

## Research Article

### Fire Analysis of Reinforced Concrete Beams with 2-D Plane Stress Concrete Model

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**Abstract:** The main purpose of this study is to investigate the nonlinear response of reinforced concrete beams under standard fire conditions. With this purpose, the 2-D nonlinear structural analysis of a chosen reinforced concrete simple beam is carried out. This beam is exposed to fire from three sides and fixed distributed loads on top of it. In these structural analyses the changes of material properties of concrete and reinforcements according to increasing temperatures are taken into account. Results drawn from these analyses are compared with the results from some simplified methods and put forward some conclusions and recommendations concerning the fire design of reinforced concrete beams.

**Keywords:** 2-D, non-linear fire analysis, reinforced concrete beam

## INTRODUCTION

It is known that all structures within economic lifetime must have a specific safety in response to collapse by becoming out of service under loading. It is also known that the structures have to maintain these characteristics during probable fires. In the circumstances, it is necessary to take into account the fire effects in design, construction and using stages of reinforced concrete structures like other structures.

The analyses show that two-way slabs have excellent fire resistance if they deform in double curvature and develop tensile membrane action (Linus *et al.*, 2004).

For the mechanical properties of reinforced concrete under high temperature with large deterioration, the reliability of reinforced concrete beams have been largely discounted. Reinforced concrete beam is usually working with cracks. Since each section with cracks has possibility of destruction, the reliability of the beam is calculated by the minimum value of *n* crack-sections' resistance. The results show that increase the reinforcement ratio and concrete cover thickness appropriately are effective measures to improve the fire resistance limit of reinforced concrete beams (Zhen *et al.*, 2010).

The results of the thermal analysis are compared to the experimental results in the literature and the analytically derived structural results are also compared

with full-scale reinforced concrete beams in previous fire exposure experiments. The comparison results indicated that the calculation procedure in this study assessed the residual bearing capabilities of reinforced concrete beams exposed to fire with sufficient accuracy. As no two fires are the same, this novel scheme for predicting residual bearing capabilities of fire-exposed reinforced concrete beams is very promising in that it eliminates the extensive testing otherwise required when determining fire ratings for structural assemblies (Hsu *et al.*, 2006).

In order to determine the fire behavior of reinforced concrete structures, there are many methods in technical literatures. These methods can be listed in order tabulated data methods which are developed based on tests and experiences, simplified methods and numerical methods which provide to be carried out thermal and structural analyses by computers. In this study it is used a developed computer code based on finite element method for both thermal and structural analyses.

## MATERIALS AND METHODS

**Thermal analysis:** Two dimensional heat flows is governed by the following partial differential equation:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + Q \quad (1)$$

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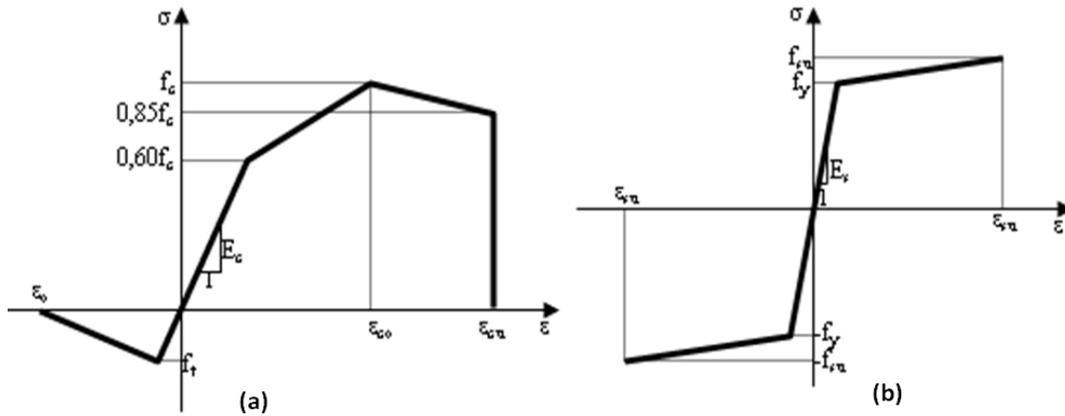


Fig. 1: Stress-strain relations of (a) concrete, (b) reinforcing steel

where,  $T(x, y, t)$ ,  $\rho$ ,  $c$ ,  $k$  and  $Q(x, y, t)$  are temperature distribution history, density, specific heat, isotropic conductivity and heat generation rate, respectively. An integral part of above equation is in its boundary and initial conditions. The initial conditions consist of the temperature of every point in the structure when the analysis begins:  $T(x, y, t_0) = T_0(x, y)$  where the temperature distribution  $T_0$  is specified. The boundary conditions must be defined at every point on the structures surface and can be a specified temperature history or a specified heat flow history.

