# Research Article Long Term Behavior of Composite Material Polyester-Wood Reinforced Glass Fiber

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Abstract: If the industry generally considers composite materials as potentially interesting, the lack of durability makes their often limited employment. Indeed, the design of composite structures often requires taking account of the long-term behaviour under various types of complex solicitations. This study is based on the development of a methodology for modeling the long term behavior under constant load for multilayer composite structures subjected to complex solicitations. This study is on the evaluation of the long-term behaviour of tri-layers glass/polyester-wood composite material. The objective is the evaluation of the long term strength of this material by the Goldenblat-Kopnov criterion. The studied model is an orthotropic tri layer plate of sandwich type with upper and lower flanges (skins) are in composite material glass/polyester KACT-B and the intermediate layer (soul) is pine wood. The character of loading of the plate is reflected in the model by the functions  $\sigma_{ik}(\xi)$ ,  $\sigma_{mn}(\xi)$  which can be of different natures. For example, to load in the long term by constant constraints in time (the weight own element for example), we will have:  $\sigma_{ik} = \text{const}$ ,  $\sigma_{mn} = \text{const}$ . The results of the calculations show that, for this type of material, the conditions of strength to the dangerous points of the plate are met.

Keywords: Goldenblat-Kopnov criterion, long term strength, tri layers composite

#### **INTRODUCTION**

Among the anisotropic materials, most widespread are composite materials. They demonstrate specific characteristics through their structure and their mechanical behaviour which is totally distinct from those of traditional materials, especially characterized, by a constitutive non-homogeneity, a clear anisotropy of their deforming and resistance properties, a low rigidity and important creeping to shearing. Their low specific weight is a major asset for their use in construction.

In practice, most used floors are either in reinforced concrete or wood. In the first case, to obtain the required capacity of resistance, we have to produce significant efforts in material (concrete + steel), which turns construction costs expensive. In the second case, we know that wooden floors have their limits in terms of their capacity to take back important long lasting exploitation loads, in spite of their advantage compared to their specific weight. For these various reasons, we must use today composite structures including more complex forms in construction (Gay, 2005; Baley, 2004).

To mitigate these disadvantages which impacts on construction costs are known, we propose the use of a sandwich" tri-layer floor "made of composite glass/polyester-wood material. This tri-layer material consists in lower and higher layers made of composite glass/polyester material and of an intermediate layer in pine wood (Fig. 1).

Many of the works are devoted to the study of issues related to the long term behaviour of orthotropic materials (Hémon *et al.*, 2011; Carrère *et al.*, 2011; Gamby and Vinet, 2008; Guedes, 2007; Krawczak, 1999; Dzenis, 1997; Kopnov and Olodo, 1992).

Including Hémon *et al.* (2011) works are devoted to holding long term composite matrix ceramics, based on a model of damage which is the modelling of the behaviour under static solicitation, by involving defects through a concept of homogenization and describing their macroscopic way developments. In this study, the authors have shown that ceramic matrix composites are orthotropic materials with a non-linear behaviour. They have also proven that life in fatigue of ceramic matrix composites depends on two factors: the mechanical and physico-chemical aspect. In total the study proposed in this section continues a model of damage in static for ceramic matrix composites to expand it then the behaviour in fatigue with the consideration of the physical and chemical phenomena and life prediction.

In Carrère *et al.* (2011) proposed an approach to damage and failure to forecast long term of composite



Fig. 1: Tri-layers polyester-wood plate

structures using a hybrid multiscale modelling damage, to predict the damage and rupture of the composite under multi-axial solicitation that is complex.

In Gamby and Vinet (2008) modelling of the behaviour in the long term of composite fiber made by compression tests that can lead to the determination of the viscoelastic parameters of the material.

In Krawczak (1999) is developed a methodology for the testing of long-term fatigue and creep behavior of polymers, test generating data required for the calculation of composite structures.

