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Research Article

Theoretical Analysis and Experimental Study of Springback Mechanism of Archwire Bending

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Abstract: Archwire is the material with high elastic and high strength property. Archwire bending forming using robot belongs to the typical big curvature bending process. In order to ensure its one-step forming, it is really important to study its spring back mechanism. This study proposes a theoretical model to predict springback in archwire bending. This model which considering the neutral line does not coincide with central line is based on the true stress and strain data obtained from tensile test. The effection of bending moment arm on springback is also considered. The springback theoretical calculation formulas of archwires whose cross-section are rectangular and circular are derived, respectively. Then an archwire bending robot is used to bend archwire into different angle and a metallurgical microscopy is used to measure formed angle. The experimental results indicate that the proposed theoretical model could accurately predict the springback angle. Furthermore, the formed angle shows a general increase with the bend angle. And in elastic deformation clearly stage, although springback angle itself is small, the proportion of it in bend angle is very large. In plastic deformation clearly stage, although springback angle itself is large, the proportion of it in bend angle is relative small.

Keywords: Archwire, archwire bending robot, experimental analysis, neutral line shift, springback, theoretical calculation

INTRODUCTION

Individual customized archwire bending is one of the key components in orthodontic treatment. The traditional way of acquiring archwire curve form is based on manual operation, which has the problems of low efficiency, high labor intensity and operator experience dependency (Muller *et al*., 2007). Some people proposed using robot to bending straight archwire into target form (Rigelsford, 2004). Springback phenomenon evidently exists in archwire bending due to its high strength and high elasticity. Elastic recovery of archwire which happens during unloading can cause dimension deviation. It is absolutely necessary to study springback mechanism of archwire bending in order to ensure its dimensional accuracy.

Many efforts have been done to reduce or compensate the springback in a lot of aspects. Kruger and Palazotto (1972) attempted to develop a simple expression for springback in which material properties are considered through the use of the Ramberg-Osgood stress-strain equation. They proposed that the existed calculation formulas of sheet metal cannot be used for springback calculation in wire forming. However, they assumed that the neutral line did not deviate from its cross-section geometric central line that actually does not conform to reality. Truke and Kalpakjian (1975) studied the springback phenomenon in round wires' bending. They pointed that as the bending force increases springback decreases rapidly, but the rate tends to stabilize with increasing force; also, springback increases with bend angle. Baragetti (2006) carried out a theoretical study on nonlinear bending of wires product. He tried his best to provide the designer with a simple model to predict the final shape of a wire by using mathematical codes and this model allows to predicting with a higher level of accuracy the final shape of wires having different cross-sections after nonlinear bending. Some researchers carried out studies regarding springback of wire products with the help of finite element analysis software (Deng *et al*., 2010; Zhang *et al*., 2010). Luis *et al*. (2005) compare analytical and FEM numerical results; the effects of friction simulation during drawing processes is taken into consideration in both analytical and numerical terms. As far as wires are concerned, some researches were addressed to the investigation of structure, element composition and

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key mechanical properties of various sizes and tempers of Australia wires (Pelsue *et al*., 2009). Also correlation research of springback error in sheet metal forming is carried out (Agus *et al*., 2012).

We can see that there is less research on archwire's springback properties. This study proposes to study the springback angles of circular archwire and rectangular archwire in different bend angles considering the influence of neutral line shift and bending moment arm. Furthermore, an archwire bending robot proposed by the authors is used to obtain the springback angles in different bend angles in the process of archwire bending.

METHODOLOGY

Bend process analysis: Archwire bending belongs to rotary bending process whose schematic diagram is shown in Fig. 1. One end of the archwire is clamped between two fixtures to prevent its motion and rotation. The moving die rotates around the center of fixed die to forming the archwire into shape. From Fig. 1 we can see that the whole bending deformation is composed of two parts which is part AB and part BC, respectively. According to plastic forming theory we know that bend angle and springback angle increase with bend moment in the initial stage of bending, which we call it elastic deformation clearly stage. With the increases of bend forming, bend angle increases continually while bend moment keeps unchanged, which we call it plastic deformation clearly stage. The following major simplifications and assumptions are used throughout the derivations in this study.

