

Research Article

Wideband Radar Target Detection Theory in Coherent K Distributed Clutter

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Abstract: The aim of this study is to analyze the influence of neglecting K distributed clutter texture on wideband radar distributed targets detection. At first, the texture and the speckle of K clutter are researched and the Probability Density Function (PDF) of K clutter and its texture are derived, then the optimal detector by Neyman-Pearson (NP) is proposed, by contrast, another detector-Suboptimum Generalized Likelihood Ratio Test (GLRT) neglecting the clutter texture is given. Next, the estimation of covariance matrix is introduced. Finally, the numerical results are presented by means of Monte Carlo simulation strategy and the simulation results highlight that the performance loss of the 2 detectors in different shaping parameter, the result shows that the performance loss of the detector in K distributed clutter less than 1 db due to the texture is neglected and adaptively estimating the covariance matrix and the K clutter texture can be neglected on wideband radar targets detection.

Keywords: Clutter texture, GLRT, K distributed clutter, Monte Carlo methods, performance loss, wideband radar target detection

INTRODUCTION

The problem of wideband radar spread targets detection has received great attention recently. It naturally arises that the detection performance loss of wideband radar spread targets. The clutter cannot be considered as Gaussian distributed in wideband radar (Yang, 2007; Kay, 1988). Recently much work has been directed towards the clutter model as compound-Gaussian distribution (Conte *et al.*, 2000, 2002a, b; Gini *et al.*, 1999, 2002).

Various adaptive detection algorithms in non-Gaussian background have been studied (Conte *et al.*, 2000, 2002a, b; Gini *et al.*, 1999, 2002; Bueno *et al.*, 2008; Robey *et al.*, 1992; Miao and Iommelli, 2008; Shuai *et al.*, 2010; Shuai, 2011; Pascal *et al.*, 2008; Bon *et al.*, 2008). The compound-Gaussian includes Weibull distributed clutter, K distributed clutter and G0 distributed clutter *et al.* (Yang, 2007). The test data denotes that K distributed clutter can effectively describe the amplitude statistics property of sea clutter and others. So, it is necessary to study the detection performance of detectors in K distributed clutter. To compound-Gaussian clutter, it is modeled as the product of 2 independent random quantities $C_t = \sqrt{\tau_t} g_t$, the clutter spikiness g_t is usually modeled as a compound-Gaussian vector (Conte *et al.*, 2000, 2002a, b; Gini *et al.*, 1999, 2002; Bueno *et al.*, 2008; Robey *et al.*, 1992; Miao and Iommelli, 2008; Shuai *et al.*, 2010;

Pascal *et al.*, 2008; Bon *et al.*, 2008). While, the PDF information of clutter texture τ_t is always neglected in the algorithms of compound Gaussian clutter and modeling it as an unknown determinate parameter in every range cell, the detector needless to estimate the shaping parameter and scaling parameter and made it easily to be operated. However, it needs to deeply research that whether the facilitation leads a large performance loss to the detector.

In the study, we derive 2 different detectors. One detector neglects the clutter texture; another regards the texture as a function of obeying certain PDF, then we give the estimation of covariance matrix. Finally the performance analyses of the 2 tests are carried out via Monte Carlo simulations.

PROBLEM STATEMENT

Assume that a coherent train of N pulses are transmitted by wideband radar and that the incoming waveform of the receiver is properly demodulated, filtered and sampled. The binary hypothesis can be written as:

$$\begin{cases} H_0 : \mathbf{z}_t = \mathbf{c}_t & t=1,2,\dots,L,L+1,\dots,L+K \\ H_1 : \begin{cases} \mathbf{z}_t = \alpha_t \mathbf{p} + \mathbf{c}_t & t=1,2,\dots,L \\ \mathbf{z}_t = \mathbf{c}_t & t=L+1,\dots,L+K \end{cases} \end{cases} \quad (1)$$

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where, $z = [z(0)...z(N-1)]^T$, N , T , p and α_t denote N -dimensional vector, the numbers of complex samples, the transpose operator, the steering vector and the unknown deterministic parameter which account for the channel propagation effects or the target reflectivity respectively. $z_1, z_2 \dots z_L$ are collected from cells under test that are referred as primary data and $z_{L+1}, z_{L+2} \dots z_{L+K}$ are secondary data which not contain any useful target echo and exhibit the same structure of the covariance matrix as the primary data. Assume the received data vectors to be independent between each range cell.