The heat flow equations for two dimensional bodies are very complex and have nearly no closed-form solution. An approximate numerical method must be used in order to obtain a solution. In this study finite element method has been used. The thermal analysis of RC cross-sections with finite element method can be seen exhaustive in reference (Burnaz, 2003).

In this study ISO-834 standard fire curve which is accepted to represent the fire around the beam is used. The curve is calculated as:

$$T_f = 345 \log_{10}(8t + 1) + T_0 \quad (2)$$

where,  $T_f$ ,  $T_0$  and  $t$  are fire environment temperature, ambient temperature and time respectively. Heat exchange at the boundaries of the fire exposed member depends on the heat transfer coefficients of both emissivity and convection. These factors and thermal properties of concrete (thermal conductivity, specific heat and density) which are function of temperature were adopted in accordance with reference (Burnaz and Durmuş, 2007).

**Structural analysis:** The finite element idealization of R.C. beams is accomplished by two dimensional elements which lie in the x-y plane of the beam

elevation. According to the assumptions of the Timoshenko beam theory the resulting stresses are assumed to be constant through the thickness of each element and stresses  $\sigma_z$ ,  $\tau_{xz}$  and  $\tau_{yz}$  are ignored. These assumptions lead to a plane stress field. In the analysis of reinforced concrete structures plane stress problems make up a large majority of practical cases. The plane stress problem theory is used for the concrete model of this study. Bar elements whose nodes can be same nodes of the concrete plane stress elements are used for the reinforcing steel model. The bond between reinforcing steel and concrete is accepted to be perfect.

Many mathematical models of the mechanical behavior of concrete are currently in use in the analysis of R.C. structures. The orthotropic model is adopted in this study. Under combinations of biaxial compressive stress concrete exhibits strength and stress-strain behavior which is different from that under uniaxial loading conditions. To this end, Kupfer biaxial strength envelope is used here. In order to simplify the concrete material model the stress-strain relation in compression is assumed piecewise linear with three branches (Fig. 1a). A single stress-strain relation is sufficient to define reinforcing steel needed in the analysis of R.C. structures. Reinforcing steel is modeled as a linear elastic, linear strain hardening material with yield stress (Fig. 1b).

By applying the virtual work principle or the theorem of minimum potential energy to the assemblage of discrete elements the following equilibrium equations result:

$$[K]\{u\} + \{F\}_{\epsilon_o} = \{R\} \quad (3)$$

where,

$[K]$  : Stiffness matrix

$\{F\}_{\epsilon_o}$  : Nodal forces due to initial strains:

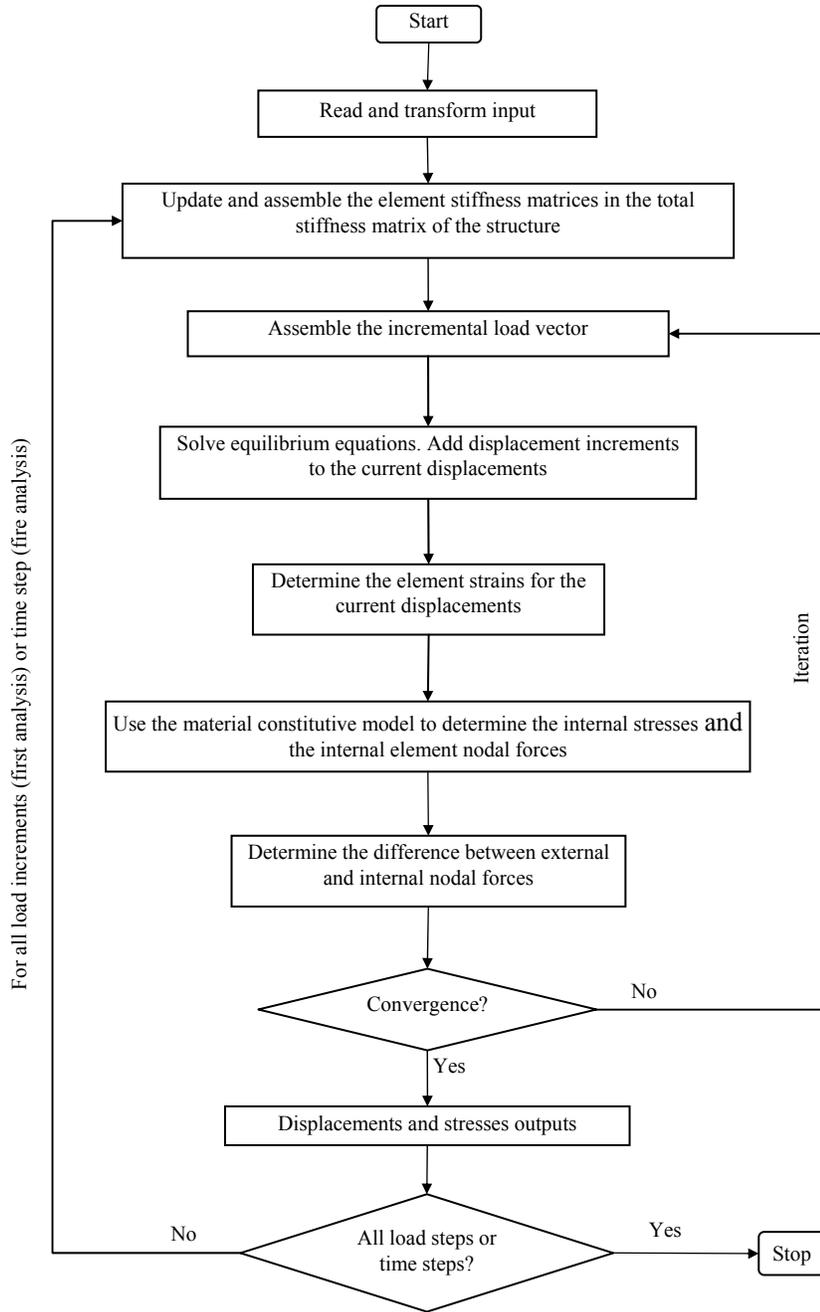


Fig. 2: Flow diagram of the nonlinear structural analysis algorithm

$$\begin{aligned}
 [K]_{conc} &= \sum_{el} \int [B]^T [D] [B] dV \\
 \{F\}_{\varepsilon_0-conc} &= \sum_{el} \int [B]^T [D] [\varepsilon_0] dV \\
 [K]_{steel} &= \frac{AE}{L} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \\
 \{F\}_{\varepsilon_0-steel} &= AE \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \varepsilon_0
 \end{aligned} \tag{4}$$

The numerical performing of the finite element model requires the solution of Eq. (3). This equation is

nonlinear equation, since the stiffness matrix  $[K]$  depends on the displacement vector  $\{u\}$ . The solution of nonlinear equations is accomplished with an iterative method. The load vector  $\{R\}$  is divided into a number of small load increments. The nonlinear solution scheme assumed in this study uses the tangent stiffness matrix.

Every nonlinear analysis algorithm consists of four basic steps: the formation of the current stiffness matrix, the solution of the equilibrium equations for the displacement increments, the state determination of all elements in the model and the convergence check.

Table 1: Stress-strain relation properties of concrete and reinforcing steel

Concrete				
$E_c$ (N/mm <sup>2</sup> )	$f_c$ (N/mm <sup>2</sup> )	$\epsilon_{co}$ (-)	$\epsilon_{cu}$ (-)	$f_t$ (N/mm <sup>2</sup> )
$3 \times 10^4$	25	0.002	0.003	3.1875
Reinforcing steel				
$E_s$ (N/mm <sup>2</sup> )	$f_y$ (N/mm <sup>2</sup> )	$f_{su}$ (N/mm <sup>2</sup> )	$\epsilon_{su}$ (-)	
$2 \times 10^5$	220	340	0.18	

These steps are presented in some detail in the flow diagram of Fig. 2 (Kwak and Filippou, 1990).

