## Research Article

# Research on the Supply Chain Buy-Back Modes Based on the Consumer's Strategic Returns with the Competition 

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#### Abstract

In this study, we have a research on the supply chain buy-back modes based on the consumer's strategic returns with the competition. In the supply chain with the manufacturer and two competing retailers, discuss the three supply chain buy-back modes focusing on the consumer's strategic returns from the point of the manufacturers: the manufacturers repurchasing the return products through the retailers; the manufacturers repurchasing the return products directly; the asymmetric return repurchasing mode. By analyzing the three models, the manufacturers' and two retailers' optimal decision and the optimal profits are obtained, which provides the suggestions for the members to choose the right supply chain buy-back mode. The results show that: in the different buy back mode, the players' profits in the supply chain are different; the choice of the buyback modes depends on the consumers' return rate, the proportion of the customers choosing to return and the repurchasing price, but the asymmetric buy back mode is non-optimal mode. Finally, the numerical analysis strongly supports the conclusions in the study.


Keywords: Buy back mode, competing environment, consumers' strategic returns, game theory, supply chain

## INTRODUCTION

In the highly competitive market environment, allowing the customers to return has become an important modern market competition strategy. To attract customers, a growing number of businesses launched the 'not satisfied then return' service strategy, which allows customers to return unconditionally within a period of time after the purchase. The customer is also used to return to the merchant to obtain a certain amount of refund or full refund. Only in the U.S, return goods from consumers each year worth more than \$ 100 billion, there are only $5 \%$ returned during to quality defects among this returns. In recent years, the academic study of the reverse supply chain is gradually increasing, some scholars have discussed remanufactured problem of the returns or recover, such as literatures. Another number of scholars focused on the return policies and the influence between manufacturers and retailers from different perspectives, but these literatures didn't consider the influence of customer's strategic return behavior (Shen et al., 2010; Stock et al., 2002; Robe, 2004; Vlachos et al., 2007; Wang and Da, 2006; Alshamrani et al., 2007; Xue et al., 2006; Zhao and Yang, 2007).

With the gradual rise of the behavior operation management, many scholars have recognized that it is necessary to consider the human factors in traditional
operational management and then the customer behavior study has gradually integrated into the supply chain-related research. In the field of supply chain management, customer strategic behavior just has been introduced to the related study. Su and Zhang (2008) introduced the customer strategic behavior to the supply chain management research firstly and studied the influence that customer's strategic behavior to the traditional newsboy model order quantity. Li et al. (2007) analyzed the influence of customer strategic behavior on the supply chain strategy of total revenue under buy-back contract, shown that the buy-back contract can coordinate the supply chain well under customer strategic behavior. Qi et al. (2010) discussed the coordinating effect of two pricing contracts on the supply chain, indicated that since the impact of customer strategic behavior, the manufacturers should set the wholesale price of the two pricing contract higher than the marginal production cost ( Su and Zhang, 2008; Li et al., 2007; Qi et al., 2010; Su, 2009; Zhai and Li, 2011; Chan, 1996).

Strategic returns means return without quality problems. Consumers do not make sure the product value and can really understand the value only after the purchase; consumers decide whether to buy and then decide whether to return through experience or after using the product. Literature which researches the impact of customer strategic return on supply chain is

[^0]fewer and does not consider the existence of competing retailers. Su (2009) takes the customer strategic return into the supply chain system firstly, researching on the supply chain performance problem in a single cycle of supply consisting by a manufacturer, providing full return a Retailers and a customer base. Based on Su X's research, Shen etc., established the customer strategic return model under the constraint of service level and determined the optimal price, order quantity and return prices under the centralized and decentralized decisionmaking models, designing the contract to coordinate the supply chain. Zhai and Li (2011) discussed the return processing strategy under the B2C mode in the supply chain which consisted of the manufacturer and on-line retailers. However, these studies of customer strategic returns didn't consider the competition. Therefore, we discuss the buy-back modes of the manufacturer in the scenario of competed retailers. And analyzing the impact on the supply chain members, then provide the guidance for the manufacturer selecting the appropriate buyback mode.

We introduced competitive factor into the customer strategic return in the supply chain system based on the researches above, analyzed the choice of the customer strategic return mode in the supply chain with two retailers.