- Any plane section remains plane after deforming and there are no aberrance occurs in the process of archwire bending.
- The stress strain diagram for compression and tension are the same.
- Membrane stresses are negligible.

Elastic deformation clearly stage: In this stage, the process of archwire bending can be idealized into a simply supported beam model which is shown in Fig. 2. θ_{ib}, θ_{is} and M_i are the bend angle, springback angle and bend moment, respectively. So, we can get the deflection equation of part BC and part AB as follows:

$$v_{BC} = -\frac{F(x-l)}{6EI}[n(3x-l)-(x-l)^2] \tag{1}$$

$$v_{AB} = \frac{Fnx}{6EIl}[l^2 - x^2] \tag{2}$$

where, F is the equivalent force of bend moment M_i. Furthermore we can get the bend angle and springback angle calculation equation as follows:

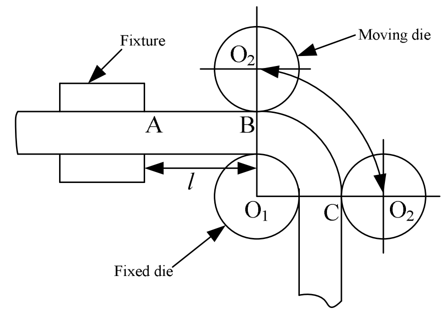


Fig. 1: Job analysis of archwire bending

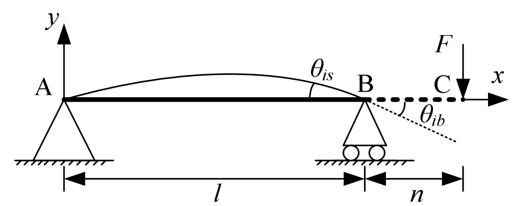


Fig. 2: Simply supported beam model

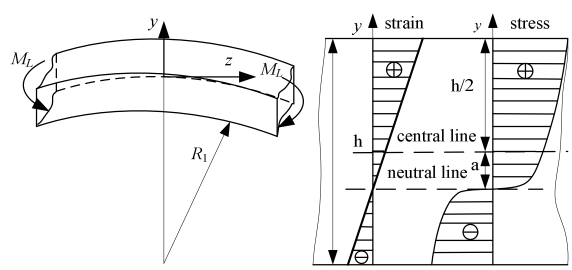


Fig. 3: Relationship of stress and strain with neutral line shift

$$\theta_{ib} = \frac{dv_n}{dx}\Big|_{x=l+n} = -\frac{Fn}{6EI}[2l+3n] = -\frac{M_i}{6EI}[2l+3n] \tag{3}$$

$$\theta_{is} = \frac{dv_l}{dx}\Big|_{x=l} = -\frac{Fnl}{3EI} = -\frac{M_i l}{3EI} \tag{4}$$

When bend moment increases to a constant number, called M_L, elastic deformation clearly stage begin to change into plastic deformation clearly stage. So, we can get the formed angle calculation equation of elastic deformation clearly stage as follows:

$$\theta_i' = |\theta_{ib}| - |\theta_{is}| = \frac{M_i}{6EI}[2l+3n] - \frac{M_i l}{3EI},\ 0 \le M_i \le M_L \tag{5}$$

Plastic deformation clearly stage: In this stage, most deformation focus on part BC and it can be considered as pure bending process, which is shown in Fig. 3. Neutral line will deviate from central line in archwire bending for its large relative bending radius. Furthermore, since strain values considered are fairly large, a true stress-strain curve is absolutely necessary. In Fig. 3 we assume that the distance between central line and neutral line is *a*. Before unloading, we assume that a line segment of initial length l_1 (equal to the length of the neutral line), situated at a distance *y* above the neutral line, after bending to an angle θ with a radius of curvature ρ, will deform to a length l_2. So, the engineering strain of the line segment can be expressed as a function of the distance *y* and the radius of curvature ρ:

$$\overline{\varepsilon} = \frac{l_2 - l_1}{l_1} = \frac{\theta(\rho + y) - \theta\rho}{\theta\rho} = \frac{y}{\rho} \tag{6}$$