In the current study two structural analyses are carried out. The first is the analysis for fixed distributed loads. The second is fire structural analysis which uses the first analysis' results and temperature distributions change in time increment. First analysis is performed with load increment; the second is performed with time increment. Also, in the second analysis the nodal forces at the each time are formed due to the strains based on temperatures, current stress and time. The strains are expressed:

$$\begin{aligned} \epsilon_{0-concrete} &= \epsilon_{th}(T) + \epsilon_{cr}(\sigma, T, t) + \epsilon_{tr}(\sigma, T) \\ \epsilon_{0-steel} &= \epsilon_{th}(T) + \epsilon_{cr}(\sigma, T, t) \end{aligned} \quad (5)$$

where,  $\epsilon_{th}$ ,  $\epsilon_{tr}$  and  $\epsilon_{cr}$  are the free thermal strain, transient strain (concrete) and classical creep strain (concrete and steel), respectively. For the calculation of transient strain and classical creep strain of concrete Anderberg-Thelandersson model and for the classical creep strain of the reinforcing steel Harmathy model is utilized (Iding *et al.*, 1977; Li and Purkiss, 2005).

The changes of thermal expansions and some characteristics of stress-strain relation of concrete and steel with temperature are taken from EN 1992-1-2 (ENV 1992-1-2, 1996).

**Numerical example:** This study was conducted from during the months of November and December in 2008 and January, February and March in 2009 at the KTU University, Trabzon, Turkey. The simple supported R.C. beam over a 5 m span considered is shown in Fig. 3, with details of the stress-strain relation properties of concrete and reinforcing steel at given in Table 1. This beam was subjected to ISO-834 standard fire ( $T_f(t)$ ) from three sides and the thermal boundary conditions (environment temperature, resultant emissivity  $\epsilon_r$  and convection factor  $h_c$ ) for fire exposed and unexposed surfaces were taken as seen in Fig. 3.

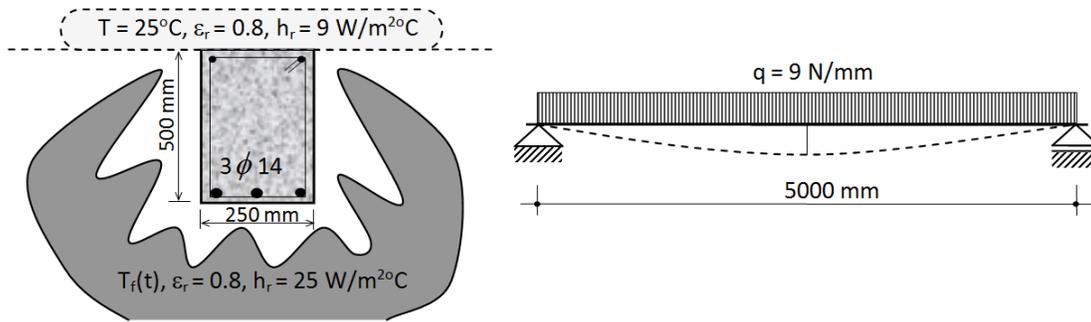


Fig. 3: Descriptions of the simple supported R.C. beam exposed fire

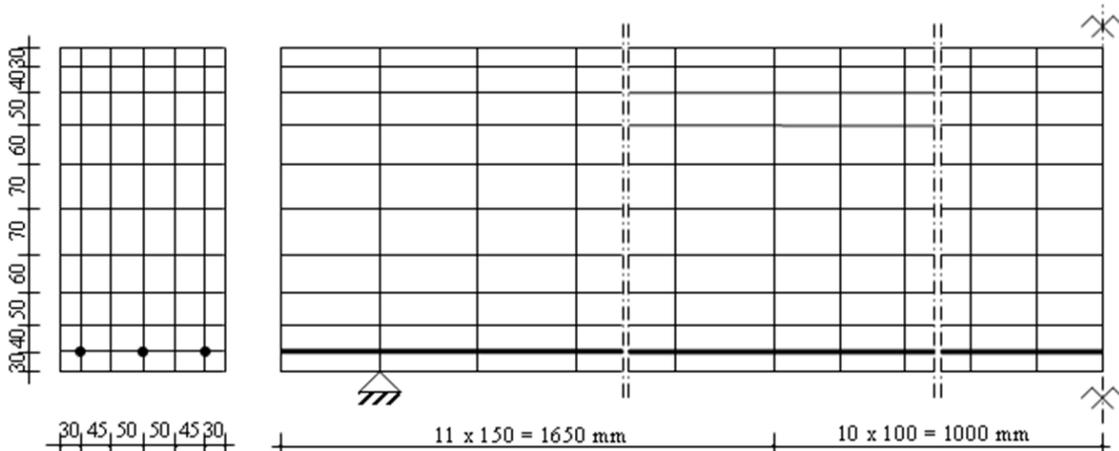


Fig. 4: Finite element discretizations for (a) thermal model and (b) structural model