The expression of K distributed clutter can be written as $C_t = \sqrt{\tau_t} g_t$, where τ_t is the texture of clutter. The speckle g_t is a zero mean complex Gaussian vector with covariance matrix M . $E\{xx^H\} = M$, where $E\{\cdot\}$ denotes statistical expectation operator. The clutter texture τ_t is a positive random variable with an unknown PDF.

The clutter amplitude can be defined as:

$$q(c_t) = c_t^H M^{-1} c_t \quad (2)$$

$$p(c_t) = |M|^{-1} \pi^{-N} \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{2\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \quad (3)$$

Let:

$$h_N(q(c_t)) \equiv \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{2\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \quad (4)$$

From Eq. (3) and (4), we can see the clutter PDF and the PDF of texture is related to each other. Assume μ and ν respectively are the scaling and shaping parameters of K distributed clutter, the PDF of K distributed clutter texture τ_t can be written as:

$$P_{\tau}(\tau) = \frac{1}{\Gamma(\nu)} \left(\frac{\nu}{\mu}\right)^\nu \tau^{\nu-1} e^{-\frac{\nu}{\mu}\tau} \quad \tau > 0 \quad (5)$$

The clutter mean value is $E\{\tau\} = \mu$, the clutter covariance is $E\{(\tau - \mu)^2\} = \frac{\mu^2}{\nu}$, the expression of $h_N(q(c_t))$ in K distributed clutter can be written as:

$$\begin{aligned} h_N^K(q(c_t)) &\equiv \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \\ &= \frac{1}{2^{\nu-N-1} \Gamma(\nu)} \left(\frac{\nu}{\mu}\right)^\nu \left(4\nu \cdot \frac{q(c_t)}{\mu}\right)^{(\nu-N)/2} \cdot \\ &\quad K_{N-\nu} \left(\sqrt{4\nu \frac{q(c_t)}{\mu}} \right) \end{aligned} \quad (6)$$

where, $\Gamma(\nu)$ denotes gamma distributed.

DETECTOR DESIGN

Optimal detector under NP criterion design in K distributed clutter: In order to get the theory form of NP detector, we assume the parameters L , p , u and v are all known, here we first assume M is known, then we estimate it. According to the optimal detection theory under NP criterion, the optimal detection statistics is the Likelihood Ratio Test (LRT) and the LRT under model (1) can be expressed as Shuai (2011):

$$\frac{\max_{\alpha_t} p(z_{1:L} | \alpha_t; H_1)}{p(z_{1:L} | H_0)} \underset{H_0}{\underset{H_1}{\geq}} \eta \quad (7)$$

Then we can obtain the PDF of target echo under H_0, H_1 which can be expressed as:

$$\begin{aligned} p(z_{1:L} | \alpha_t; H_1) &= |M|^{-L} \pi^{-NL} \prod_{t=1}^L h_N^K(q_1(z_t)) \\ &= \frac{1}{2^{-L} \Gamma^L(\nu)} \left(\frac{\nu}{\mu}\right)^{L(\nu+N)/2} \prod_{t=1}^L q_1^{(\nu-N)/2}(z_t) \cdot \\ &\quad K_{N-\nu} \left(\sqrt{4\nu \frac{q_1(z_t)}{\mu}} \right) \end{aligned} \quad (8)$$