In a similar way, the engineering strain $\overline{\varepsilon}'$ of the line segment after unloading can be expressed as follows:

$$\overline{\varepsilon}' = \frac{l_2' - l_1'}{l_1'} = \frac{\theta'(\rho' + y) - \theta'\rho'}{\theta'\rho'} = \frac{y}{\rho'} \tag{7}$$

where, variable l_2', l_1' are the length of the neutral line and the line segment after unloading, respectively. Variable θ' and ρ' are the formed angle and the radius of curvature after unloading. The engineering strain difference Δε before and after unloading becomes:

$$\Delta\varepsilon = \overline{\varepsilon} - \overline{\varepsilon}' = y \cdot (\frac{1}{\rho} - \frac{1}{\rho'}) \tag{8}$$

The stress-strain relationship of the engineering strain difference follow Hooke's law as the engineering strain difference is caused by elastic unloading. So, the unloading stress σ can be described as follows:

$$\sigma = E \cdot \Delta\varepsilon = E \cdot y \cdot (\frac{1}{\rho} - \frac{1}{\rho'}) \tag{9}$$

The unloading moment can be calculated from:

$$M_U = \int_A \sigma \cdot y dA = \int_A E \cdot y^2 \cdot (\frac{1}{\rho} - \frac{1}{\rho'}) dA \tag{10}$$

In a similar way, the loading moment can be calculated from:

$$M_L = \int_A \sigma(\varepsilon) \cdot y dA \tag{11}$$

After unloading, archwire is free. So we can get following equation:

$$M_U = M_L \tag{12}$$

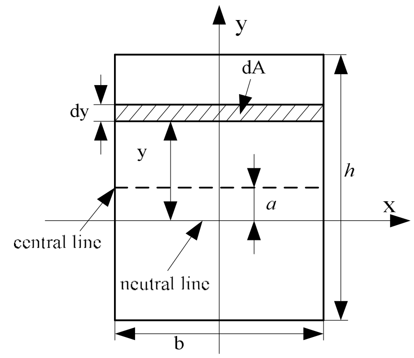


Fig. 4: Cross section of rectangular archwire

Because the arc length of neutral line is changeless in both loading process and unloading process, we can get following equation:

$$\rho\theta = \rho'\theta' \tag{13}$$

Also, according to plane bent rod theory we can obtain the calculation equation of neutral radius of curvature as follows:

$$\rho = \frac{\int_A dA}{\int_A \frac{1}{R} dA} = \frac{A}{\int_A \frac{1}{R} dA} \tag{14}$$

Combining Eq. (10) to Eq. (14) we will obtain the springback angle calculation formula. It is obviously that springback can be solved once the neutral line radius of curvature ρ, unloading moment M_U and loading moment M_L are found. Furthermore, we can see that they are all related with inertia moment. That is to say, they are all related with cross section shape of archwire.

Springback theory: In this study, two kinds of archwire commonly used in orthodontic treatment, which are rectangular archwire and circular archwire, are studied. For the rectangular archwire, the cross-section schematic diagram is shown in Fig. 4. The expression of area differential can be described as follows:

$$dA = bdy \tag{15}$$

Combining Eq. (14) and Eq. (15) gives the neutral line radius of curvature of rectangular archwire:

$$\rho = \frac{h}{\ln \frac{R_1 + h}{R_1}} \tag{16}$$

where, R_1 and h are the radius of the fixed die and the thickness of the archwire, respectively. Assumed that the constitutive equation of rectangular archwire is as follows:

$$\sigma = m_i\varepsilon^i + m_{i-1}\varepsilon^{i-1} + \cdots + m_1\varepsilon^1 + m_0,\ i \in N \tag{17}$$

