory.

#### STRUCTURE OF SOME POROUS MEDIA

The structure of porous media is highly complex and difficult to describe. It is even more complicated to analyze the transfer behaviors within porous systems, especially when different transport mechanisms happen together. Moreover, the range of the scale from nano to macro within one porous media is huge, which adds further complexity of the system. Modeling the transport phenomena of fibrous media is therefore a great challenge. In many applications, textile materials

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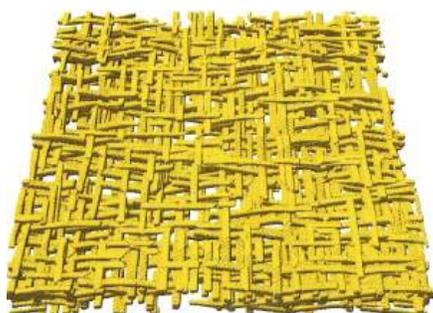


Fig. 1: A fiber sample constructed with the deposition model

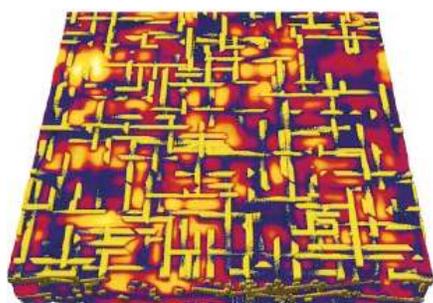


Fig. 2: The velocity field of fluid flow through the fiber web

serve as porous media, which air, vapor, particles, heat or electricity pass through. Therefore, it is of importance to understand the transport properties of fibrous media.

**Fibers:** Synthetic fibers can either be modified during manufacture, e.g. by producing fibers with irregular cross-section, or be optimally blended with natural fibers to improve their thermo-physiological properties. It was adjustable by blending a desired amount of a compound having a high water ratio with the synthetic fiber of introducing a desired amount of hygroscopic functional group into polymer. It may be covered, entangled or twisted with other fibers to form a composite yarn which then is knit or woven to form a fabric. The cross-sectional shape of the fiber may be circular, triangular, L-shaped, T-shaped, Y-shaped, W-shaped, flat-shaped, dog-bone shaped, hollow or indefinite. Fabrics were produced by knitting or weaving of bicomponent conjugate filaments. After that the fabrics were exposed to an alkaline solution in combination with thermal and mechanical treatment, which separates the bicomponent conjugate filaments. Hence split type micro-fiber fabrics were formed. As these contain numerous capillaries, it can absorb the sweat rapidly and transport moisture faster because of these capillaries.

Textiles are fibrous porous media made up of textile fibers in different structural forms. Figure 1 is a fiber sample constructed with the deposition model (Koponen *et al.*, 1998). Figure 2 is the velocity field of fluid flow through the fiber web. Koponen *et al.* (1998)

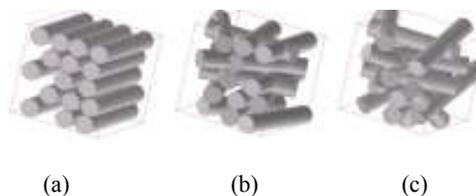


Fig. 3: (Color online) Structures with different fibers Orientation, (a) 1 directional (1D), (b) 2 directional (2D) and (c) 3 directional (3D)

reported the results of essentially *ab initio* simulations of creeping flow through large three dimensional random fiber webs that closely resemble fibrous sheets such as study and nonwoven fabrics (Koponen *et al.*, 1998). Since the limitations of postulated mechanisms, the method can apply specify fiber webs. Textile is certainly a complex multi-pore structure, which can be described as parallel pore structure or pellets accumulation pore structure (Park, 2006). The heat and moisture transfer through textile is affected by its structure.

**Fiber web:** Fibrous materials can be divided into one-, two- and three-directional media (Fig. 3) (Tamayol and Bahrami, 2011). In One-Directional (1D) structures, the axes of fibers are parallel to each other. In Two Directional (2D) fibrous matrices, the axes of fibers are located on planes parallel to each other with random positions and orientations on these planes. The axes of fibers in Three-Directional (3D) structures are randomly positioned and oriented in space. 1D and 2D materials are anisotropic, e.g., see for more details. However, 3D structures can be considered isotropic, e.g., metal-foam samples studied in this study. The considered microstructures are indeed anisotropic; thus, this study can only be applied to transverse flows. Transverse direction for 1D fiber is normal to the fibers' axes and for 2D materials is perpendicular to the fibers' planes. In isotropic 3D structures, transverse flow is perpendicular to any plane (Tamayol and Bahrami, 2011).

