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Research Article

Research of Micro-Rectangular-Channel Flow Based on Lattice Boltzmann Method

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Abstract: The source codes of Lattice Boltzmann Method (LBM) based on the D3Q15 model were developed in the current study. In the simulation process, the pressure boundary conditions were developed and the rectangular micro-channel flow was investigated. The Width-to-Height ratio (W/H) is the main influencing parameter of the rectangular micro-channel flow in low Reynolds number condition. A smaller W/H of the rectangular micro-channel results in a greater difference of drag coefficient between LBM simulation data and empirical formula data. On the empirical data of drag coefficient approximating laminar flow, when the Reynolds number is more than 10, the drag coefficients of LBM simulation and empirical data of laminar flow are in substantial agreement. Thus, in more than 10 conditions of Reynolds number, the empirical data on laminar flow can be used in rectangular micro-channel flow.

Keywords: D3Q15, drag coefficient, lattice boltzmann method, rectangular micro-channel flow

INTRODUCTION

Micro-channel flow is the typical flow in a Micro-Electronics-Mechanism-System (MEMS). Owing to the difficulty of conducting an experimental study in micro flow, we rely on a numerical research one of the main study methods in this field. However, micro flow is out of continuous medium flow and Computational Fluid Dynamics (CFD) cannot be applied in micro flow simulation because it causes a bottleneck in numerical research. In discontinuous medium flow, the Direct Simulation of Monte Carlo (DSMC) and Lattice Boltzmann Method (LBM) are two kinds of advantageous numerical methods. In DSMC simulation, velocity boundary is a well-considered inlet boundary condition. The velocity of simulation molecules entering the flow field is a combination of inlet flow velocity with molecular thermal velocity. In low speed flow, especially in an inlet velocity lower than the molecular thermal velocity, DSMC will cause computational instability. Therefore DSMC is often used in rare hypersonic flow. However the lattice model of LBM is only applicable for subsonic condition (Chun and Ladd, 2007)' thus LBM is the main numerical method in micro flow simulation.

Lattice Boltzmann Method (LBM) is a mesoscopic approach that assumes the fluid flow to be composed of a collection of particles which represented by a distribution function (Jing and Hu, 2012). And these particles in a discrete lattice migrate and collide with each other in accordance with a simple movement rules. By the statistical particle can get the macro movement of fluid characteristics (He *et al*., 2012). In last decade, because of its attractive simplicity of programming and capability of simulating complex fluid systems, LBM has rapidly emerged as a powerful technique with potential for fluid system simulation (Chen and Doolen, 1998).

Although LBM has been extensively applied in a large number of simulations of complex fluid system, it has difficulty in treating of boundary condition. The popular boundary condition of LBM is velocity inlet boundary and it is not convenient for fluid system simulation including micro-channel flow. As the scale of the domain is reduced in micro-channel flow, which has been developed rapidly in recent years, the continuum assumption is no longer applicable (Tang *et al*., 2005). Though some analytical approaches and traditional numerical methods have been conducted for micro channel flow investigation, most of the studies are limited to treating boundary (Tang *et al*., 2004). Therefore, to change the popular method of setting velocity boundary, pressure boundary was developed in LBM in current study. Rectangular micro-channel flow characteristic was investigated. The purpose of the current research is to define the principles of micro-flow resistance and provide the theoretical basis for the design of MEMS flow or a new type of heat channel in aerospace.

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LBM MODEL AND OPERATION LAW

LBM is now as important as CFD in flow simulation. By solving Navier-Stokes equations, CFD helps achieve flow simulation. Entirely different from CFD, LBM allow flow simulation by solving Boltzmann-BGK equations. Kinetic equations of particle distribution function are solved in LBM. Flow macroscopic parameters can be obtained by integrating of distribution function.

Based on LBM theory, physical flow space is discrete to some ordered sets of lattices and velocity space is discrete to some ordered velocity vectors. Boltzmann equations and BGK approximation can be expressed as follows (Renwei *et al*., 2002):

$$f_\alpha\left(\vec{r}+\vec{e_\alpha}\delta_t, t+\delta_t\right)-f_\alpha\left(\vec{r},t\right)=-\frac{1}{\tau}\left[f_\alpha\left(\vec{r},t\right)-f_\alpha^{eq}\left(\vec{r},t\right)\right]+\delta_t F_\alpha\left(\vec{r},t\right) \quad (1)$$

δ_t : Time step
$\vec{e}_\alpha \delta_t$: Space step
f_α : Velocity distribution function in the direction of α
f_α^{eq} : Equilibrium state velocity distribution function in the direction of α
τ : Relaxation Time

D3Q15 model: D3Q15 is a typical discrete model of LBM. The progress of space discrete and velocity discrete is shown in the Fig. 1 (Chen *et al*., 2008).