In this study, true strain expressions are used to calculate unloading moment and loading moment. The expression for true strain can be stated as:

$$\varepsilon = \ln(1+\bar{\varepsilon}) \tag{18}$$

where, $\bar{\varepsilon}$ is the engineering strain which can be given as follows:

$$\bar{\varepsilon} = y/\rho \tag{19}$$

Combining Eq. (10) and (11), Eq. (14), (16) to Eq. (19) gives the unloading moment and loading moment of the rectangular archwire, respectively:

$$M_U = \frac{Ebh^3 + 12Eba^2h}{12}\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \tag{20}$$

$$M_L = \rho^2 b\int_{\ln(1+\frac{-h/2+a}{\rho})}^{\ln(1+\frac{h/2+a}{\rho})} \sigma(\varepsilon)\cdot(e^{2\varepsilon} - e^{\varepsilon})d\varepsilon \tag{21}$$

Combining Eq. (12), Eq. (13), Eq. (20) and Eq. (21) gives the springback angle of plastic deformation clearly stage of rectangular archwire:

$$\theta_{ps} = \frac{12M_L\rho}{Ebh^3 + 12Eba^2h}\theta \tag{22}$$

So, the final formed angle of rectangular archwire combining elastic deformation clearly stage and plastic deformation clearly stage is as follows:

$$\theta' = \begin{cases} \frac{M_i}{6EI}[2l+3n] - \frac{M_i l}{3EI},\ 0 \le \theta \le \theta_{ib}, 0 \le M_i \le M_L \\ \theta - \frac{12M_L\rho}{Ebh^3 + 12Eba^2h}\theta - \left(\frac{M_L}{6EI}[2l+3n] - \frac{M_L l}{3EI}\right) + K, \theta \ge \theta_{ib} \end{cases} \tag{23}$$

where, K is the correction factor. In a similar way, for the circular archwire, the cross-section schematic diagram is shown in Fig. 5. The expression of area differential can be described as follows:

$$dA = 2\sqrt{\frac{h^2}{4} - (y-a)^2}\,dy \tag{24}$$

So, the calculation equation of neutral radius of curvature, unloading moment and loading moment can be expressed as follows, respectively:

$$\rho = \frac{h^2}{4[2(R_1 + \frac{h}{2}) - \sqrt{4(R_1 + \frac{h}{2})^2 - h^2}\,]} \tag{25}$$

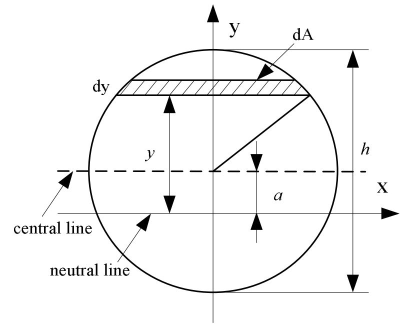


Fig. 5: Cross section of circular archwire

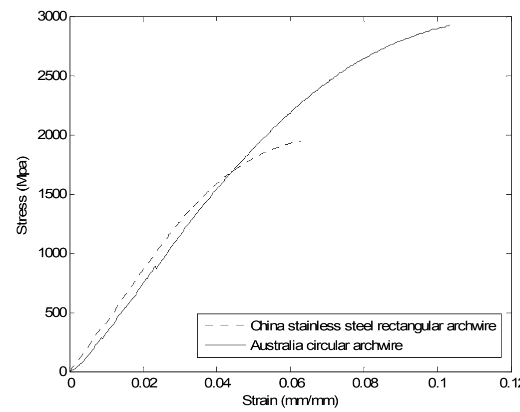


Fig. 6: Stress-strain curve of archwire

$$M_U = \frac{Eh^2(\pi h^2 + 16\pi a^2)}{64}\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \tag{26}$$

$$M_L = 2\rho^2\int_{\ln(1+\frac{-h/2+a}{\rho})}^{\ln(1+\frac{h/2+a}{\rho})} \sigma(\varepsilon)\cdot(e^{2\varepsilon} - e^{\varepsilon})\sqrt{\frac{h^2}{4} - \rho^2(e^{\varepsilon}-1)^2}\,d\varepsilon \tag{27}$$

