

## Research Article

### Mathematical Modeling of Fuel Pressure inside High Pressure Fuel Pipeline of Combination Electronic Unit Pump Fuel Injection System

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**Abstract:** In order to completely understand the trend of pressure variations inside High Pressure (HP) fuel pipeline of Combination Electronic Unit Pump (CEUP) fuel injection system and study the impact of two major physical properties of fuel i.e., density and dynamic viscosity on pressure a 1D nonlinear dynamic mathematical model of fuel pressure inside pipeline using Wave Equation (WE) has been developed in MATLAB using finite difference method. The developed model is based on the structural parameters of CEUP fuel injection system. The impact of two major physical properties of the fuel has been studied as a function of pressure at various operating conditions of diesel engine. Nearly 13.13 bars of increase in pressure is observed by increasing the density from 700 kg/m<sup>3</sup> to 1000 kg/m<sup>3</sup>. Whereas an increase of viscosity from 2 kg/m.s to 6 kg/m.s results in decrease of pressures up to 44.16 bars. Pressure corrections in the mathematical model have been incorporated based on variations of these two fuel properties with the pressure. The resultant pressure profiles obtained from mathematical model at various distances along the pipeline are verified by correlating them with the profiles obtained from simulated AMESim numerical model of CEUP. The results show that MATLAB mathematical results are quite coherent with the AMESim simulated results and validate that the model is an effective tool for predicting pressure inside HP pipelines. The application of the this mathematical model with minute changes can therefore be extended to pressure modeling inside HP rail of Common Rail (CR) fuel injection system.

**Keywords:** Density, dynamic viscosity, finite difference, mathematical model, wave equation

## INTRODUCTION

Variation in injected fuel quantity and injection timing adversely affect the fuel efficiency and pollutant emissions (Future Diesel Engines, Society of Automotive Engineers, 1997) and therefore, development efforts of the diesel engine are addressed to provide increased control, high efficiency and less toxic emission products (An *et al.*, 2010). Pressure fluctuations inside the HP fuel injection system are one of the several factors liable for variations of injected fuel quantity. CEUP fuel injection system is being used in variety of heavy duty vehicles and marine engines in China. This system meets the China's newest emission legislations (Liyun *et al.*, 2008; Liyun *et al.*, 2010) by providing flexible control of injection timing and injection duration with the help of a high speed solenoid. HP pipeline between HP pump and injector is one of the major components in CEUP as shown in Fig. 1. In this study mathematical model of fuel pressure inside HP fuel pipeline using a wave equation has been developed not only to study the trend of

pressure wave but also the effects of key fuel properties like density and dynamic viscosity on pressure wave. Many researches (Rakopoulos and Hountalas, 1996; Lee *et al.*, 2002; Catania *et al.*, 2008) have used principles of mass continuity and momentum conservation to model pressure inside fuel pipeline in their mathematical models. Whereas one of the two methods used in modeling of pressure wave inside Common Rail (CR) fuel injection system (Kristina, 2000) is wave equation.

Developed mathematical model correctly predicts the pressures at various locations along the pipeline and at different operating conditions. The results have been validated by correlating the results with those obtained from simulated AMESim numerical model of CEUP. The results are quite coherent and validate that the developed mathematical model as an effective base tool for further investigations. Moreover the model predicts that the density and dynamic viscosity changes as a function of pressure during fuel injection cycle and define the trend of pressure profile in the HP fuel pipeline.

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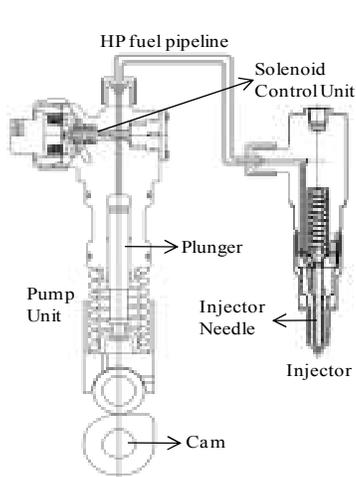