Difference with the existing literature is:

- The reason that the consumers returning the product is not because of the quality problem, but there has not consistent utility of value estimate between before and after purchase, which also called strategic return.
- The environment we studied has the competitive retailers in the supply chain and the refund providing to the customer is not full refund.
- From the view of the manufacturers, considering the customer strategic returns, research on the optimal buy-back mode choice through the three supply chain modes.

In this study, we have a research on the supply chain buy-back modes based on the consumer's strategic returns with the competition. In the supply chain with the manufacturer and two competing retailers, discuss the three supply chain buy-back modes focusing on the consumer's strategic returns from the point of the manufacturers: the manufacturers repurchasing the return products through the retailers; the manufacturers repurchasing the return products directly; the asymmetric return repurchasing mode. By analyzing the three models, the manufacturers' and two
retailers' optimal decision and the optimal profits are obtained, which provides the suggestions for the members to choose the right supply chain buy-back mode. The results show that: in the different buy back mode, the players' profits in the supply chain are different; the choice of the buyback modes depends on the consumers' return rate, the proportion of the customers choosing to return and the repurchasing price, but the asymmetric buy back mode is nonoptimal mode. Finally, the numerical analysis strongly supports the conclusions in the study.

Basic assumptions and parameter description: We focused on a single-cycle supply chain system which is consisted by a manufacturer, two competitive retailers and a group of customers, in which the manufacturer produced one product.

Before the start of the selling season, the manufacturer gives the wholesale price $\mathrm{w}_{\mathrm{i}}, \mathrm{i}=1,2$ according to the historical data and the product cost; subsequently, the retailers decide the unit retail price $\mathrm{p}_{\mathrm{i}}$; finally, the customers decide whether to purchase the goods based on the retail price and after purchasing, then decide whether to return. We assume that the proportion that the customers choosing to return is $k$ for easier analysis, without considering the cost of the product returns process and the product residual values assumed as 0 .

The decision-making goal of the manufacturer and the retailers both are expected maximized profits. The time series of the events is shown as Fig. 1.

The market demand of the products sold by retailers depends on the retail prices of their own and the competitor, which is $Q_{i}=1-p_{i}+\alpha\left(p_{j}-p_{i}\right)$. In which, the parameter $\mathrm{Q}_{\mathrm{i}}$ is the sell amount of the retailer $i ; p_{i}$ is the retail price of the retailer $i ; \alpha$ is the substitute quality of retailers, the larger $\alpha$ value is, the stronger substitute quality of two retailer is.

Because the analysis focuses on the choice of the buy-back modes for the manufacturer, we assume the coefficient of variation of two retailers as $\alpha=1$. Then the market demand function of two retailers based on Chan (1996) is as follows:

$$
\begin{equation*}
Q_{1}=1-p_{1}+\left(p_{2}-p_{1}\right) ; Q_{2}=1-p_{2}+\left(p_{1}-p_{2}\right) \tag{1}
\end{equation*}
$$

## BUY BACK MODES

To face the customer strategic returns, we discussed three customer strategic returns decision-


Fig. 1: The time series diagram of supply chain even


Fig. 2: Buy-back mode I: cooperative buy-back mode back
making modes that the manufacturers may make: the manufacturer buys the customer returns through the retailers; the manufacturer buys customer returns directly; in the asymmetric buy-back mode, the manufacturer buys one retailer's customer returns directly, while the other is through the retailer's transit to buy-back. The following will analyze the decisions of the retailers and the manufacturer in the three buyback modes.

## Buy-back mode I:

Cooperative buy-back mode: Many companies adopted the cooperative buy-back mode, which means two retailers and the manufacturer cooperating to deal with the customers strategic returns: When customer has the return wishes, they must return the product back to the purchase retailer first, the return compensation is $r$ and the compensation is related to selling price $r=\theta p_{i}$; then the retailer return the corresponding returns to the manufacturer, the manufacturer buy the return product with the market repurchase price $t$. Buy-back mode is shown in Fig. 2.

In this buy-back mode, the two retailers' profit functions are:

$$
\begin{align*}
& \pi_{r 1}^{1}=\left(p_{1}-w-k\left(\theta p_{1}-t\right)\right)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)  \tag{2}\\
& \pi_{r 2}^{1}=\left(p_{2}-w-k\left(\theta p_{2}-t\right)\right)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right) \tag{3}
\end{align*}
$$

The manufacturer's profit function is:

$$
\begin{align*}
& \pi_{m}^{1}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+  \tag{4}\\
& (w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right)
\end{align*}
$$

According to the events time series diagram, the game relationship between the retailers and the manufacturer can be described as the Stackelberg games model that the manufacturer acts as the leader while retailers act as the follower. Therefore, we use backward induction to solve the optimal decision of the retailers and manufacturer.