In Dzenis (1997) for the prediction of the long-term strength layered plates, the authors have developed a probabilistic model to analyze the evolution of the damage in the material; This model assumes that each layer of the laminate consists of a large number of mesovolumes for which the probabilistic functions are used for the analysis of rigidities and strengths induced in these micronutrients. These strengths are calculated by the laminated plate theory and the theory of random functions. The deterioration of the rigidities of the layers is calculated on the basis of the likelihood of damage to the mesovolumes using the panoply of random processes.

In Kopnov and Olodo (1992) it was developed a method of construction of the phenomenological criteria of long term strength of anisotropic materials. This method is based on two approaches in the theory of strength criteria: the Mises-Hill, Goldenblat-Kopnov approach and the A.A. Ilyushin approach. The coupling of these two approaches resulted in a criterion of strength, taking into account the particularities of the mechanical properties of the material.

The objective of our study will assess long term . strength floors made in tri layer composites, subjected to static loading with a sinusoidal loading law, by a macroscopic model that takes into account anisotropy of the mechanical properties of the material and especially the difference in behaviour in traction, compression in one direction any anisotropy and in shearing.

#### **MATERIALS AND METHODS**

This study was conducted in 2011 by a multidisciplinary team of the Polytechnic School of Abomey-Calavi/University of Abomey-Calavi i (UAC) Benin, University Institute of Technology-IUT/University of Abomey-Calavi (UAC) Benin, University Institute of Technology IUT/UT, University of Thies, Senegal with the assistance of Professor Krivochapko S.N. (Chairman of the Chair of strength of materials and structures of Russian Friendship University-Moscow) and especially of Professor Kopnov V.A. of the Military Academy Piotr Veleky in Moscow.

The experimental part of the study was carried out in the Laboratory of the Chair of strength of materials of the Russian Friendship University in Moscow and simulation studies are conducted at the Polytechnic School of Abomey-Calavi (EPAC)/University of Abomey-Calavi (UAC), Benin.

The material in study is a sandwich tri-layers plate: Both lower and higher soles are made in composite woven 2D orthogonal with glass KACT-B fibres and epoxy matrix. The reinforcements are laid out according to the orthogonal directions of a Cartesian coordinates, the unit constituting a simplified geometry with internal ordered architecture. The intermediate layer (the heart) is a pine wood rolled product which constant rubber bands are:  $E = 10.10^3$  MPa;  $\mu = 0.5$ (Fig. 1). The thicknesses of the layers are linked by the relation  $h_1 = h_2/4$ . The elastic characteristics of the composite of the plate are presented in Table 1.

For behaviour study in the long-term of the plate of glass/polyester type "KACT-B" (Fig. 1), we will use the Goldenblat-Kopnov long-term strength criterion of the anisotropic materials and the curve of the long-term strength of this material presented in Fig. 2. This curve



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Fig. 2: Experimental data and layers for approximation of the KACT-B glass composite long-term resistance

Table 1: Elastic characteristics of composite KACT-B

Characteristics of the composite		Long term
KACT-B	Static loading	loading
E <sub>x</sub> , MPa	$2, 22.10^4$	$1,95.10^4$
E <sub>y</sub> , MPa	$1,75.10^4$	$1,53.10^4$
E <sub>z</sub> , MPa	$0, 4.10^4$	$0,32.10^4$
G <sub>xy</sub> , MPa	$4.10^{3}$	$4.10^{3}$
G <sub>yz</sub> , MPa	$2.10^{3}$	$2.10^{3}$
G <sub>zx</sub> , MPa	$1.10^{3}$	$1.10^{3}$
$\mu_{xv}$	0.13	0.23
μ <sub>vz</sub>	0.45	0.6
H <sub>2X</sub>	0.1	0.12

is obtained in this study by means of compression in the long-term uniaxial test.

