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**Research Article**

# Dynamics Analysis of Nonlinear Stiffness Rotor-Bearing System with Crack Fault

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**Abstract:** The model of nonlinear stiffness rotor-bearing system with crack fault was set up, and the nonlinear dynamic characteristics of the system were analyzed by numerical method. In critical rotational speed range, there are frequency division motions when the crack depth is sallow and chaotic motion when the crack is deep. In supercritical rotational speed range, the main motions are periodical frequency divisions. The numbers increase along with the depth of crack. In ultra-supercritical rotational speed range, there are quasi-periodic motion decreasing when the crack depth is sallow and periodical-3 motion when the crack is deep. The conclusions provide a theoretic basis reference for the failure diagnosis to the nonlinear stiffness rotor-bearing systems with crack fault.

**Keywords:** Crack, dynamic characteristics, nonlinear, rotor-bearing system, stiffness

## INTRODUCTION

The crack in shaft is the severe fault that difficult to detect in time, which bring enormous intimidate to running safety of rotor system. Many researchers do plenty of studies on the dynamic characteristics of rotor system with crack fault. Henry (1976) set up motion differential equation in circumrotate coordinates adopting asymmetry stiffness single disk rotor model. They found that there are subcritical resonance of 1/2, 1/3, 1/5 et al, especially subcritical resonance of 1/2. Kevin and Yin (2000) studied the vibration characteristics and stability of rotor adopting three kinds of models including the open crack model, close crack model and breathing crack model. Darpe *et al*. (2004) studied the transient response and breathing behaviors in critical rotate and sub-humorous resonance regions of a Jeffcott rotor adopting breathing crack model, open-close crack model and open crack model. They found that breathing crack model can simulate the breathing behaviors of crack in shaft better, which are affected by imbalance direction angle, acceleration rate, damping and crack depth. The breathing behaviors of crack are different at acceleration process and deceleration process. He *et al*. (2003) used Poincaré map to analyze the three known types of bifurcation response. In using Poincaré map, they adopt the cosine model for opening and closing of crack with time. They briefly reviews the well known view point that the bifurcation and stability of equilibrium point of Poincaré map can reflect the transformation of orbit of rotor. They found that for crack-containing rotor, the second sub-harmonic motion corresponds to the period doubling bifurcation, which may give rise to super-harmonic resonance, the jump phenomenon of response with variation of parameter corresponds to the saddle-node bifurcation, and the occurrence of quasi-periodic motion corresponds to the Naimark-Sacker bifurcation. Zheng *et al*. (2003) deduced the motion differential equations of a horizontal cracked Jeffcott rotor applying the model of breath crack and considering the variation of moment of inertia at the crack, and the nonlinear dynamic response and swing vibration of the system were theoretical and experimental investigated. From numerical simulation, they found that when crack is sallow the response of the system is periodic motion at some rotating speeds; it will appear periodic doubling motion and harmonic components. When crack is deep, it will appear many nonlinear responses. On the other hand, there will appear harmonic components in swing response of disk. From experimental results, they found obvious that when crack exists, there will be high exponent harmonic components in responses of swing vibration and transverse oscillation. Although the frequency components of transverse oscillation are the same as swing vibration, but the amplitudes of two cases are apparently different. Sekhar and Prasad (1997) developed a flexibility matrix for a slant crack and later the stiffness matrix of a slant cracked element to use subsequently in the FEM analysis of the rotor-bearing system. The frequency spectrum of the steady state response of the cracked rotor was found to have sub-harmonic frequency components at an interval frequency corresponding to the torsional frequency, which can be used for crack detection. The model of nonlinear stiffness rotor-bearing system with crack fault

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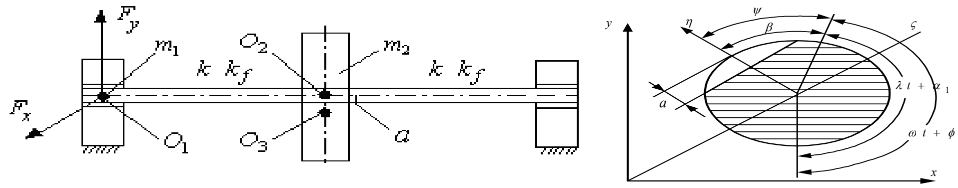


Fig. 1: Model of nonlinear stiffness rotor-bearing system with crack fault

was set up, and the effect of crack depth to the nonlinear behaviors of rotor at critical and super-critical rotating range was investigated.