The matrix of velocity discrete in D3Q15 model is shown in the following equations:

$$E=\begin{bmatrix} 0 & +1 & -1 & 0 & 0 & 0 & 0 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ 0 & 0 & 0 & 0 & 0 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \end{bmatrix} \quad (2)$$

Equilibrium state velocity distribution function is given as follows:

$$\begin{aligned} f_\alpha^{eq} &= \rho\omega_\alpha\left[1-\frac{u^2}{2c_s^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \alpha=0 \\ f_\alpha^{eq} &= \rho\omega_\alpha\left[1+\frac{\vec{e_\alpha}\cdot\vec{u}}{c_s^2}+\frac{\left(\vec{e_\alpha}\cdot\vec{u}\right)^2}{2c_s^4}-\frac{u^2}{2c_s^2}\right] \ldots\ldots \alpha=1\ldots14 \end{aligned} \quad (3)$$

$$\omega_0=\frac{2}{9}, \omega_1=\frac{1}{9}, \omega_3=\frac{1}{72}, c_s=\frac{1}{\sqrt{3}}$$

Macroscopic density and momentum are determined based on the following equations:

$$\rho=\sum_{\alpha=0}^{14} f_\alpha \quad (4)$$

$$\rho\vec{u}=\sum_{\alpha=0}^{14} f_\alpha \vec{e_\alpha} \quad (5)$$

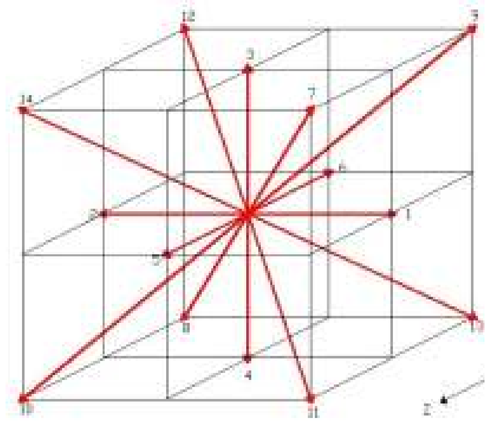

Fig. 1: Schematic diagram of space discrete and velocity discrete in the D3Q15 model

Using Chapman-Enskog Expansion, Eq. (1) can be resumed to second order Navier-Stokes equations. Kinematical viscosity can be determined by:

$$\nu=c_s^2\left(\tau-\frac{1}{2}\right)\delta_t \quad (6)$$

In the LBM standard, Eq. (1) can be divided into two parts: the local collision process and the migration process.

Local collision process:

$$\tilde{f}_\alpha\left(\vec{r},t\right)=f_\alpha\left(\vec{r},t\right)-\frac{1}{\tau}\left[f_\alpha\left(\vec{r},t\right)-f_\alpha^{eq}\left(\vec{r},t\right)\right] \quad (7)$$

Migration process:

$$f_\alpha\left(\vec{r}+\vec{e_\alpha}\delta_t, t+\delta_t\right)=\tilde{f}_\alpha\left(\vec{r},t\right) \quad (8)$$

Pressure boundary for LBM: The object of the current study is to determine the rectangular micro-channel flow. The selected calculation domain is a rectangular-channel. The original parameters are in the following equation:

$\mathrm{w} = 3 \times 10^{-6}\mathrm{m}$ (Width of channel)

$\mathrm{H} = 3 \times 10^{-6}\mathrm{m}$ (Heigh of channel)

$\mathrm{L} = 30 \times 10^{-6}\mathrm{m}$ (Length of channel)

On studying the influence of width-to-height ratio (W/H) to micro flow, W/H was changed from 1 to 2:

W/H = 1~2

These configuration parameters were chosen because flow of L/H>100 conditions had been studied by other researchers and W/H = 1-2 is a common configuration design in MEMS.

The no-slip bounce-back rule was used in the solid boundary setting. Likewise, any particle will spring back to its original direction when it collides with the solid boundary (Verhaeghe *et al*., 2009). The flow inlet was set as a pressure boundary. The boundary can be described with the following equation:

$$P = \rho c_s^2 \tag{9}$$

ρ : Relative density
C_s : Lattice sound speed

The set of inlet pressure P_{inlet} converts into inlet density ρ_{inlet} setting because the lattice sound speed is constant. Moreover, the set of outlet pressure P_{outlet} converts into the outlet density ρ_{outlet} setting. Besides these conditions, we also learned that there is only vertical velocity in the inlet and outlet of the rectangular micro-channel flow (Zhang *et al*., 2005).