$$\begin{aligned} p(z_{1:L} | H_0) &= |M|^{-L} \pi^{-NL} \prod_{t=1}^L h_N^K(q_0(z_t)) = \\ &= \frac{1}{2^{-L} \Gamma^L(\nu)} \left(\frac{\nu}{\mu}\right)^{L(\nu+N)/2} \prod_{t=1}^L q_0^{(\nu-N)/2}(z_t) \cdot \\ &\quad K_{N-\nu} \left(\sqrt{4\nu \frac{q_0(z_t)}{\mu}} \right) \end{aligned} \quad (9)$$

where,

$$\begin{aligned} q_1(z_t) &= (z_t - \alpha_t P)^H M^{-1} (z_t - \alpha_t P) \\ q_0(z_t) &= z_t^H M^{-1} z_t \end{aligned}$$

substitute Eq. (8) and (9) into (7) and estimate the target information α_t , because the PDF in Eq. (8) is monotonically decreasing function of $q_1(z_t)$, the $\max_{\alpha_t} p(z_{1:L} | \alpha_t; H_1)$ equals to $\min_{\alpha_t} q_1(z_t)$, that is the Maximum Likelihood Estimation (MLE) of α_t can be written as $\hat{\alpha}_t = \arg\min_{\alpha_t} q_1(z_t)$, by simple calculating, it can be expressed as $\hat{\alpha}_t = \frac{P^H M^{-1} z_t}{P^H M^{-1} P}$, substitute $\hat{\alpha}_t$ into $q_1(z_t)$, and let $\hat{q}_t(z_t) = (z_t - \hat{\alpha}_t P)^H M^{-1} (z_t - \hat{\alpha}_t P)$, after simple calculating, Eq. (7) can be written as:

$$\begin{aligned} &\frac{\prod_{t=1}^L h_N(\hat{q}_t(z_t))}{\prod_{t=1}^L h_N(q_0(z_t))} \\ &= \frac{\prod_{t=1}^L \left(q_1(z_t)^{(\nu-N)/2} \cdot K_{N-\nu} \left(\sqrt{4\nu \frac{q_1(z_t)}{\mu}} \right) \right)_{H_1}}{\prod_{t=1}^L \left(q_0(z_t)^{(\nu-N)/2} \cdot K_{N-\nu} \left(\sqrt{4\nu \frac{q_0(z_t)}{\mu}} \right) \right)_{H_0}} \underset{H_0}{\underset{H_1}{\geq}} \eta \end{aligned} \quad (10)$$

where, $\hat{q}_t(z_t) = (z_t - \hat{a}_t P)^H M^{-1} (z_t - \hat{a}_t P)$, Eq. (10) is the optical detector in K distributed clutter.

Suboptimum GLRT detector design: The suboptimum GLRT is proposed by Robey *et al.* (1992) and Shuai (2011):

$$T_G = -N \sum_{t=1}^L \ln \left(\frac{|P^H M^{-1} z_t|^2}{(P^H M^{-1} P)(z_t^H M^{-1} z_t)} \right) \quad (11)$$

Covariance matrix estimation: In Eq. (10) and (11), the covariance matrix M is unknown; we need to estimate it via secondary data vector. The estimator is obtained by maximizing the likelihood function by M, which can be written as:

$$\hat{M}_{MLE} = \arg \max_M p(Z_{1:K} | t_{1:K}, M) \quad (12)$$

In formula (12), to obtain \hat{M}_{MLE} via the likelihood function about covariance matrix partial differential equals to zero:

$$\sum_{t=1}^k \left[\frac{\partial \ln |M|}{\partial M} - \frac{1}{\tau_t} \frac{z_t^H M^{-1} z_t}{\partial M} \right] = 0 \quad (13)$$