In porous textiles the liquid water transfer coupled with moisture and heat transfer is a complicated process involving simultaneous, coupled heat and mass transfers.

**Yarns:** Composite yarns also were developed for making moisture management fabrics. These novel yarns yielded fabrics capable of quickly absorbing perspiration from a wearer's skin and yet capable of quickly releasing that moisture, resulting in surprising levels of wearer comfort and wearer preference. A composite yarn was made (Peters *et al.*, 1999). The yarn comprising hydrophilic fibers embedded at the yarn center within a matrix of hydrophobic fibers, in which there was a gradual transition between fiber types. The transition began at the center of the composite yarn where hydrophilic fibers were concentrated and progressed to the outermost layer

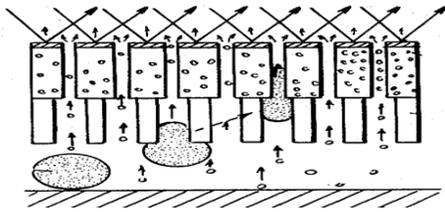


Fig. 4: Schematically illustrates the composite fabric and how it functions

where hydrophobic fibers predominate. An increase in the area of the inter face between hydrophobic and hydrophilic components did not increase the total amount of moisture that can be taken up by the composite yarn, however, enhanced the kinetics of absorption so that moisture transfers become more rapid and effective in the moisture management fabric.

**Knitted fabrics:** There are many fabrics made by weaving, knitting and non-weaving. Traditional knits such as pique, honeycomb or ribbed raise textures trap a certain amount of air between the body and knits. By reducing the contact points between the skin and garment, air circulates freely and lets the body breathe. Figure 4 schematically illustrates the composite fabric and how it functions. Knitted fabric had at least two layers made from yarns mainly composed of non-hygroscopic fiber of at least 1 denier, characterized in that the inter-fiber space of a yarn in one layer differs from that in the other layer. To achieve hygroscopic property, an inter-fiber space of a first yarn forming one layer of the fabric was made with a different size from that of a second yarn forming the other layer. The difference in inter-fiber space size can be impacted by suitably selecting the structure of the yarn forming the respective layers and the fineness of the fiber composing the yarn and the fabric structure itself.

**Coated and laminated fabrics:** Thomas *et al.* (2005) invented a method for providing a laminate of at least two layers to provide breathability and moisture barrier properties without appreciably reducing elastic properties. A first material comprised a woven or nonwoven web and a second material comprises a cellular elastomeric film or fiber. Such materials maybe integrated into a laminate by forming first materials onto second material, or by joining the two materials through chemical or physical means such as the use of adhesives. The second material can be prepared by mixing a cell opening agent with the elastomeric polymer resin and extruding the mixture at appropriate conditions whereby the cell opening agent decomposes or reacts to release agents that forms cells in the elastomeric film. The material was useful in personal care products such as diapers, feminine care products, child care products, incontinence products and health care products where such properties are desirable for

attributes such as comfort, body shaping, conformance, dryness and the like. Hall (2009) invented a multilayer breathable moisture management and compressive device, having a laminated material that including thin, hydrophilic polyurethane foam coated on both surfaces with an adhesive.

### FRactal MODEL

The cumulative size-distribution of pores in porous media whose sizes are greater than or equal to  $\lambda$  have also been proven to follow the fractal scaling law (Yu and Cheng, 2002):

$$N(L \geq \lambda) = \left(\frac{\lambda_{max}}{\lambda}\right)^{D_f} \quad (1)$$

Where N is the number of pores or islands(on earth) or spots (on engineering surfaces) whose sizes are greater than or equal to the size  $\lambda$ ,  $\lambda$  and  $\lambda_{max}$  are the pore size and the largest pore size, respectively and  $D_f$  is the fractal dimension for pore area. The total number of the pores or islands or spots from the smallest diameter  $\lambda_{min}$  to the largest diameter  $\lambda_{max}$  can be obtained from Eq. (1) as Yu and Cheng (2002):

$$N_t(L \geq \lambda_{min}) = \left(\frac{\lambda_{max}}{\lambda_{min}}\right)^{D_f} \quad (2)$$