The expression of Goldenblat-Kopnov long-term strength criterion of anisotropic materials presents as follows:

$$\Phi(t) = (1)$$

$$\int_{0}^{t} \Omega_{ik}(t-\xi)\sigma_{ik}(\xi)d\xi + \left[\int_{0}^{t}\int_{0}^{t} \Omega_{ikmn}(t-\xi_{1}, t-\xi_{2})\sigma_{ik}(\xi_{1})\sigma_{mn}(\xi_{2})d\xi_{1}d\xi_{2})\right]^{\frac{1}{2}} \le 1$$

where,

t = The moment considered after the loading of the body

 $\xi$  = The time span in the interval  $0 \le \xi \le t$ 

 $\Omega$ ik,  $\Omega$ ikm -Influence functions (cores), taking into account of the rheological character of the material resistance properties; the nature and parameters of these functions are established though experimentation

 $\sigma$ ik ( $\xi$ ),  $\sigma$ mn( $\xi$ ) - Functional characterizing the loading program of the body

the argument  $(t - \xi)$  assumes their invariance at the beginning of the loading.

One of the possible shapes of the cores  $\Omega$ ik,  $\Omega$ ikmn can be, for example, the following expressions

$$\Omega_{ik}(t-\xi) = \prod_{ik} f_1(t-\xi)$$
  

$$\Omega_{ikmn}(t-\xi_1, t-\xi_2) = \prod_{ikmn} f_2(t-\xi_1, t-\xi_2)$$
(2)

In this study, we will use the exponential cores in the type:

$$\Omega_{ik}(t-\xi) = b \prod_{ik} e^{-\alpha(t-\xi)}$$

$$\Omega_{ikmn}(t-\xi_1, t-\xi_2) = b^2 \prod_{ikmn} e^{-\alpha(2t-\xi_1-\xi_2)}$$
(3)

where, b and  $\alpha$  - are experimental parameters for the given material? They are expressed in time unit. Ilik, Ilikmn Are the components of the material static resistance tensors?

#### **RESULTS AND DISCUSSION**

Evaluation of the long-term strength of the plate:

**Determining the components IIik, and IIikmn:** To determine the components of the static resistance tensors IIik, and IIikmn, the pattern best adapted is the Goldenblat-Kopnov pattern which takes into account the sensitivity of material on sign variation of normal constraints ( $\sigma$ +b  $\neq \sigma$ -b), as well as sign variation of shearing constraints ( $\tau$ +b  $\neq \tau$ -b):

$$\prod_{ik} \sigma_{ik} + \sqrt{\prod_{ikmn} \sigma_{ik} \sigma_{mn}} \le 1$$
(4)

In fact, the components of the resistance tensors  $\Pi$ ik, and  $\Pi$ ikmn are expressed according to the resistance constants of the material (Table 1). In pure traction and compression, in the first main direction, warp direction of the glass fibre, it is necessary to introduce into the relation (4):

- 
$$\sigma_{ik} = \sigma_{11} = \sigma_1^+$$
 in traction

-  $\sigma_{ik} = -\sigma_{11} = -\sigma_1^-$  in compression.

In that case we will have:

$$\begin{cases} \prod_{11} \sigma_1^+ + \sqrt{\prod_{1111} (\sigma_1^+)} = 1 \\ -\prod_{11} \sigma_1^- + \sqrt{\prod_{1111} (\sigma_1^-)^2} = 1 \end{cases}$$