Based on these conditions, the pressure boundary equations are drawn as follows:

$$\rho_{inlet} = \sum_{\alpha=0}^{14} f_\alpha (x_{inlet}, y_{inlet}, z_{inlet}) \tag{10}$$

$$\rho_{inlet}\vec{u} = \sum_{\alpha=0}^{14} f_\alpha (x_{inlet}, y_{inlet}, z_{inlet})\overrightarrow{e_\alpha} \tag{11}$$

After the migration process is finished, f_i (i = 0, 2, 3, 4, 5, 6, 8, 10, 12, 14) can be obtained in the D3Q15 model. These values are concluded from the neighbor lattice point. Here, f_i (i = 1, 7, 9, 11, 13) and velocity should be defined as x direction u_x. Based on the above equations, the following can obtained:

$$f_1+f_7+f_9+f_{11}+f_{13}=\rho-\left(f_0+f_2+f_3+f_4+f_5+f_6+f_8+f_{10}+f_{12}+f_{14}\right) \tag{12}$$

$$f_1+f_7+f_9+f_{11}+f_{13}=\rho u_x+\left(f_2+f_8+f_{10}+f_{12}+f_{14}\right) \tag{13}$$

According to additional functions, the following result can be derived:

$$f_1 = f_2 + \frac{2}{3}\rho_{inlet}u_x \tag{14}$$

$$u_x = 1 - \frac{1}{\rho_{inlet}}\left[f_0+f_3+f_4+f_5+f_6+2\left(f_2+f_8+f_{10}+f_{12}+f_{14}\right)\right]$$

$$f_i = f_{i+1} + \frac{1}{12}\rho_{inlet}u_x - \frac{1}{4}\left[e_{iy}\left(f_3-f_4\right)+e_{iz}\left(f_5-f_6\right)\right] \quad i=7,9,11,13 \tag{15}$$

LBM program process: Figure 2 shows the LBM program process, used in study. Figure 3 shows the data transfer process.

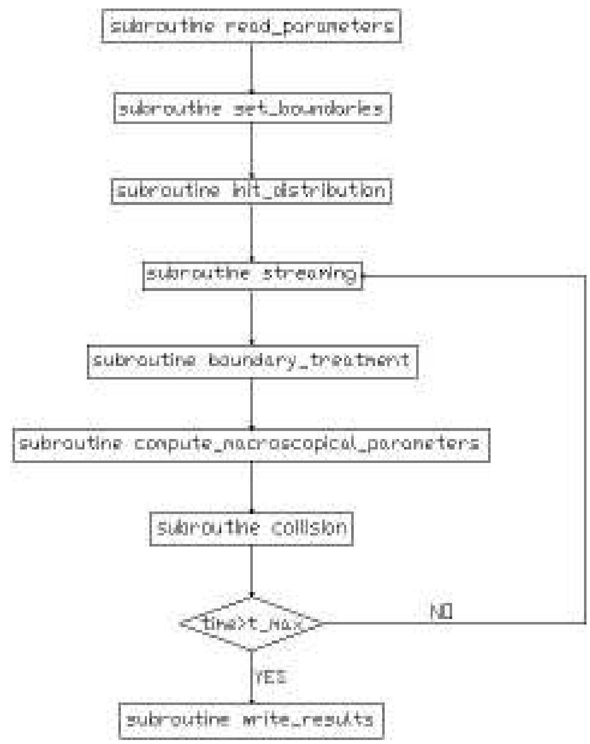

Fig. 2: LBM program process

SIMULATION RESEARCH OF RECTANGULAR MICRO-CHANNEL FLOW

Based on the LBM program developed in this study, the simulation research of rectangular micro-channel flow was conducted. The research focused on drag coefficient of air flow. The convergence condition of simulation is given as follows:

$$\sum_{\vec{r}}\sum_{i=0}^{14} f(i,\vec{r},t) - \sum_{\vec{r}}\sum_{i=0}^{14} f(i,\vec{r},t-1) \le 10^{-11} \tag{16}$$