## NONLINEAR STIFFNESS, CRACK MODELING AND THE SYSTEM EQUATION OF MOTION

The model of nonlinear stiffness rotor-bearing system with crack fault and section of crack are shown in Fig. 1. Shaft coupling connects the motor and rotor. The system mass is equivalently concentrated on the center of the disk and every bearing support. The torsional vibration and gyro moment are neglected and only the lateral vibration of system is considered. Both ends of rotor are supported by journal bearings with symmetrical structures; $O_1$ is geometric center of bearing; $O_2$ is geometric center of rotor; $O_3$ is center of mass of rotor; $m_1$ is lumped mass of rotor at bearing; $m_2$ is equivalently lumped mass at disk. Massless elastic shaft connect disc with bearing. $k$ is the stiffness of elastic shaft, $k_f$ is the nonlinear stiffness of shaft; there is a transversal crack with deepness of a in the middle of shafts. $F_x$ and $F_y$ are nonlinear oil film forces.

Assuming the radial displacements of left axes center of rotor system are $x_1$, $y_1$ respectively, the elongate of neutral axis and physics nonlinear factors of shaft are expressed by the sum of linear term and cubic term, then the radial displacements of disk are $x_2$, $y_2$ respectively, then non-dimensional motion differential equation of system can be expressed as:

$$\begin{cases}
\ddot{x}_1 + \xi_1\dot{x}_1 + \eta_{f1}[(x_1 - x_2)^2 + (y_1 - y_2)^2](x_1 - x_2) \\
\quad + \eta_1[1 - \frac{\varepsilon\delta}{2}F(\psi)](x_1 - x_2) + \frac{\varepsilon\eta_1}{2}F(\psi)[(x_1 - x_2) \\
\cos 2t + (y_1 - y_2)\sin 2t] = \frac{1}{M_1} f_x(x_1, y_1, \dot{x}_1, \dot{y}_1) \\
\ddot{y}_1 + \xi_1\dot{y}_1 + \eta_{f1}[(x_1 - x_2)^2 + (y_1 - y_2)^2](y_1 - y_2) \\
\quad + \eta_1[1 - \frac{\varepsilon\delta}{2}F(\psi)](y_1 - y_2) + \frac{\varepsilon\eta_1}{2}F(\psi)[(x_1 - x_2) \\
\sin 2t - (y_1 - y_2)\cos 2t] = \frac{1}{M_1} f_y(x_1, y_1, \dot{x}_1, \dot{y}_1) - G \\
\ddot{x}_2 + \xi_2\dot{x}_2 + 2\eta_{f2}[(x_2 - x_2)^2 + (y_2 - y_1)^2](x_2 - x_1) \\
\quad + 2\eta_2[1 - \frac{\varepsilon\delta}{2}F(\psi)](x_2 - x_1) + \varepsilon\eta_2 F(\psi)[(x_2 - x_1) \\
\cos 2t + (y_2 - y_1)\sin 2t] = u\cos\tau \\
\ddot{y}_2 + \xi_2\dot{y}_2 + 2\eta_{2f}[(x_2 - x_2)^2 + (y_2 - y_1)^2](y_2 - y_1) \\
\quad + 2\eta_2[1 - \frac{\varepsilon\delta}{2}F(\psi)](y_2 - y_1) + \varepsilon\eta_2 F(\psi)[(x_2 - x_1) \\
\sin 2t - (y_2 - y_1)\cos 2t] = u\sin\tau - G
\end{cases} \tag{1}$$

where, $\varepsilon$ and $\delta$ are relative stiffness parameters about crack deepness *a* (Yang *et al*., 2002). $F(\psi)$ is open and close function of crack, $F(\psi)=1+\cos(\psi)/2$, $\psi= t - ø_0+\beta- \arctan y/x$, $\beta$ is contained angle between crack direction and eccentricity; $ø_0$ is initial phase place; $\zeta_i = c_i/m_i \omega$, $c_1$ is damping coefficient of bearing respectively; $c_2$ is damping coefficient of disc respectively. $\eta^i$ $k/m_i \omega^2$, $\eta_{fi}$ $k_f/m_i \omega^2$, $M_i = c\omega^2/\delta_i$, $G = g/c\omega^2$, $x_i = X_i/c$, $y_i=Y_i/c$ are non-dimensional displacement relative to bearing clearance c. $f_x = F_x/\delta$, $f_y = F_y/\delta$ are non-dimensional nonlinear oil-film force components (Luo *et al*., 2007), $\tau$ is non-dimensional time, $\tau = \omega t$, *u* is non-dimensional eccentricity of rotor.