Differentiating Eq. (1) with respect to  $\lambda$  yields:

$$-dN = D_f \lambda_{max}^{D_f} \lambda^{-(D_f+1)} d\lambda \quad (3)$$

Equation. (3) gives the pore number between the pore sizes  $\lambda$  and  $\lambda + d\lambda$ . The negative sign in Eq. (3) implies that the island or pore number decreases with the increase of island or pore size and  $-dN > 0$ . Dividing Eq. (3) by Eq. (2) results in:

$$-\frac{dN}{N_t} = D_f \lambda_{min}^{D_f} \lambda^{-(D_f+1)} d\lambda = f(\lambda) d\lambda \quad (4)$$

where,  $f(\lambda) = D_f \lambda_{min}^{D_f} \lambda^{-(D_f+1)}$  is the probability density function. According to probability theory, the probability density function,  $f(\lambda)$ , should satisfy the following normalization condition:

$$\int_{-\infty}^{\infty} f(\lambda) d\lambda = \int_{\lambda_{min}}^{\lambda_{max}} f(\lambda) d\lambda = 1 - \left(\frac{\lambda_{min}}{\lambda_{max}}\right)^{D_f} = 1 \quad (5)$$

It is clear that Eq. (5) holds if and only if (Yu and Cheng, 2002):

$$\left(\frac{\lambda_{\min}}{\lambda_{\max}}\right)^{D_f} \cong 0 \quad (6)$$

Eq. (6) can be considered as a criterion whether a porous medium can be characterized by fractal theory and technique. In general<sup>7</sup>, porous media is  $\lambda_{\min} / \lambda_{\max} < 10^{-2}$ , thus Eq. (6) approximately holds for porous media. Thus, the fractal theory and technique can be used to analyze the characters of porous media. In Eq. (6),  $D_f$  can be obtained by Yu and Cheng (2002):

$$D_f = d - \frac{\ln \phi}{\ln \frac{\lambda_{\min}}{\lambda_{\max}}} \quad (7)$$

where,  $d$  is the Euclidean dimension and equal to 2 and 3 in the two- and three-dimensional spaces, respectively.  $\phi$  is porosity of a porous medium.

The capillary pressure of porous media is given by Ahn *et al.* (1991):

$$P = \frac{FT \cos \theta}{\lambda} \frac{1-\phi}{\phi} \quad (8)$$

In Eq. (8),  $T$  is surface tension of fluids,  $\theta$  is contact angle between liquid and solid and  $F$  is shape factor depending on geometry of a medium and on flow direction ( $F = 4$ ) when the capillary is cylindrical, (Ahn *et al.*, 1991).

The average capillary pressure of porous media can be obtained by:

$$P_{av} = \int_{\lambda_{\min}}^{\lambda_{\max}} Pf(\lambda)d\lambda = \int_{\lambda_{\min}}^{\lambda_{\max}} \frac{FT \cos \theta}{\lambda} \frac{1-\phi}{\phi} f(\lambda)d\lambda = \frac{FT \cos \theta}{\lambda_{\min}} \frac{1-\phi}{\phi} \frac{D_f}{1+D_f} \quad (9)$$

For unsaturated porous media, the average capillary pressure of porous media can be expressed by:

$$P_u = \frac{FT \cos \theta}{\lambda_w} \frac{1-\phi_w}{\phi_w} \quad (10)$$

where,  $\lambda_w$  is the equivalent diameter for wetting phase,  $\phi_w$  is the volume fraction of the wetting phase:

$$P_{u,av} = \int_{\lambda_{\min,w}}^{\lambda_{\max,w}} P_u f(\lambda_w) d\lambda_w = \frac{FT \cos \theta}{\lambda_{\min,w}} \frac{1-S_w \phi}{S_w \phi} \frac{D_{f,w}}{1+D_{f,w}} \quad (11)$$

where,  $D_{f,w}$  is the fractal dimension for wetting phase, which can be obtained by extending the fractal dimension for saturated fluid as Yu and Li (2004):

$$D_{f,w} = d - \frac{\ln \phi_w}{\ln \frac{\lambda_{\min,w}}{\lambda_{\max,w}}} = d - \frac{\ln(S_w \phi)}{\ln \frac{\lambda_{\min}}{\lambda_{\max}}} \quad (12)$$