Substitute (12) into (11), \hat{M}_{MLE} can be written as:

$$\hat{M}_{MLE} = \frac{1}{K} \sum_{t=1}^K \frac{z_t z_t^H}{\tau_t} \quad (14)$$

In formula (14), the estimator cannot be directly got because the unknown parameters τ_t $t = 1, 2, \dots, k$, we obtain the maximum likelihood estimation $\hat{\tau}_t$ to replace, τ_t which can be written as:

$$\hat{\tau}_t = \frac{z_t^H M^{-1} z_t}{N} \quad t = 1, 2, \dots, K \quad (15)$$

Substitute (15) into (14), we can get the MLE of covariance matrix, which can be written as:

$$\hat{M}_{MLE} = \frac{N}{K} \sum_{t=1}^K \frac{z_t z_t^H}{z_t^H M^{-1} z_t} \quad (16)$$

In formula (16), we can obtain the estimator by iterative method (Bon *et al.*, 2008; Conte *et al.*, 2002a, b; Shuai, 2011):

$$\hat{M}_{MLE}(i) = \frac{N}{K} \sum_{t=1}^K \frac{z_t z_t^H}{z_t^H \hat{M}_{MLE}(i-1)^{-1} z_t} \quad i = 1, \dots, N_{it} \quad (17)$$

In Eq. (17), we can set the initial value of \hat{M}_{MLE} , as:

$$\hat{M}_{MLE}(0) = \frac{N}{K} \sum_{t=1}^k \frac{z_k z_k^H}{z_k^H z_k}$$

Substitute \hat{M}_{MLE} in formula (17) into (10) and (11), we can obtain the NP-K detector and GLRT detector as follows:

$$\frac{\prod_{t=1}^L h_N(\hat{q}_1(z_t))}{\prod_{t=1}^L h_N(q_0(z_t))} = \frac{\prod_{t=1}^L \left(q_1(z_t)^{(\nu-N)/2} \cdot K_{N-\nu} \left(\sqrt{\frac{4\nu q_1(z_t)}{\mu}} \right) \right)^{H_1}}{\prod_{t=1}^L \left(q_0(z_t)^{(\nu-N)/2} \cdot K_{N-\nu} \left(\sqrt{\frac{4\nu q_0(z_t)}{\mu}} \right) \right)^{H_0}} \geq \eta \quad (18)$$

where,

$$\hat{q}_1(z_t) = (z_t - \hat{a}_t P)^H (\hat{M}_{MLE})^{-1} (z_t - \hat{a}_t P)$$

$$\hat{q}_0(z_t) = z_t^H (\hat{M}_{MLE})^{-1} z_t$$

$$T_G = N \sum_{t=1}^L \ln \left(1 - \frac{|P^H (\hat{M}_{MLE})^{-1} z_t|^2}{(P^H (\hat{M}_{MLE})^{-1} P)(z_t^H (\hat{M}_{MLE})^{-1} z_t)} \right) \quad (19)$$

PERFORMANCE ASSESSMENT

In the following simulation, both the Probability Detection (PD) and Probability of False Alarm (PFA) of NP-K detector and GLRT detector are got by Monte Carlo methods. We mainly review the texture information lost of the K distributed clutter leads to the preference loss of NP-K detector. Assume $PFA = 10^{-4}$, $L = 4$, $N = 8$ and $\mu = 1$ The mean value of K distributed clutter is μ , so the Signal-Clutter-Ratio (SCR) can be define as:

$$SCR = \frac{\sum_{t=1}^L (\alpha_t P)^H (\hat{M}_{MLE})^{-1} (\alpha_t P)}{\mu}$$

The detection performance of the NP-K detector and GLRT detector show as follows:

In Fig. 1 to 4, we can see that the smaller of the shaping parameter ν , the better detection performance of the 2 detectors, especially when a low SCR, the reason is that the clutter is very acuity when shaping parameter is small and the clutter energy mainly occupies in several range cells of spread targets, which cases the