Firstly, obtain the first derivative of the retailer's profit function respect to the variables, which are:

$$
\begin{align*}
& \frac{\partial \pi_{r 1}^{1}}{\partial p_{1}}=(1-k \theta)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)-  \tag{5}\\
& 2\left(p_{1}-w-k\left(\theta p_{1}-t\right)\right)=0 \\
& \frac{\partial \pi_{r 2}^{1}}{\partial p_{2}}=(1-k \theta)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right)-  \tag{6}\\
& 2\left(p_{2}-w-k\left(\theta p_{2}-t\right)\right)=0
\end{align*}
$$

From formula (5-6), we can get:

$$
\begin{equation*}
p_{1}=\frac{1}{3}+\frac{2(w-k t)}{3(1-k \theta)} ; p_{2}=\frac{1}{3}+\frac{2(w-k t)}{3(1-k \theta)} \tag{7}
\end{equation*}
$$

Then the manufacturer obtains its own best decisions based on the retailers' decision. And the manufacturer's decision function is:

$$
\begin{align*}
& \operatorname{Max}_{m}^{1}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+  \tag{8}\\
& (w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right) \\
& \text { s.t. } p_{1}=\frac{1}{3}+\frac{2(w-k t)}{3(1-k \theta)} ; \quad p_{2}=\frac{1}{3}+\frac{2(w-k t)}{3(1-k \theta)}
\end{align*}
$$

From formula (8), we can get the manufacturers optimal wholesale price, which is:

$$
\begin{equation*}
w^{1^{*}}=k t+\frac{1-k \theta}{2} \tag{9}
\end{equation*}
$$

Integrating formula (9) into (7), we can get the optimal price of retailers is:

$$
\begin{equation*}
p_{1}^{1^{*}}=p_{2}^{1^{*}=}=\frac{2}{3} \tag{10}
\end{equation*}
$$

From formula (10), we can get that the retail price in the cooperative buy-back mode is not related with the manufacture's repurchase price.

Due to the optimal decision variables of the retailers and manufacture, the optimal profits can be calculated as:

$$
\begin{align*}
& \pi_{m}^{1^{*}}=\frac{(1-k \theta)}{3}  \tag{11}\\
& \pi_{r 1}^{1^{*}}=\pi_{r 2}^{1^{*}}=\left(p_{1}-w-k\left(\theta p_{1}-t\right)\right)\left(1-p_{1}+\right.  \tag{12}\\
& \left.\left(p_{2}-p_{1}\right)\right)=\frac{1-k \theta}{18}
\end{align*}
$$

## Buy-back mode II:

Independent buy-back mode: Many manufacturers adopt the buy-back mode that customers return the product back to the manufacturer directly, that is independent buy-back mode. Buy-back mode II is shown as Fig. 3.
In this mode, the retailers' profit functions are:


Fig. 3: Buy-back mode II: independent buy-back mode

$$
\begin{align*}
& \pi_{r 2}^{2}=\left(p_{1}-w\right)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)  \tag{13}\\
& \pi_{r 2}^{2}=\left(p_{2}-w_{2}\right)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right) \tag{14}
\end{align*}
$$

The manufacturer's profit function is:

$$
\begin{equation*}
\pi_{m}^{2}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+(w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right) \tag{15}
\end{equation*}
$$

Adopt backward induction to obtain the best decisions of the retailers and the manufacturer. Firstly, for the retailers, the optimal decision variables are:

$$
\begin{equation*}
p_{1}=\frac{1+2 w}{3} ; p_{2}=\frac{1+2 w}{3} \tag{16}
\end{equation*}
$$

For the manufacturer, we can obtain the optimal decision based on the retailers', which is:

$$
\begin{align*}
& M a x \pi_{m}^{2}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+  \tag{17}\\
& (w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right) \\
& \text { s.t. } p_{1}=\frac{1+2 w}{3} ; \quad p_{2}=\frac{1+2 w}{3}
\end{align*}
$$