Table 2: Components	of the resistance te	nsors							
Resistance tensors	$\Pi_{11}(MPa^{-1})$	П <sub>22</sub> ( М	Pa <sup>-1</sup> )	$\Pi_{1111}(MPa^{-2})$	$\Pi_{2222}(N)$	MPa <sup>-2</sup> )	$\Pi_{1122}(MPa^{-2})$	Π <sub>1212</sub>	(MPa <sup>-2</sup> )
Values	$-1,35.10^{-3}$ $-1,232.10^{-3}$ $7,83.10^{-5}$ $6,30.10^{-5}$		0-5	-5,51 .10 <sup>-5</sup>	0				
Table 3: Material spec	tific breaking stren	gth							
Specified breaking str	ength (MPa)	$\sigma_{b1}^{+}$	$\sigma_{b1}$	$\sigma_{b2}^{+}$	$\sigma_{b2}$	$\tau_{b0}$	$\tau_{b45}^+$	$\tau_{b45}$	$\tau^{\rm b}_{\ ck}$
Values		132	98	149	109	0	64	62	40

or

$$\Pi_{11} = \frac{1}{2} \left( \frac{1}{\sigma_1^+} - \frac{1}{\sigma_1^-} \right); \quad \Pi_{1111} = \frac{1}{4} \left( \frac{1}{\sigma_1^+} + \frac{1}{\sigma_1^-} \right)^2$$
(5)

Similarly,

$$\Pi_{22} = \frac{1}{2} \left( \frac{1}{\sigma_2^+} - \frac{1}{\sigma_2^-} \right); \ \Pi_{2222} = \frac{1}{4} \left( \frac{1}{\sigma_2^+} + \frac{1}{\sigma_2^-} \right)^2 \tag{6}$$

Let us consider two different states of pure shearing:

$$\sigma_{11} = \tau_{b_{45}}^+; \, \sigma_{22} = -\tau_{b_{45}}^+ \tag{7}$$

and

$$\sigma_{11} = -\tau_{b_{45}}^{-}; \ \sigma_{22} = \tau_{b_{45}}^{-} \tag{8}$$

In introducing the expressions (7) and (8) in criterion (4), we can obtain:

$$\left[ \left( \prod_{11} - \prod_{22} \right) + \sqrt{\prod_{1111} + \prod_{2222} - 2\prod_{1122}} \right] r_{b_{45}}^+ = 1$$
(9)

$$\left(\left(\prod_{22} - \prod_{11}\right) + \sqrt{\prod_{1111}} + \prod_{2222} - 2\prod_{1122} \mathbf{r}_{b_{45}}^{-} = 1 \quad (10)\right)$$

The compatibility of these last two expressions requires:

$$2(\prod_{11} - \prod_{22}) = \frac{1}{\tau_{b_{45}}^+} - \frac{1}{\tau_{b_{45}}^-}$$
(11)

Considering the relations (5) and (6), we can have:

$$\frac{1}{\sigma_1^+} - \frac{1}{\sigma_2^+} - \frac{1}{\sigma_1^-} + \frac{1}{\sigma_2^-} = \frac{1}{\tau_{b_{45}}^+} - \frac{1}{\tau_{b_{45}}^-}$$
(12)

In the specific case where  $\tau^+_{b45} = \tau^-_{b45}$ , we have:

$$\frac{1}{\sigma_1^+} - \frac{1}{\sigma_1^-} = \frac{1}{\sigma_2^+} - \frac{1}{\sigma_2^-} = 0$$
(13)

The formula (12) expresses the compatibility condition of the material's resistance constants. When this condition is not met, the criterion (4) could not be applied to this material.

Table 4: Values of the parameters  $\alpha$  and b $\alpha$  (hour <sup>-1</sup>)

Assuming then that the compatibility condition (12) is met, from (9) or (10) we can determine the component

b (hour

1.70

Π<sub>1122</sub>:

1.20

$$\Pi_{1122} = \frac{1}{8} \left[ \left( \frac{1}{\sigma_1^+} + \frac{1}{\sigma_1^-} \right)^2 + \left( \frac{1}{\sigma_2^+} + \frac{1}{\sigma_2^-} \right)^2 - \left( \frac{1}{\tau_{b_{45}}^+} + \frac{1}{\tau_{\bar{b}_{45}}^-} \right)^2 \right]$$
(14)