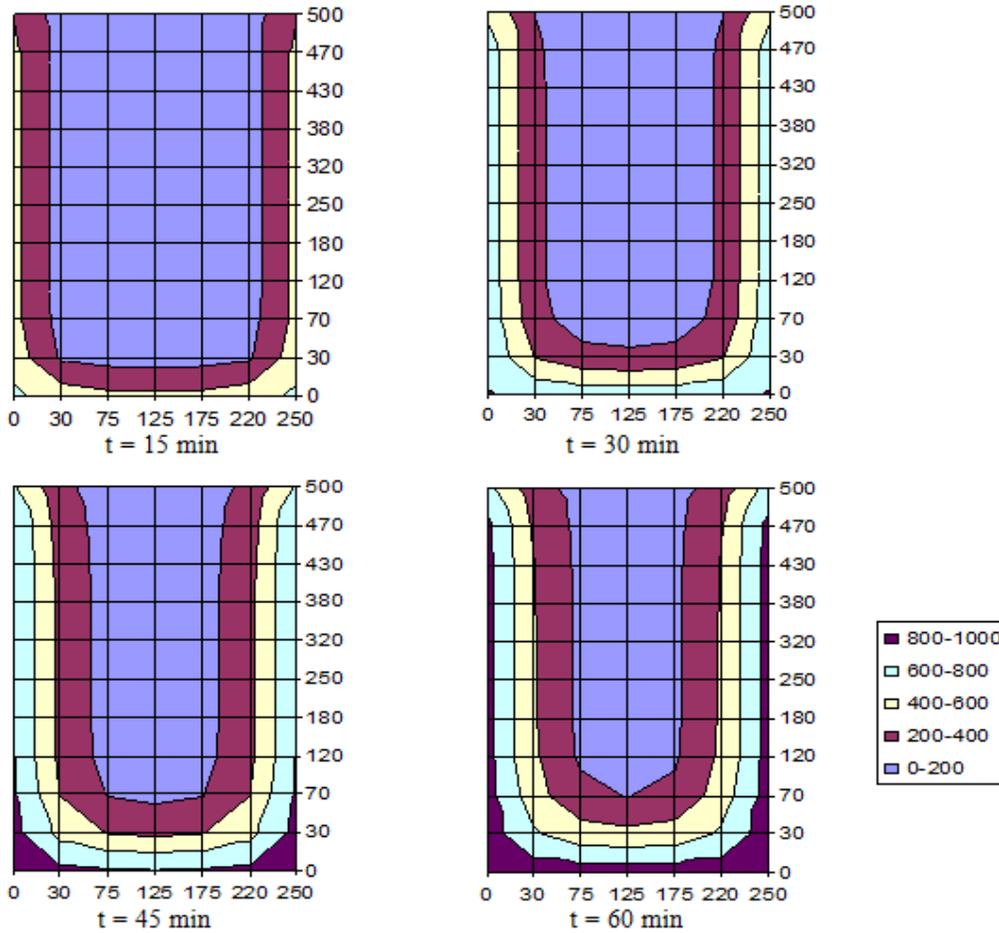


Fig. 5: Temperature distributions (°C) of cross-sections for 15, 30, 45 and 60 min

Figure 4a and 4b shows thermal cross-section model and structural model respectively. In the structural analyses only longitudinal reinforcing bars were taken into account but these were ignored for thermal analysis because of not effecting results too much. Within the models, concrete was modeled by rectangular elements; truss bar element was used for longitudinal reinforcing bar. The diameter and compression bars were ignored in these models.

### RESULTS AND DISCUSSION

The thermal analysis was carried out in order to obtain temperature distribution history of beam cross-section. This was determined by using a developed computer code (Burnaz and Durmuş, 2007) based on finite element method. During thermal analysis 0.005 h was used as a time step. Finally the temperatures of all elements of beam cross-section were calculated according to each time steps. Some distributions obtained were given in Fig. 5 for 15, 30, 45 and 60 min. As it is seen, the temperatures increase with time increment.