Combining Eq. (12), (13), (26) and (27) gives the springback angle of plastic deformation clearly stage of circular archwire:

$$\theta_{ps} = \frac{64M_L\rho}{Eh^2(\pi h^2 + 16\pi a^2)}\theta \tag{28}$$

So, the final formed angle of circular archwire combining elastic deformation clearly stage and plastic deformation clearly stage is as follows:

$$\theta' = \begin{cases} \frac{M_i}{6EI}[2l+3n] - \frac{M_i l}{3EI},\ 0 \le \theta \le \theta_{ib}, 0 \le M_i \le M_L \\ \theta - \frac{64M_L\rho}{Eh^2(\pi h^2 + 16\pi a^2)}\theta + \frac{M_L}{6EI}[2l+3n] - \frac{M_L l}{3EI} + K, \theta \ge \theta_{ib} \end{cases} \tag{29}$$

Springback calculation: Two different archwires, China stainless steel rectangular archwire and Australia circular archwire, were examined (Table 1) and their experimental stress-strain curves can be observed in Fig. 6. Their constitutive equation is as follows, respectively:

Table 1: Material properties data of archwire

	Stainless steel rectangular archwire	Australia circular archwire
Elastic modulus E (Gpa)	208.88	191.07
Yield stress $\sigma_{0.2}$ (Mpa)	1719.58	1934.06
Tensile stress σ_b (Mpa)	1822.95	2408.66
Cross-section (mm)	0.3×0.5	0.4

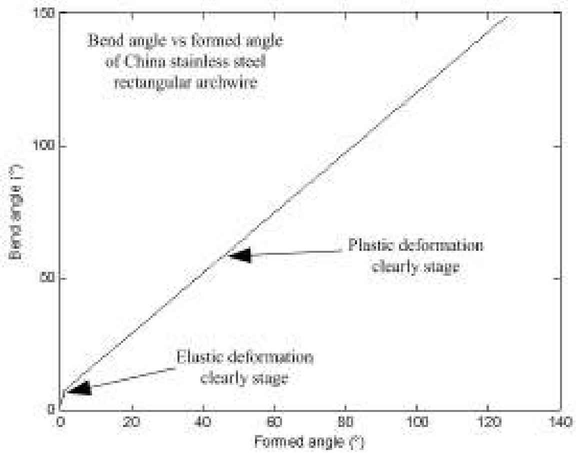


Fig. 7: Theoretical curve of bend angle vs formed angle of China stainless steel rectangular archwire

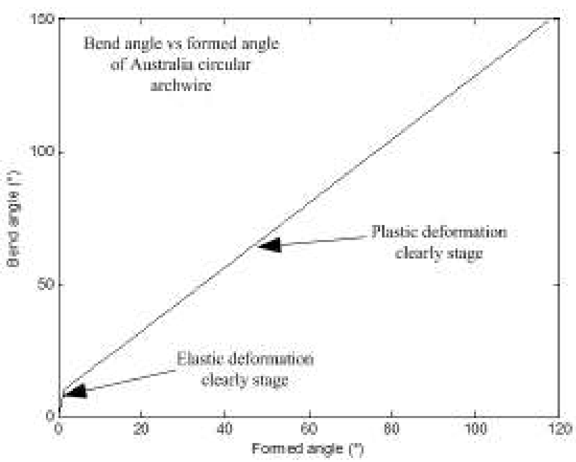


Fig. 8: Theoretical curve of bend angle vs formed angle of Australia circular archwire

$$\sigma = -2.8377e^6\varepsilon^3 - 0.1092e^6\varepsilon^2 + 0.0454e^6\varepsilon^1 \quad (30)$$

$$\sigma = 1.8348e^7\varepsilon^4 - 0.4436e^7\varepsilon^3 + 0.0134e^7\varepsilon^2 + 0.0036e^7\varepsilon^1 \quad (31)$$