## RESULTS AND DISCUSSION

Aiming at the strong nonlinear characteristics, the numerical simulation method is used to analysis the vibration response of rotor at different parameter factors. Figure 2 is the response of time-domain waveform, chart of axes track, amplitude spectrum and Poincaré map at different crack depths when the rotate speed is $\omega$ = 880rad/s (The first critical rotate speed of the system (with no faults) is $\omega_0$ = 882.5 rad/s). When the non-dimensional crack depth $b$ = 0.4 ($b = a/R$, $R$ is the radius of shaft), it can be seen from time-domain waveform and chart of axes track that it corresponds to periodic-2 motion, there is 1/2 frequency division in amplitude spectrum and two fixed points in Poincaré map, which illuminate the periodic-2 frequency division motion. When the non-dimensional crack depth $b$=0.7, it can be seen from time-domain waveform and chart of axes track that it corresponds to periodic-4 motion, there are 1/4, 1/2 and 3/4 frequency divisions in amplitude spectrum and four fixed points in Poincaré map, which illuminate the periodic-4 frequency division motion. Along with the increase of crack depth, when the non-dimensional crack depth $b$ = 1.0, it can be seen from time-domain waveform and chart of axes track that the response of the rotor are quite disorder, there are some obvious continuous spectrum in amplitude spectrum and chaotic motion surround four "equilibrium point", which illuminate the chaotic motion. When the non-dimensional crack depth $b$ = 1.3, the motion characteristics are similar to $b$=1.0, there are some obvious continuous spectrum in amplitude spectrum and chaotic motion surround two "equilibrium point".

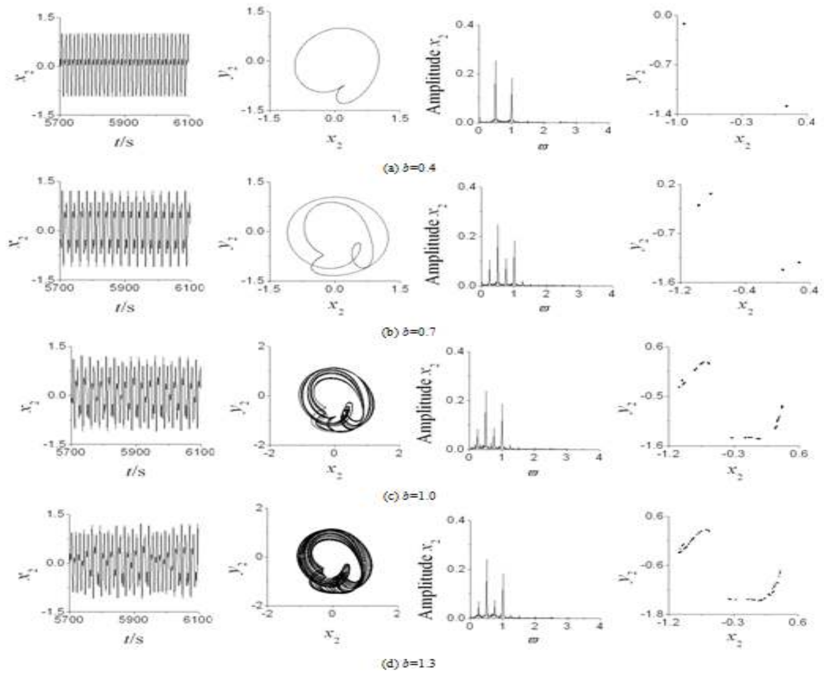


Fig. 2: Responses of rotor system with the changing of crack depths when $\omega$ = 880rad/s