The structural analysis under fire conditions was carried out according to the flow diagram in Fig. 2. With this purpose a computer code which use nonlinear finite element method was developed. The temperature values of cross section obtained from thermal analysis was assigned to structural model by calculating the arithmetic mean of temperatures of each element in each height level. It was assumed that the temperature doesn't change along span of the beam.

After running the aforementioned structural analyzing program the stresses, displacements, so the deflections of the beam was obtained for load and time increments. On the other hand the collapsing time of the beam exposed to fire was determined. The mid-span deflection and moment diagrams of the R. C. beam according to load and time increments were given in Fig. 6. As it is seen from the figure the deflection and moment values are increased proportionally in the analysis related to load increment. In the section of time increments (fire analysis) the deflection is increased and the moment is constant until the collapsing time which was found approximately 33 min. This time

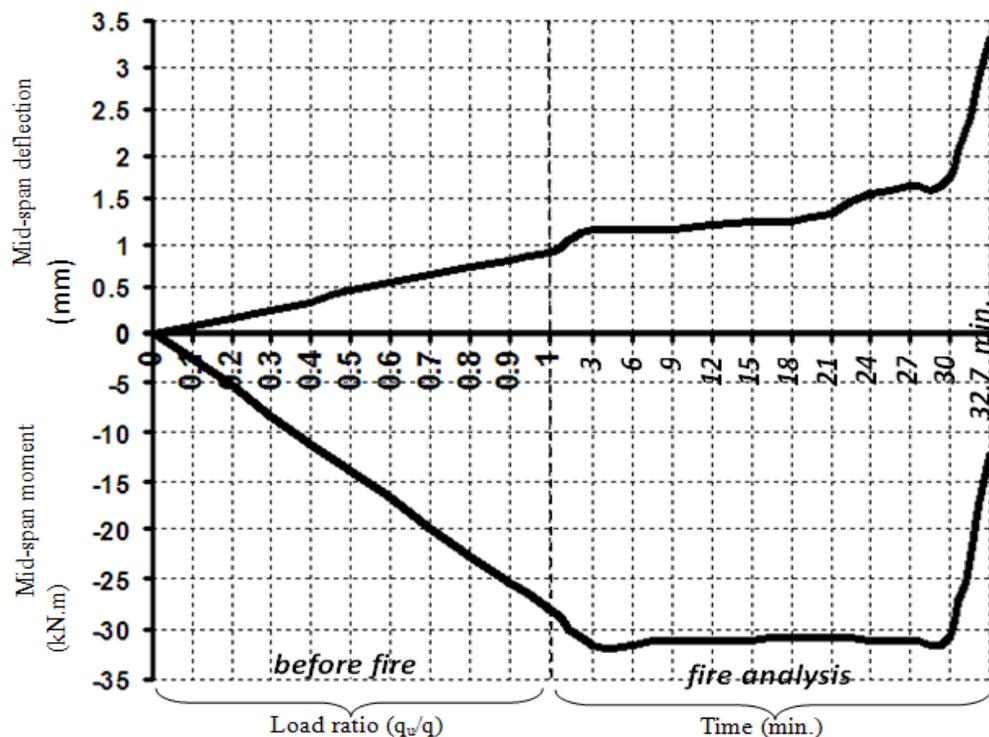


Fig. 6: The mid-span deflection and moment diagrams of the R.C. beam according to load and time increments

value represents fire resistance value of the simple supported R.C. beam loaded uniform distributed.

The fire resistance of this beam was also calculated by using the zone simplified method in EN 1992-1-2 (ENV 1992-1-2, 1996). It was found approximately 42 min. Furthermore referring to tabulated data in the same standard it was seen that the fire resistance of the beam is below 60 min for  $b = 250$  mm and  $a = 30$  mm which are width of the beam and axis distance of corner bars to the side of beam, respectively.

### CONCLUSION

The comparison which has been made between proposed nonlinear model's result and Eurocode's results shows that there is an acceptable correlation. The zone simplified method and tabulated data in EN 1992-1-2 are on the safe side, because the fire resistance of simple supported R.C. beam found from nonlinear structural analysis is more little than the others'. The computer codes which were developed by using nonlinear finite element method and plane stress problem theory can be used for analyzing the R.C. beams exposed to fires.

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