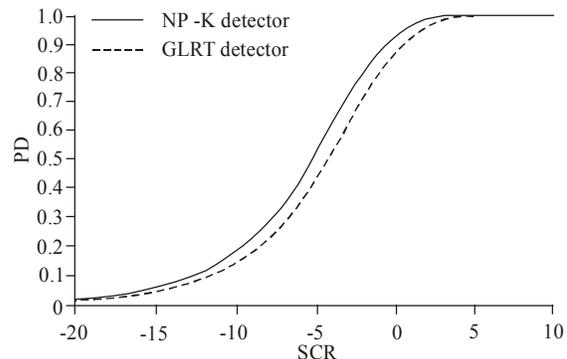


Fig. 1: The detection performance of NP-K and GLRT when $\nu = 0.8$

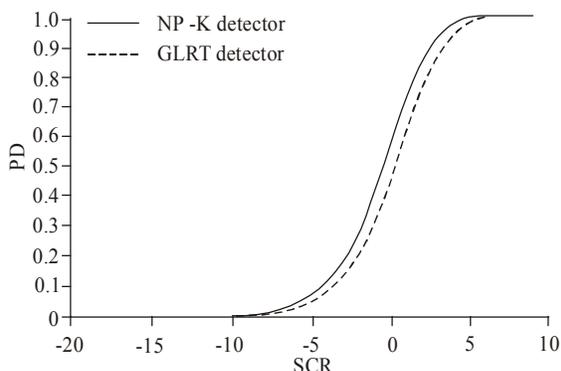


Fig. 2: The detection performance of NP-K and GLRT when $\nu = 1.6$

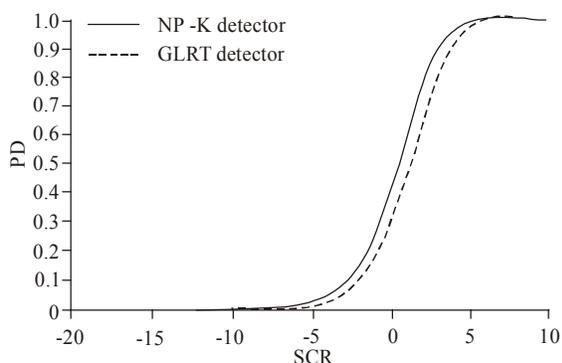


Fig. 3: The detection performance of NP-K and GLRT when $\nu = 2.4$

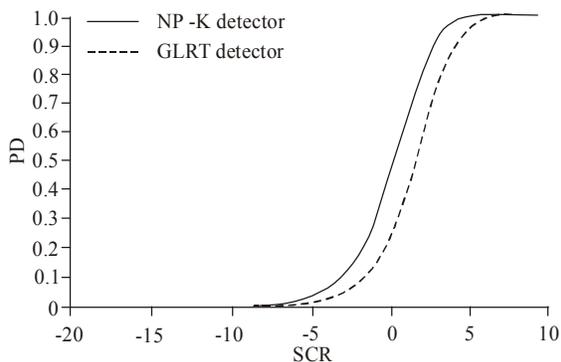


Fig. 4: The detection performance of NP-K and GLRT when $\nu = 3.0$

small target easily to be detected. Moreover, we can see that the difference between the 2 detectors less than 1 db with the increasing of SCR.

CONCLUSION

In this study, we have addressed the problem of adaptive detection of range-spread targets in K distributed clutter. More precisely, we first give the expression of K distributed clutter, analyze the texture

of K clutter and propose the PDF of K clutter texture. Next, we introduce 2 detectors and give the expression of each detector and then give the covariance matrix estimation of detector in K distributed clutter. Finally, by Monte Carlo simulation, we propose the performance loss due to adaptively estimate the covariance matrix and texture component is neglected. The results show that the K clutter texture can be neglected due to a small loss performance compare to the optical detector. We can see the expression of NP-detector is very complex and it needs to know many parameters, so this result is meaningful to wideband radar target detection in coherent K distributed clutter.

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