For every calculated condition with different W/H, the convergence condition was filled.
The drag coefficient is defined as:

$$\lambda = \Delta P / \frac{L}{D} / \frac{\rho V^2}{2} \tag{17}$$

L : Length of the channel
D : Equivalent diameter of the rectangular-channel
ΔP : Differential pressure between the inlet and the outlet

The curves of W/H that influence the drag coefficient in Re = 1.5, 4, 6, 8, 10 conditions are given in Fig. 4 to 8. A bigger W/H of the rectangular micro-channel corresponds a greater drag coefficient. The reason is that wall area increase with W/H and it cause

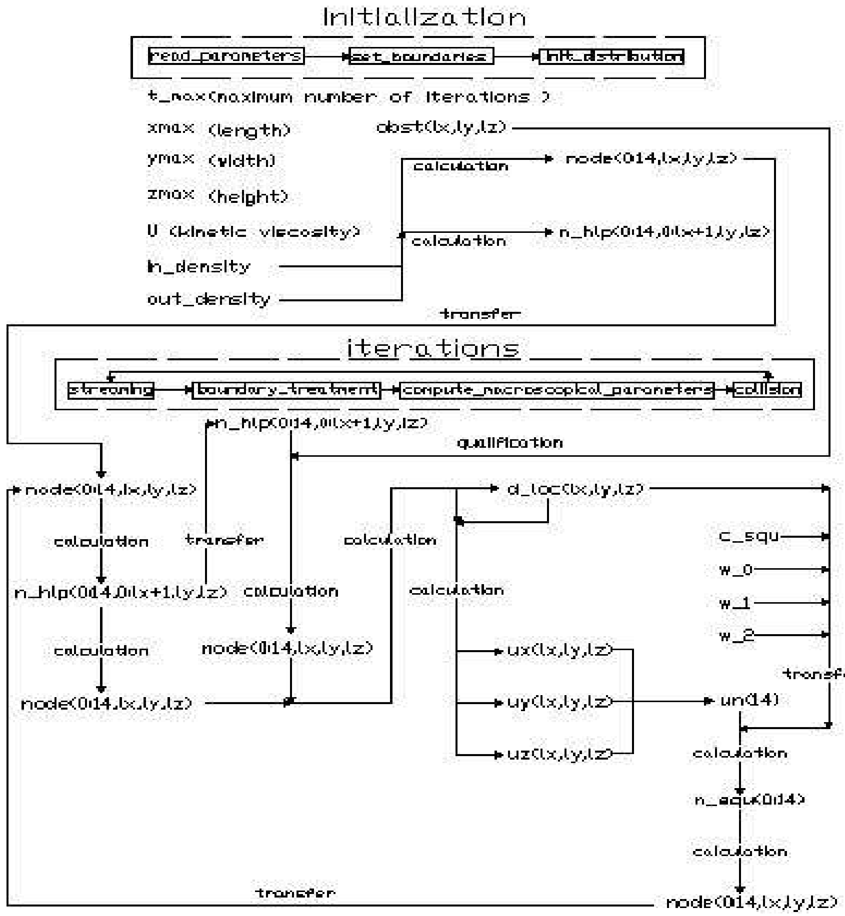

Fig. 3: Data transfer process

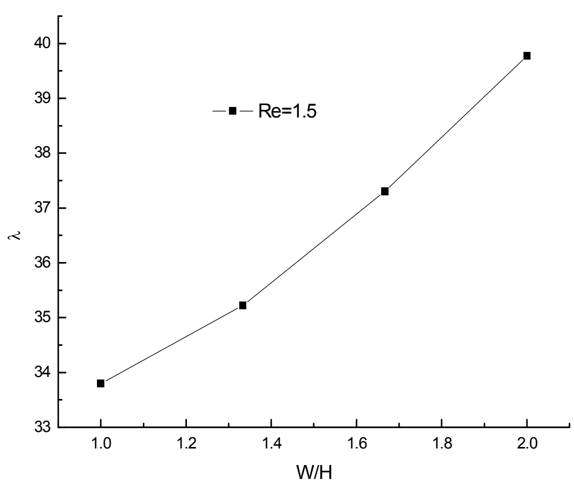

Fig. 4: Re = 1.5 drag coefficient curve

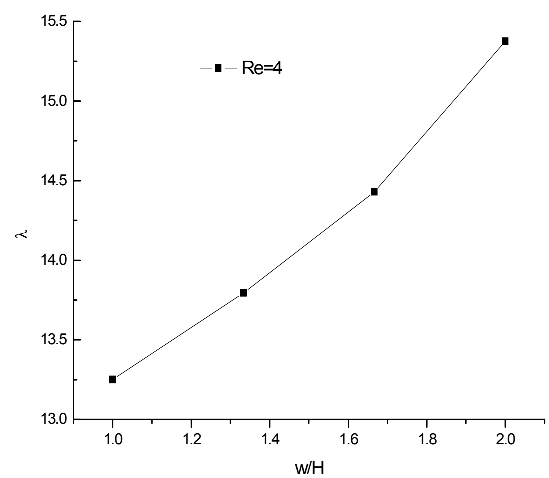

Fig. 5: Re = 4 drag coefficient curve

increasing of drag coefficient. Moreover, as the W/H decreases, the drag coefficient also decreases. Therefore a smaller W/H of the rectangular micro-channel corresponds a greater difference in drag coefficient between LBM simulation data and empirical formula data.

For example, in the condition of Re = 1.5, drag coefficient can be obtained through empirical formula data:

$$\lambda = \frac{64}{\mathrm{Re}} = 42.6(\mathrm{Re} = 1.5) \tag{18}$$

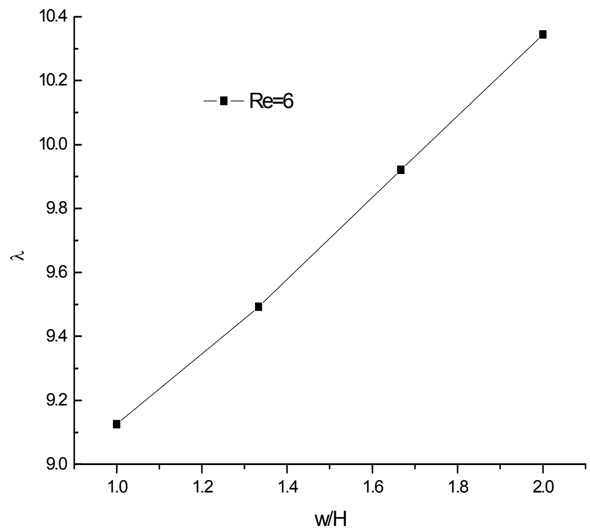

Fig. 6: Re = 6 drag coefficient curve

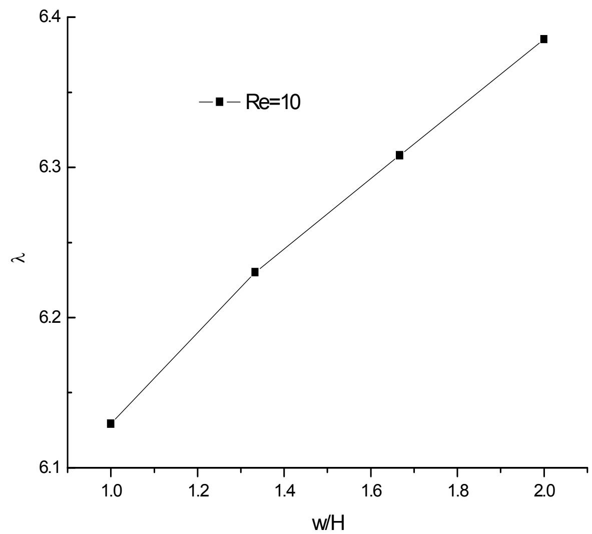

Fig. 8: Re = 10 drag coefficient curve

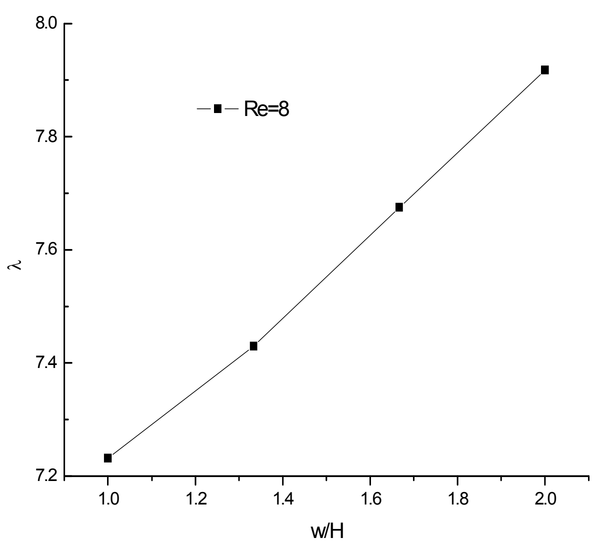

Fig. 7: Re = 8 drag coefficient curve

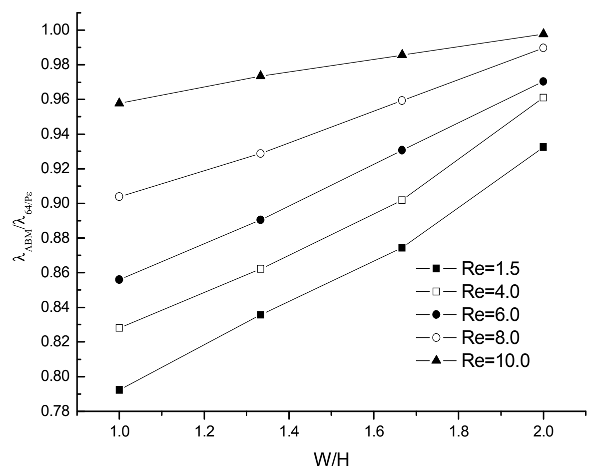

Fig. 9: Curve of ratio between LBM data and empirical formula data

From Fig. 4, we can see that the drag coefficient of the LBM data is $\lambda = 39.77$. When W/H = 2.0, two kinds of data are in substantial agreement. This law is also shown in other Re conditions from Fig. 5 to 8.

The comparison between LBM data and empirical formula data is shown in Fig. 9. In this figure, the ratio between LBM data and empirical formula data is given. Thus the data shown in Fig. 9 can be computed as follows:

$$\lambda_{\mathrm{LBM}}/\lambda = \lambda_{\mathrm{LBM}}/\frac{64}{\mathrm{Re}} \tag{19}$$

Therefore, the ratio approaches 1.0 with Re number increasing and the micro-channel flow agrees with the continuous medium hypothesis.

CONCLUSION

Pressure boundary conditions were developed in the proposed LBM D3Q15 model and the rectangular micro-channel flow was investigated. The following conditions can be concluded:

- W/H is the main influencing parameter of rectangular micro-channel flow in low Reynolds number condition.
- A smaller W/H of the rectangular micro-channel corresponds to a greater the difference in drag coefficient between LBM simulation data and empirical formula data. On the empirical data of drag coefficient approximating the laminar flow, when the Reynolds number is more than 10, the drag coefficients of LBM simulation and empirical data of laminar flow are in substantial agreement.