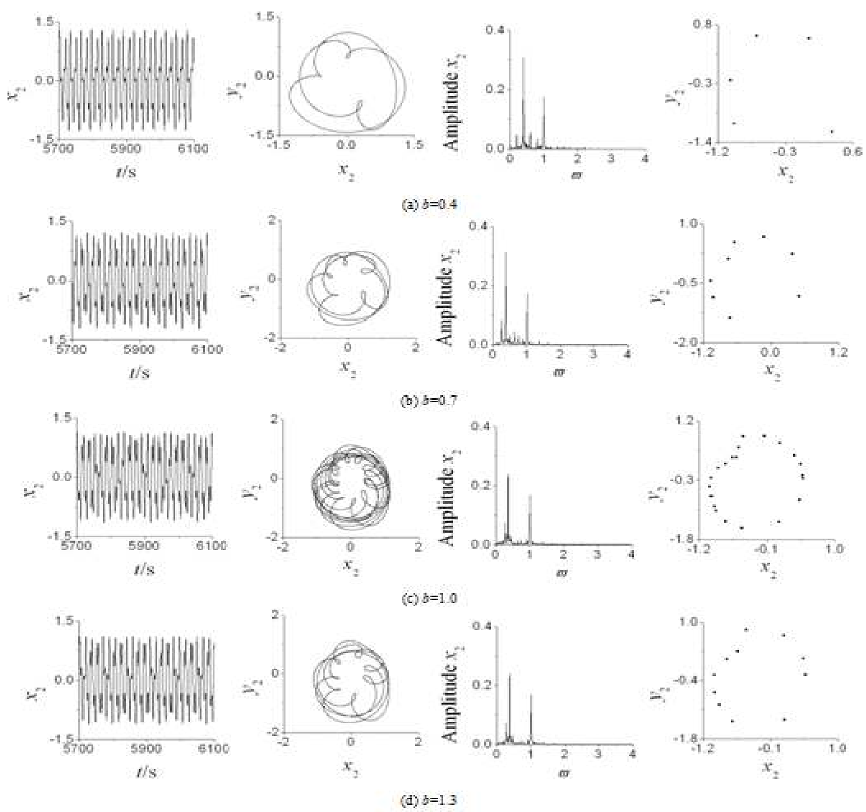


Fig. 3: Responses of rotor system with the changing of crack depths when $\omega$ = 1390rad/s

Figure 3 is the response of rotor system at different crack depths when the rotate speed is $\omega$=1390rad/s. It can be seen that in supercritical rotational speed range, the main motion form of the rotor system are

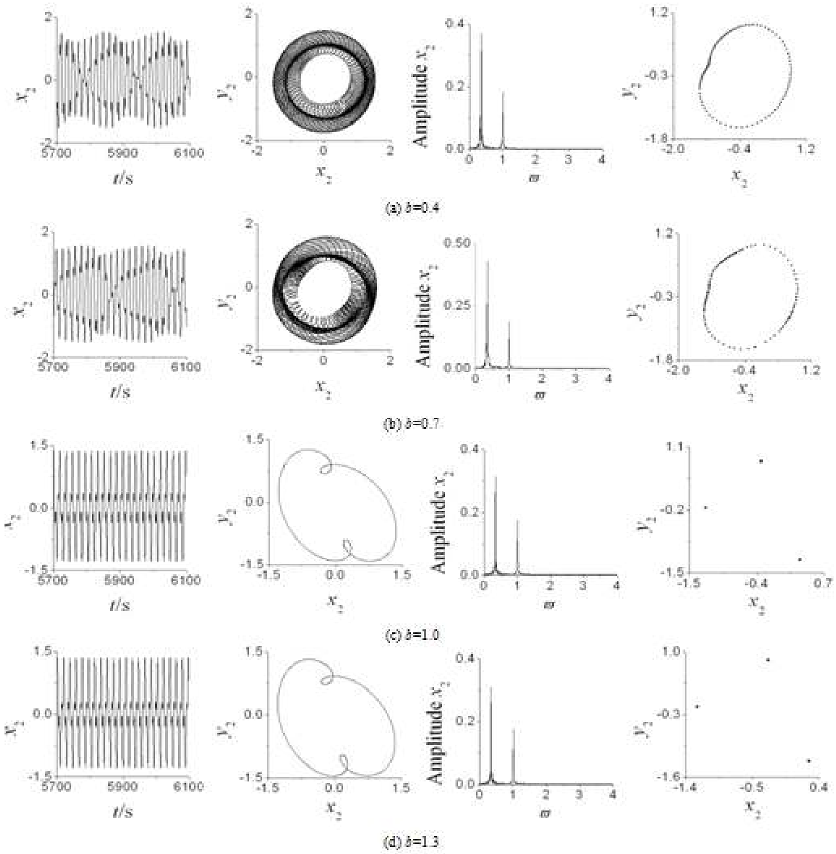


Fig. 4: Responses of rotor system with the changing of crack depths when $\omega$ = 1920rad/s

frequency division motions. The responses are respectively for periodic-5, periodic-8, periodic-21 and periodic-11 along with the increase of crack depth.

Figure 4 is the response of rotor system at different crack depths when the rotate speed is $\omega$ = 1920 rad/s. In ultra-supercritical rotational speed range, when the non-dimensional crack depth $b$ = 0.4 and 0.7, there are “beat vibration” response in time-domain waveform, periodic response similarity in chart of axes track, higher harmonic and incommensurability frequencies in amplitude spectrum and obvious closed curve in Poincaré map, which illuminate the quasi-periodic motions. When the non-dimensional crack depth $b$ = 1.0 and 1.3, it can be seen from time-domain waveform and chart of axes track that it corresponds to periodic-3 motion, there is 1/3 frequency division in amplitude spectrum and three fixed points in Poincaré map, which illuminate the periodic-3 frequency division motion.

## CONCLUSION

The model of nonlinear stiffness rotor-bearing system with crack fault was set up, and the nonlinear dynamic characteristics of the system were analyzed by numerical method. The motion forms are different in critical rotational speed range, supercritical rotational speed range and ultra-supercritical rotational speed range along with the increase of crack depth, which can be used as the basis for fault diagnosis to the nonlinear stiffness rotor-bearing systems with crack fault.

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