Concerning both states of pure shearing  $\tau$ +b0 and  $\tau$ -b0. The criterion (4) suggests:

$$2\left(\prod_{12} + \sqrt{\prod_{1212}}\right) \tau_{b_0}^+ = 1; \ 2\left(-\prod_{12} + \sqrt{\prod_{1212}}\right) \tau_{b_0}^- = 1 \ (15)$$

To solve the identities (15), considering that in the main system +b0 and  $\tau$ -b0 we will have:

$$\begin{cases} \Pi_{12} = \frac{1}{4} \left( \frac{1}{\tau_{b_0}^+} - \frac{1}{\tau_{b_0}^-} \right) = 0, \\ \Pi_{1212} = \frac{1}{16} \left( \frac{1}{\tau_{b_0}^+} + \frac{1}{\tau_{b_0}^-} \right)^2 = \frac{1}{4\tau_{b_0}^2} \end{cases}$$
(16)

In the main system of coordinates these components are zero.

For the material studied, the values of the resistance tensors components  $\Pi$ ik, and  $\Pi$ ikmn are reported to Table 2.

The values of the limits of glass KACT-B composite static resistance following the main directions of anisotropy are reported to Table 3,

**Determination of the parameters**  $\alpha$  and b: The parameters  $\alpha$  and b of the exponential cores are given using the mathematical pattern of the experimental long-term resistance curve. The expression of this pattern is as follows:

$$t_* = -\frac{1}{\alpha} \ln(1 - \frac{\alpha}{b} \frac{\sigma_b}{\sigma_{b*}}) \tag{17}$$

t\* = The breakthrough time of the material

 $\sigma b^*$  = The corresponding value of the constraint  $\sigma b$  = The elastic resistance limit of the material i

= The elastic resistance limit of the material in uniaxial compression warp direction

		Constraints tensor (MPa)						
								Long-term resistance
Side of the plate	Coordinates Most requested points	$\sigma_{11}$	$\sigma_{22}$	$\sigma_{33}$	$\sigma_{12}$	$\sigma_{23}$	$\sigma_{31}$	$\Phi(t)$
Superior side	x = 0.5a; y = 0.5a; z = 0	-96.09	-80.88	-4	0	0	0	0.707
Inferior side	x = 0.5a; y = 0.5a; z = 0.1a	96.27	81.09	0	0	0	0	0.614

Table 5: Long-term behaviour of the plate

The values of the  $\alpha$  and *b* parameters are obtained by solving systems of equations resulting from the Eq. (17) in the interval.

The digital values of the  $\alpha$  and *b* parameters of the glass/polyester KACT-B composite are given in the Table 4:

# Wording the evaluation criterion for the plate's long term strength considering:

The functions σik(ξ), σmn(ξ) which constitute the material's loading nature are constant over time i.e.:

$$\sigma_{ik} = const, \sigma_{mn} = const$$

- The exponential cores and the loading law σik(ξ),σik(0) ξr in the criterion (1),
- the specific case r = 0 which corresponds in practice to a static loading in the long term through constant constraints over time σik = const

The Goldenblat-Kopnov long-term strength criterion of anisotropic materials takes the following shape:

$$\Phi(t) = \int_{0}^{t} b \prod_{ik} e^{-\alpha(t-\xi)} \sigma_{ik_{(0)}} d\xi + \left[ \int_{0}^{t} \int_{0}^{t} b^{2} \prod_{ikmn} e^{-\alpha(2t-\xi_{1}-\xi_{2})} \sigma_{ik_{(0)}} \sigma_{mn_{(0)}} d\xi_{1} d\xi_{2} \right]^{0.5} \le 1$$
(18)