From formula (17), we can calculate the optimal wholesale price of the manufacturer, which is:

$$
\begin{equation*}
w^{2^{*}}=\frac{1+k t}{2} \tag{18}
\end{equation*}
$$

Substituting formula (18) into (16), the retailers' optimal decision is:

$$
\begin{equation*}
p_{1}^{2^{*}}=\frac{2+k t}{3} ; p_{2}^{*}=\frac{2+k t}{3} \tag{19}
\end{equation*}
$$

Due to the optimal decisions of the manufacturers and retailers, we can obtain the optimal profit functions:

$$
\begin{align*}
& \pi_{m}^{2^{*}}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+(w-k t)(1-  \tag{20}\\
& \left.p_{2}+\left(p_{1}-p_{2}\right)\right)=\frac{(1-k t)^{2}}{3} \\
& \pi_{r 1}^{2^{*}}=\pi_{r 2}^{2^{*}}=\left(p_{1}-w\right)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)=\frac{(1-k t)^{2}}{18} \tag{21}
\end{align*}
$$



Fig. 4: Buy-back mode III: asymmetric buy-back mode

## Buy-back mode III:

Asymmetric buy-back mode: In buy-back mode III, the manufacturer sells the product to the retailers. But when the manufacturer buys back the customer strategic returns, it buys one of the retailer's strategic returns back directly and buy the other's return back through the retailer, that means asymmetric buy-back mode, shown in Fig. 4.

In this mode, the retailer and the manufacturer's profit functions are:

$$
\begin{align*}
& \pi_{r 1}^{3}=\left(p_{1}-w-k\left(\theta p_{1}-t\right)\right)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)  \tag{22}\\
& \pi_{r 2}^{3}=\left(p_{2}-w\right)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right)  \tag{23}\\
& \pi_{m}^{3}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)+  \tag{24}\\
& (w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right)
\end{align*}
$$

For the retailers:

$$
\begin{align*}
& \frac{\partial \pi_{r 1}^{3}}{\partial p_{1}}=(1-k \theta)\left(1-p_{1}+\left(p_{2}-p_{1}\right)\right)-  \tag{25}\\
& 2\left(p_{1}-w-k\left(\theta p_{1}-t\right)\right)=0 \\
& \frac{\partial \pi_{r 2}^{3}}{\partial p_{2}}=1-p_{2}+\left(p_{1}-p_{2}\right)-2\left(p_{2}-w\right)=0 \tag{26}
\end{align*}
$$

From formula (25-26), we can obtain:

$$
p_{1}=\frac{1}{3}+\frac{2}{15} w+\frac{8(w-k t)}{15(1-k \theta)} ; p_{2}=\frac{1}{3}+\frac{8}{15} w+\frac{2(w-k t)}{15(1-k \theta)}
$$

The manufacturer's decision function based on the retailer's decision is:

$$
\begin{align*}
& M a x \pi_{m}^{3}=(w-k t)\left(1-p_{1}+\left(p_{2}-p_{1}\right)+\right. \\
& (w-k t)\left(1-p_{2}+\left(p_{1}-p_{2}\right)\right)  \tag{27}\\
& \text { s.t. } p_{1}=\frac{1}{3}+\frac{2}{15} w+\frac{8(w-k t)}{15(1-k \theta)} \\
& \quad p_{2}=\frac{1}{3}+\frac{8}{15} w+\frac{2(w-k t)}{15(1-k \theta)}
\end{align*}
$$

From formula (27), the optimal wholesale price of manufacture can be calculate as:

$$
\begin{equation*}
w^{3^{*}}=\frac{(1+2 k t)(1-k \theta)+k t}{2-k \theta}=1+2 k t-\frac{1+k t}{2-k \theta} \tag{28}
\end{equation*}
$$

Substituting formula (28) into (25-26), the optimal retail price is:

$$
\begin{align*}
& p_{1}^{3^{*}}=\frac{1}{3}+\frac{2(1+2 k t)(1-k \theta)+10 k t+8}{15(2-k \theta)}  \tag{29}\\
& p_{2}^{3^{*}}=\frac{1}{3}+\frac{8(1+2 k t)(1-k \theta)+10 k t+2}{15(2-k \theta)} \tag{30}
\end{align*}
$$