Fig. 1: Schematic of the CEUP system

**MATERIALS AND METHODS**

**CEUP fuel injection system and experimental setup:**

CEUP fuel injection system is a combination of several units of Fig. 1 consisting of a pump, a solenoid control unit, a fuel pipeline and an injector. Four pump units are combined together on a low pressure combination pump box as shown in Fig. 2 to make one CEUP. Low pressure (~5 bars) fuel is pumped into plunger chamber by a supply fuel pump. A solenoid control unit determines the timing and duration of fuel injection. Upward motion of plunger develops pressure required for fuel injection. Fuel in the plunger chamber returns back to the fuel tank if the solenoid is de-energized otherwise fuel pressure inside the plunger chamber, HP

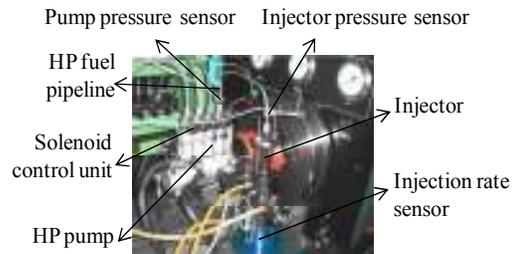


Fig. 2: Testbench for CEUP system

Table 1: Combinations of cam speed, cam angle and pipe length

Cam rotational speed (rpm)	Cam Angle (°CaA)	Pipe Length (m)
500, 700, 900,	2, 6, 10	0.27, 0.47
1100 and 1300	and 14	and 0.67

pipeline and injector starts increasing till it exceeds the closing pressure of injector needle. After which the injector needle is lifted and fuel is injected into the cylinder. Fuel injection process is stopped by de-energizing the solenoid again.

During laboratory experiments pump and injector pressures at both ends of HP fuel pipeline were recorded using KISTLER 4067 sensors along with other required parameters as shown in Fig. 2. Laboratory tests were conducted at all combinations of cam rotational speeds, cam angles (°CaA) and fuel pipe lengths mentioned in Table 1. Measured pump and injector pressures were then used not only to validate the CEUP AMESim numerical model shown in Fig. 3 but also as boundary conditions of the mathematical model. Pressures inside HP pipeline were calculated

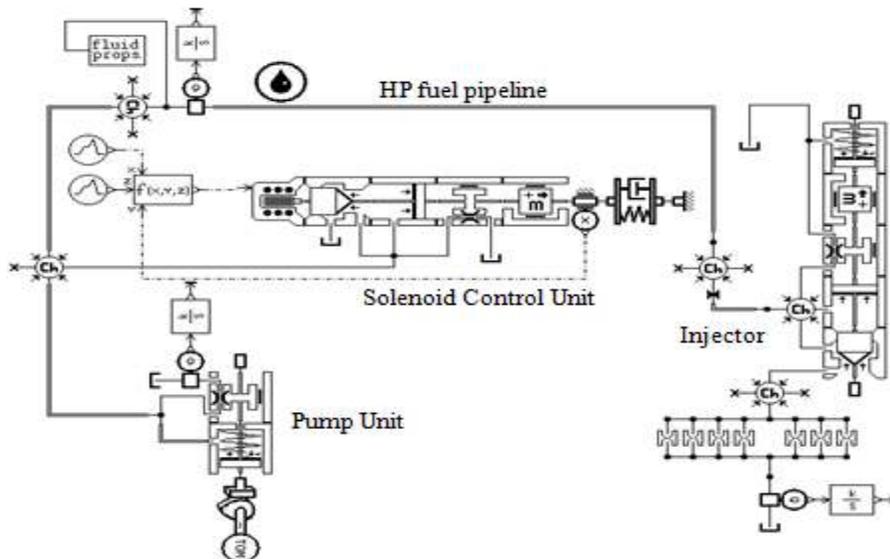


Fig. 3: CEUP AMESim simulation model

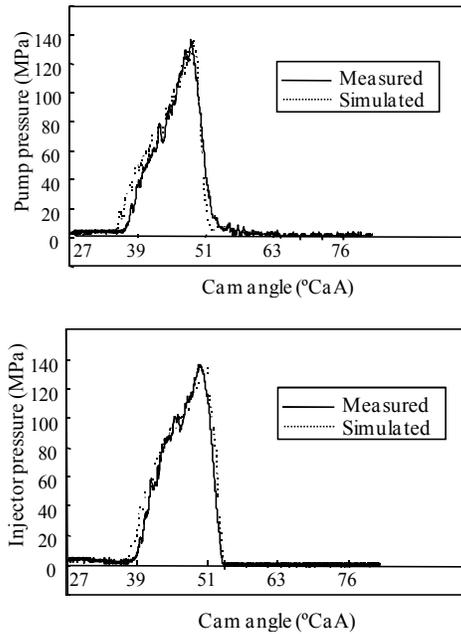


Fig. 4: Comparison of experiment and AMESim simulation results (a) pump end pressures and (b) injector end pressures