For China stainless steel rectangular archwire, the final formed angle can be obtained according to Eq. (23). The relationship of bend angle and formed angle of China stainless steel rectangular archwire is shown in Fig. 7:

$$\theta' = \begin{cases} 0.0174M_i,\ 0 \le \theta \le 7.3079, 0 \le M_i \le 66.3618 \\ 0.8776\theta - 5.2544, \theta \ge 7.3079 \end{cases} \quad (32)$$

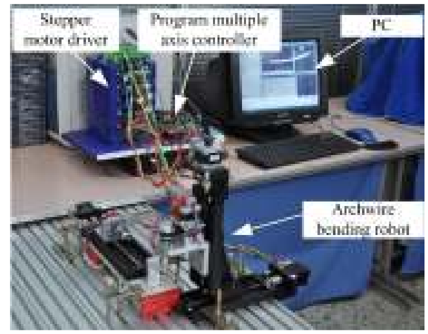


Fig. 9: Archwire bending robot experimental system

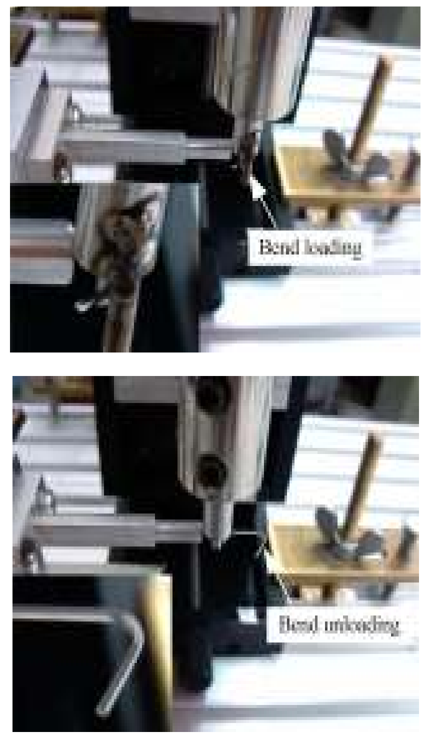


Fig. 10: Process of archwire bending by robot

For Australia circular archwire, the final formed angle can be obtained according to Eq. (29). The relationship of bend angle and formed angle of Australia circular archwire is shown in Fig. 8:

$$\theta' = \begin{cases} 0.0456M_i,\ 0 \le \theta \le 10.3126, 0 \le M_i \le 35.6948 \\ 0.8319\theta - 6.9512, \theta \ge 10.3126 \end{cases} \quad (33)$$

EXPERIMENT

It has been previously shown that springback can be theoretically expressed. However, the expression found to determine experimental springback is not a function of measured moment quantities but of geometrical angular measurements. Figure 9 illustrates the archwire bending robot used for archwire bending.