After integration, we obtain the condition of longterm strength for the constant given loading (Fig. 1):

$$\frac{b}{\alpha}(1 - e^{-\alpha t}) \Big[ \prod_{ik} \sigma_{ik_{(0)}} + \sqrt{\prod_{ikmn} \sigma_{ik_{(0)}} \sigma_{mn_{(0)}}} \Big] \le 1 \quad (19)$$

or

$$\prod_{ik} \sigma_{ik_{(0)}} + \sqrt{\prod_{ikmn} \sigma_{ik_{(0)}} \sigma_{m\eta_{(0)}}} \le \frac{\alpha}{b(1 - e^{-\alpha})}$$
(20)

Taking into account in the formula (20), the values of the parameters presented in Table 2 and 4 and after having established the components of the constraint tensor (Table 5) at most requested points of the plate of coordinates: X = 0.5a; y = 0.5a; Z = 0 and X = 0.5a; y = 0.5a; Z = h = 0.5a, for the long-term loading of the shape:

$$q(x, y) = q_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$
(21)

where, q0 = 5MPa, maximum long-term strength to the superior and inferior sides of the plate appears in Table 5

In terms of quality, we find a space problem in the superior side of the plate. However the normal constraint  $\sigma z$  ( $\sigma 33$ ) shows a value negligible enough compared to  $\sigma x$  and to  $\sigma y$  ( $\sigma 11$  and  $\sigma 22$ ). This component  $\sigma z$  could then be neglected in calculations. With the inferior side, the normal constraint  $\sigma z$  is null leading to a bi-axial request, that is why, the evaluation of the long-term resistance which is in theory a space problem can be simplified and brought back to a planning problem.

In addition, the and b parameters play a major role from the point of view of the durability of the material. In fact, compared to the criterion (19), calculations show that the factor  $b/\alpha$  impacts on the safety margin of the material submitted to a long-term loading. This represents a considerable advantage and shows that we can improve long-term strength of material while having a close look at the evolution of  $\alpha$  and b over time.

The main interest of the results obtained in this study lies in the fact that first up to now, issues related to the prediction of the long term behaviour of composites are resolved by models proposed in thermodynamic form (Schapery, 1997), with very often approaches of homogenization does not allow to establish explicitly a model of macroscopic behavior of the material. While most of the polymeric materials well enough meet models based on the nonlinear Schapery viscoelastic approach (Schapery, 2000). The approach taken in this study is based on elasto-viscoplastic model of Goldenblat-Kopnov (Kopnov and Olodo, 1992) which is based on a coupling of the Mises-Hill plastic potential and Ilyushin A.A. kinetic approach. By this model, the results obtained in this study reflect rigorously the difference in behavior in traction and compression ( $\sigma_{bo}^+ \neq \sigma_{bo}^-$ ). in a direction any anisotropy and the difference in the sign of the shear stresses  $(\tau_{bo}^{+} \neq \tau_{bo})$ . What is true in the calculation of the components of static resistance tensors  $\Pi_{ik}$  and  $\Pi_{ikmn}$  of the material.

These results are improving existing homogenization methods and can be used in the future in the creation of normative documents for the calculation of the composite structures under different solicitations.

## CONCLUSION

In this study it is proposed a methodology for modeling of the long-term behaviour of multilayer composite structures under constant load generating complex loading. This methodology is based on a tensor model of the long-term strength criterion anisotropic materials taking into account .too features mechanical properties of the material, anisotropy, the duration and the nature of the loading variant at the time, the influence of the previous loading on the strength of the material at the moment considered (heredity) and the type of stress state.

On the basis of the experimental data obtained from tests of long term strength, were determined the numerical values of parameters of the exponential type kernels for the establishment of these influence functions of the long term strength criterion.

It has carried to the determination of the tensor of constraints for this layered orthotropic plate. These calculations being necessary for the assessment of long term strength of the plate.

Numerical simulation was conducted to assessment of long-term strength layered orthotropic plates. Could see that the conditions of strength to the dangerous points of the plate are met under the proposed model.

The results demonstrate the effectiveness of the method proposed in this study for the evaluation of long-term strength structures in anisotropic materials.

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