**CEUP numerical model and validation:** A numerical model of CEUP system shown in Fig. 3 was also developed in AMESim. The model was validated by comparing pump and injector pressures with those obtained through laboratory experiments at operating conditions mentioned in Table 1. The comparison results at 1100 rpm cam rotational speed, 10°CaA and 0.47 m length of HP pipeline are shown in Fig. 4a and b. The results are quite coherent and therefore validate the AMESim CEUP numerical model.

## RESULTS AND DISCUSSION

**Mathematical modeling of HP pipe and its validation:** The flow of fuel for mathematical modeling is considered laminar and assumed to be one-directional. Fuel is considered homogenous and without any cavitations and no losses within the fuel and at boundaries are considered. At first a 1D mathematical model is developed using the ideal WE (1). This second-order linear partial differential equation is given along with initial and boundary conditions:

$$\left. \begin{aligned}
 \frac{d^2 p(x,t)}{dt^2} &= c^2 \frac{d^2 p(x,t)}{dx^2} & x \in (0,L), t \in (0,T) \\
 p(x,0) &= \text{initial\_pressure} & x \in (0,L) \\
 \frac{dp(x,0)}{dt} &= 0 & x \in (0,L) \\
 p(0,t) &= \text{pump\_pressure} & t \in (0,T) \\
 p(L,t) &= \text{injector\_pressure} & t \in (0,T)
 \end{aligned} \right\} (1)$$

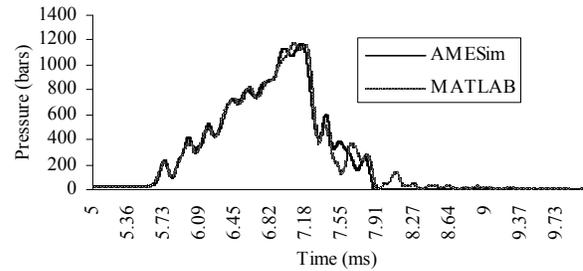


Fig. 5: Comparison of pressure predictions between AMESim and MATLAB ideal WE

where,  $p$  is pressure,  $c$  is speed of sound and  $L$  is total length of pipe. Eq. (1) is solved by first discretizing it in space and time by using centered difference approximation. Mesh sizes of time  $t$  and distance  $x$  have been taken as  $1\mu s$  and  $L/100$  respectively but within the stability criteria of  $(c\Delta t/\Delta x) \leq 1$ . Fuel pressure inside pipe have been sought out at these mesh points. It is very clear from Eq. (1) that by keeping the  $c$  constant it relates second time derivative of pressure to its spatial laplacian  $\Delta p$ .

Pressure obtained from mathematical model at atleast 10 equidistant locations inside pipe have been compared with those obtained from AMESim numerical model. This comparison has been made at all combinations mentioned in Table 1. All the results are quite coherent thus validating the mathematical model. Maximum deviation among the compared results is observed around middle of the pipe lengths. Therefore pressures only in the middle of pipe lengths are discussed in the rest of this study for the sake of space. As an example both AMESim and MATLAB pressure curves at the centre of 0.47 m long HP pipeline with operating conditions of 1100 rpm cam speed and 10° CaA are shown in Fig. 5.

Figure 5 shows that the both curves have nearly the same trend with some difference towards the end of fuel injection cycle. The prediction of mathematical model varies differently from AMESim numerical model after 7.44 ms. The reason for this difference might be phase change during pressure wave propagation or due to harmonics of main frequency component in pump signal. Considering only the trends before this difference a maximum of 58.15 bars of pressure difference is observed at 7.04 ms. Moreover a delay in mathematical model is also observed due to the reason that the speed of sound is considered constant in the mathematical model whereas in reality it varies with the change in pressure (Jun-Xiao *et al.*, 2001).