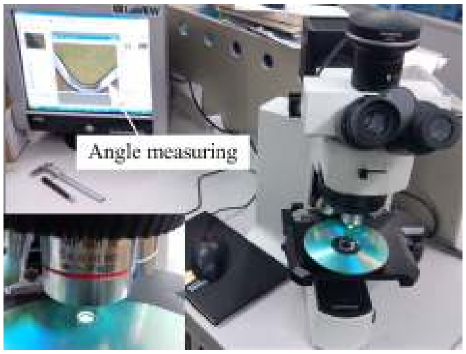


Fig. 11: Formed angle measuring

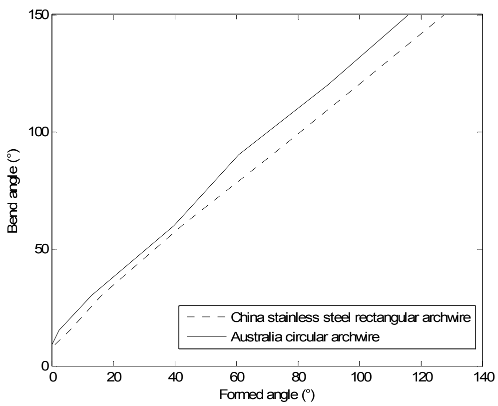


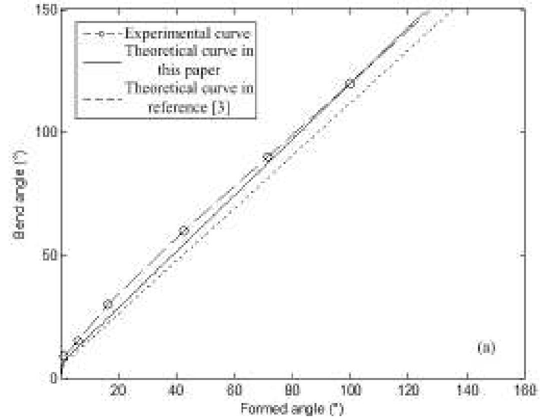
Fig. 12: Experimental curve of bend angle vs formed angle of two archwires

Archwire is installed in the robot and then be bent into specified angle. The bending process is shown in Fig. 10. After unloading, springback phenomenon appearance and the final formed angle are obtained. The formed angles are measured with Olympus metallographic microscope which is shown in Fig. 11.

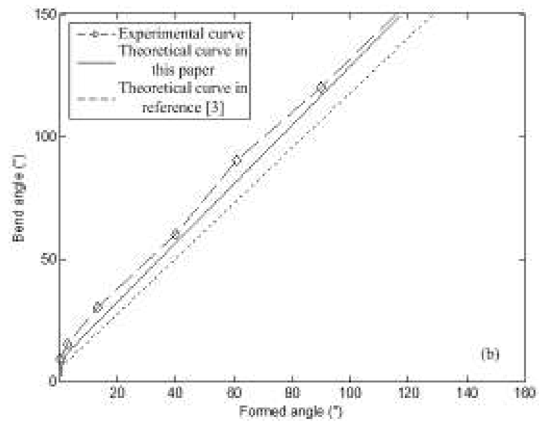
From the experiment we can get a series of data including bend angle and formed angle. Fitting the data into curve we can get the bend angle vs formed angle curve of China stainless steel rectangular archwire and Australia circular archwire, which is shown in Fig. 12, respectively.

DISCUSSION

The comparison of theoretical results in this study, theoretical results in reference (Kruger and Palazotto, 1972) and experimental results was shown in Fig. 13. From Fig. 13 we can see that, for both China stainless steel rectangular archwire and Australia circular archwire, the theoretical results in this study can accord better with the experimental results than the theoretical results in reference (Kruger and Palazotto, 1972) with experimental results. It shows that the theoretical model proposed in this study, which considering the influence of



(a)



(b)

Fig. 13: Comparison of theoretical results in this paper, experimental results and theoretical results in reference (Kruger and Palazotto, 1972), (a) China stainless steel rectangular archwire, (b) Australia circular archwire

neutral line shift and bending moment arm, is an accurate model to predict springback in archwire bending

Also, from Fig. 13 we can see that, the formed angle for archwire bending shows a general increase with the bend angle. Furthermore, in elastic deformation clearly stage, although springback angle itself is small, the proportion of it in bend angle is very large because elastic deformation plays a main role in total deformation. In plastic deformation clearly stage, although springback angle itself is large, the proportion of it in bend angle is relative small because plastic deformation plays a main role in total deformation.

CONCLUSION

This study proposed a theoretical model which considering the influence of central line shift and bending moment arm. Springback calculation equations of China stainless steel rectangular archwire and Australia circular archwire were derived based on the proposed model. An archwire bending experiment was carried out on an archwire bending robot proposed by the authors. Then springback angle and formed angle were measured in a metallographic microscope. There will be the following decision conclusions:

- Formed angle of archwire bending shows a general increase with bend angle.
- In elastic deformation clearly stage, the proportion of springback angle in bend angle is very large. In other words, formed angle is small in this stage.
- In plastic deformation clearly stage, the proportion of springback angle in bend angle is relative small. That is to say, formed angle is large in this stage.

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