- In more than 10 conditions of Reynolds number, the empirical data of laminar flow can be used in rectangular micro-channel flow.

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REFERENCES

Chen, S.Y. and G.D. Doolen, 1998. Lattice Boltzmann method for fluid flows. Ann. Rev. Fluid Mech., 30: 329-364.

Chen, S., J. Tolke, S. Geller and M. Krafczyk, 2008. Lattice Boltzmann model for incompressible ax symmetric flows. Phys. Rev. Eng., 78: 046703.

Chun, B. and A.J.C. Ladd, 2007. Interpolated boundary condition for lattice Boltzmann simulations of flows in narrow gaps. Phys. Rev. Eng., 75(6 pt 2): 066705.

He, B., Y.C. Chen, W. Feng, Q. Li, A. Song, Y. Wang, M. Zhang and W. Zhang, 2012. Compressible lattice Boltzmann method and applications. Int. J. Numer. Anal. Mod., 9: 410-418.

Jing, F. and C.H. Hu, 2012. Application of lattice Boltzmann method for simulation of turbulence diffusion from a CO_2 lake in deep Ocean. J. Novel Carbon Res. Sci., 5: 10-18.

Renwei, M., S. Wei, Y. Dazhi and L. Li-Shi, 2002. Lattice Boltzmann method for 3-D flows with curved boundary. NASA/CR-2002-211657, ICASE Report No. 2002-17.

Tang, G.H., W.Q. Tao and Y.L. He, 2004. Lattice Boltzmann method for simulating gas flow in micro channels. Int. J. Modern Phys., 15: 335-341.

Tang, G.H., W.Q. Tao and Y.L. He, 2005. Lattice Boltzmann method for gaseous micro flow using kinetic theory boundary conditions. Phys. Fluids, 17(5): 058101-158104.

Verhaeghe, F., L.S. Luo and B. Blanpain, 2009. Lattice Boltzmann modeling of microchannel flow in slip flow regime. J. Comput. Phy., 228: 147-157.

Zhang, Y., R. Qin and D.R. Emerson 2005. Lattice Boltzmann simulation of rarefied gas flows in micro channels. Phys. Rev. Eng., 71(4 pt 2): 047702.