Losses inside fuel are due to viscosity, heat conduction and molecular level energy losses and viscous losses in medium can adequately define most of

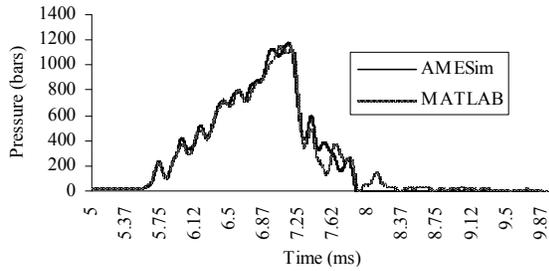


Fig. 6: Comparison of pressure predictions between AMESim and MATLAB WE with viscous damping

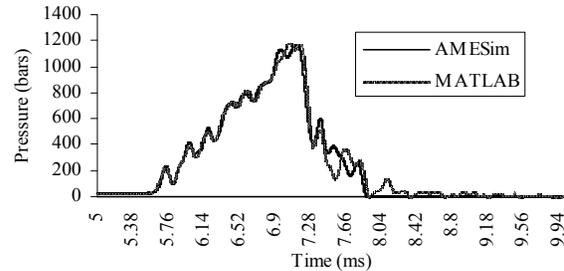


Fig. 9: Comparison of pressure predictions between AMESim and MATLAB WE with viscous damping, density and pressure corrections

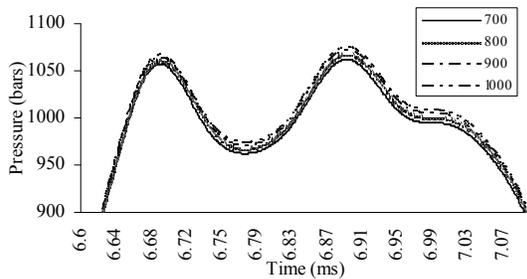


Fig. 7: Effect of change of density on pressure

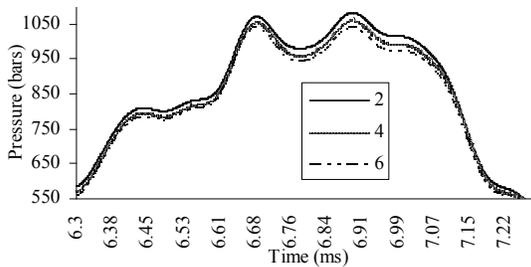


Fig. 8: Effect of change of viscosity on pressure

the losses (Bertram, 1973). A wave equation with viscosity damping has been derived which is given below:

$$\left. \begin{aligned} \frac{d^2 p(x,t)}{dt^2} &= c^2 \frac{d^2 p(x,t)}{dx^2} + \frac{4\eta}{3\rho} \frac{d^3 p(x,t)}{dx^2 dt} & x \in (0,L), t \in (0,T) \\ p(x,0) &= \text{initial\_pressure}, \frac{dp(x,0)}{dt} = 0 & x \in (0,L) \\ p(0,t) &= \text{pump\_pressure} & t \in (0,T) \\ p(L,t) &= \text{injector\_pressure} & t \in (0,T) \end{aligned} \right\} \quad (2)$$

where,  $\rho$  is density and  $\eta$  is dynamic viscosity. Same procedure for solving and validation of Eq. (2) is carried out as for Eq. (1) for all operating conditions mentioned in Table 1. The results are coherent and validate the model. Both MATLAB and AMESim pressure curves with losses at centre of 0.47 m high pressure pipe line with operating conditions of 1100 rpm cam speed and 10°CaA are shown in Fig. 6. Change in the pressure

trends towards the end of fuel injection cycle after 7.38 ms and a delay in mathematical model are observed. A maximum of 55.4 bars of pressure difference is recorded at 6.95 ms time when ignoring change in trend.

So far  $c$ ,  $\rho$  and  $\eta$  have been considered constants in the mathematical modeling. During fuel injection process in CEUP pressure inside HP pipeline rises as high as 1500 bars. This variation in pressure has definite impact on  $\rho$  and  $\eta$  of the diesel fuel (Jun-Xiao *et al.*, 2001; Andre *et al.*, 2004). The effects of change of  $\rho$  and  $\eta$  on pressures predicted by mathematical model have also been studied at all operating conditions mentioned in Table 1. It has been observed that pressure increases with the increase in density and the change is more prominent at high pressures. A maximum increase of nearly 13.13 bars of pressure has been observed at 6.96 ms by increasing density from 700 kg/m<sup>3</sup> to 1000 kg/m<sup>3</sup> at operating conditions of 1100rpm cam speed and 10°CaA is shown in Fig. 7.