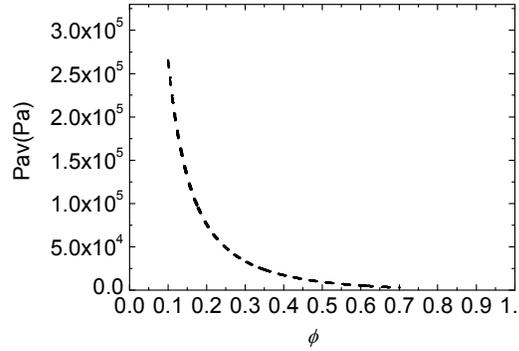


Fig. 5: The average capillary pressure versus porosity

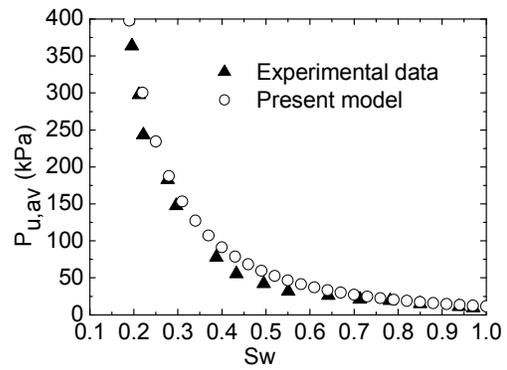


Fig. 6: The average capillary pressure of unsaturated porous media predicted by Eq. (11) compared with experimental data (Dana and Skoczylas, 2002) at porosity 0.18

where  $\lambda_{\min,w}$  and  $\lambda_{\max,w}$  are related by Yu and Li (2004):

$$\lambda_{\min,w} = \lambda_{\min} \sqrt{S_w}, \lambda_{\max,w} = \lambda_{\max} \sqrt{S_w} \quad (13)$$

where,  $S_w$  is the saturation of the wetting phase and is related to porosity by  $\phi_w = S_w \phi$  (Bear, 1972).

## RESULTS AND DISCUSSION

In porous media, capillary pressure is the force necessary to squeeze a hydrocarbon droplet through a pore throat (works against the interfacial tension between oil and water phases) and is higher for smaller pore diameter. Accurately predicting capillary pressures in a porous medium is central to understanding the movement and distribution of fluids within such systems. The capillary pressure of porous media relates the pressures in the two fluid phases. The capillary pressure in a porous medium is an increasing function of the non-wetting phase saturation or, alternately, a decreasing function of the wetting phase saturation.

Figure 5 is a plot of the average capillary pressure of porous media versus porosity. Figure 5 shows that

the average capillary pressure of porous media decreases as the porosity increases.

Figure 6 compares the predicted average capillary pressure of unsaturated porous media by Eq. (11) with the available experimental data (Dana and Skoczylas, 2002). The present model predictions show good agreement with the experimental data (Dana and Skoczylas, 2002). It is also found from Fig. 6 that the average capillary pressure of unsaturated porous media increases with the decrease of saturation  $S_w$  and at low saturation the capillary pressure difference increases remarkably with the decrease of saturation  $S_w$ . These are all expected and consistent with the practical physical phenomena.

### CONCLUSION

In this study, the formula of calculating the capillary pressure of porous media is given by taking into account the fractal property of pore in porous media. In the present approach, the proposed fractal model is explicitly related to porosity, fractal dimension of pore and saturation. It is found that the capillary pressure of porous media increases with decreasing of the porosity. In addition, based on the parametric effect analysis, we conclude that the capillary pressure of porous media is negatively correlated with the saturation. This model contains less empirical constants than the conventional correlations. No additional empirical constant is introduced. A good agreement between the proposed model predictions and experimental data is found. The validity of the present model for the capillary pressure of porous media is thus verified. The proposed fractal model can reveal the transport property of porous media.

### ACKNOWLEDGMENT

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