Pressure decreases with the increase of dynamic viscosity and this change too is more prominent at high pressures. A maximum decrease of 44.16 bars in pressure observed at 6.97 ms with the increase of viscosity from 2 kg/m.s to 6 kg/m.s at operating conditions of 1100 rpm cam speed and 10°CaA is shown in Fig. 8.

In order to accommodate the pressure corrections due to changes in density and dynamic viscosity in the derived mathematical model experimental relations in Eq. (3) derived by Jun-Xiao *et al.* (2001) have been used:

$$\left. \begin{aligned} \rho(x,t) &= \rho_0(x,t) \times \left( 1 + \frac{0.6 \times 10^{-9} p(x,t)}{1 + (1.7 \times 10^{-9} p(x,t))} \right) & x \in (0,L) \\ \eta(x,t) &= \eta_0(x,t) \times \exp\left[ (\ln \eta_0(x,t) + 9.67) \left\{ (1 + 5.1 \times 10^{-9} p(x,t))^z \times \left( \frac{T-138}{T_0-138} \right)^{-s} - 1 \right\} \right] & \end{aligned} \right\} \quad (3)$$

where  $\rho_0$ ,  $\eta_0$  are recorded at reference room temperature  $T_0$  and  $z$  and  $s$  are indexes between viscosity and pressure and viscosity and temperature respectively. Comparison of AMESim model and viscous damped, density and viscosity corrected MATLAB model is shown in Fig. 9 at operating conditions

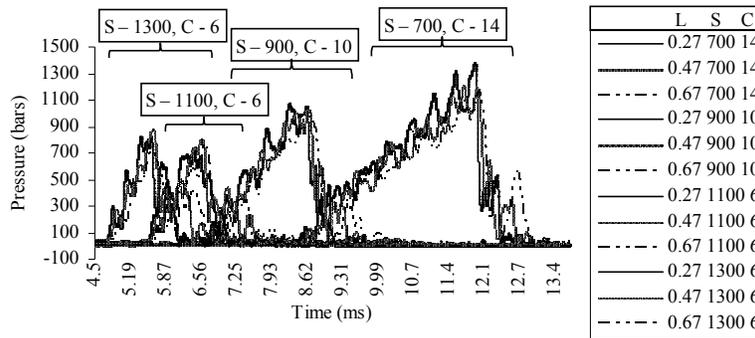


Fig. 10: Pressure predictions of mathematical model in the middle of pipe lengths at some of operating conditions mentioned in Table 1. Where  $L$ ,  $S$  and  $C$  are length of pipeline (m), cam speed (rpm) and cam angle ( $^{\circ}$ CA) respectively

of 1100 rpm cam speed and  $10^{\circ}$ CaA. A maximum of 54.3 bars of pressure difference is recorded at 6.96 ms time when change in trends towards the end of fuel injection process ( $t > 7.44$  ms) is ignored. Figure 10 shows pressure predictions of mathematical model of viscous damped WE with density and viscosity corrections in the middle of pipes at some of operating conditions mentioned in Table 1.

### CONCLUSION

- Based on the structural parameters of the CEUP a numerical model of entire CEUP system in AMESim has been developed and validated against laboratory experiments by comparing pump and injector pressures at various operating conditions.
- A 1D nonlinear dynamic mathematical model of fuel pressure inside pipeline using wave equation has been developed in MATLAB. The mathematical model with laboratory measured pump and injector pressures as boundary conditions has been validated using numerical model of CEUP at various operating conditions.
- The effect of changes in two major fuel properties i.e., density and dynamic viscosity as a function of pressure have also been studied and implemented. Nearly 13.13 bars of increase in pressure is observed by increasing density from  $700 \text{ kg/m}^3$  to  $1000 \text{ kg/m}^3$  at operating conditions of 1100 rpm cam speed and  $10^{\circ}$ CaA. Whereas an increase of viscosity from  $2 \text{ kg/m.s}$  to  $6 \text{ kg/m.s}$  results in decrease of pressures up to 44.16 bars at same operating conditions.

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