From formula (28-30), we can get the optimal profit functions of the retailers and manufacture:

$$
\begin{align*}
& \pi_{r 1}^{3^{*}}=\left(p_{1}^{3^{*}}-w^{3^{*}}-k\left(\theta p_{1}^{3^{*}}-t\right)\right)\left(1-p_{1}^{3^{*}}\right)  \tag{31}\\
& \pi_{r 2}^{3^{*}}=\left(\frac{1}{3}+\frac{2-7(1+2 k t)(1-k \theta)-5 k t}{15(2-k \theta)}\right)\left(\frac{2}{3}-\right. \\
& \left.\frac{8(1+2 k t)(1-k \theta)+10 k t+2}{15(2-k \theta)}\right)  \tag{32}\\
& \pi_{m}^{3^{*}}=\frac{2(1+k t)(1-k \theta)(1-2 k t)}{3(2-k \theta)} \tag{33}
\end{align*}
$$

## COMPARATIVE ANALYSES

Through the analysis above, we can compare relevant optimal decisions under three different modes, to obtain some meaningful management sights and then provide guidance for the manufacturers to choose the effective buy-back strategies.

Theorem 1: Under the three repurchase modes, the contrast of the wholesale price is shown:
when $0<t<\frac{\theta}{4-3 k \theta}, w_{2}^{*}>w_{3}^{*}>w_{1}^{*}$
when $\frac{\theta}{4-3 k \theta}<t<\theta, w_{3}^{*}>w_{2}^{*}>w_{1}^{*} ;$
when $\theta<\mathrm{t}<1, \mathrm{w}^{*}{ }_{3}>\mathrm{w}^{*}{ }_{1}>\mathrm{w}^{*}{ }_{2}$
Proof: Due to $w_{1}^{*}-w_{2}^{*}=k t+\frac{1-k \theta}{2}-\frac{1+k t}{2}>0$, we can obtain:
when $1>t>\theta, w_{1}^{*}>w_{2}^{*} ; 0<t \leq \theta, w_{1}^{*} \leq w_{2}^{*}$

Due to $w_{1}^{*}-w_{3}^{*}=k t+\frac{1-k \theta}{2}-\frac{(1+2 k t)(1-k \theta)+k t}{2-k \theta}>0$, we can get that $w_{1}^{*}<w_{3}^{*}$;

Due to $w_{2}^{*}-w_{3}^{*}=\frac{1+k t}{2}-\frac{(1+2 k t)(1-k \theta)+k t}{2-k \theta}>0$,
we can obtain $0<t<\frac{\theta}{4-3 k \theta}, w_{2}^{*}>w_{3}^{*} ;$
For $\theta / 4-3 \mathrm{k} \theta<\theta$, we can obtain the theorem 1 .

## Theorem 2:

- In buy-back mode I and buy-back mode II, the retail price of the two retailer's are equal; while in buy-back mode III, when the buy-back price of the manufacturer providing is lower, the retail price that the retailer buy the returns back as intermediary role is higher than that the manufacturer adopt buy-back strategy directly; Otherwise, when the buy-back price manufacturer provide is higher, the retail price that retailer buy the return back as intermediary role is lower than that manufacturer adopts buy-back strategy directly.
- Under these three buy-back modes, for retailer 1, the retail price which provide is:

$$
p_{1}^{1^{*}}<p_{1}^{2^{*}}<p_{1}^{3^{*}}
$$

- Under these three buy-back modes, for retailer 2, the retail price is:
when $_{0<t<\frac{3 \theta}{18-8 k \theta}}, \quad p_{2}^{3^{*}}<p_{2}^{1^{*}}<p_{2}^{2^{*}}$
when $\frac{3 \theta}{18-8 k \theta} \leq t \leq \frac{3 \theta}{16-11 k \theta}, p_{2}^{1^{*}}<p_{2}^{3^{*}}<p_{2}^{2^{*}}$
when $\frac{3 \theta}{16-11 \mathrm{k} \theta}<t<1, p_{2}^{1^{*}}<p_{2}^{2^{*}}<p_{2}^{3^{*}}$
Proof: From formula (10, 16, 29, 30), Theorem 2 (1-2) is easy to prove.

For

$$
\begin{aligned}
& p_{2}^{2^{*}}-p_{2}^{3^{*}}=\frac{2+k t}{3}-\frac{1}{3}-\frac{2(1+2 k t)(1-k \theta)+10 k t+8}{15(2-k \theta)} \\
& p_{2}^{1^{*}}-p_{2}^{3^{*}}=\frac{5(2-k \theta)(1+k t)-8(1+2 k t)(1-k \theta)-10 k t-2}{15(2-k \theta)}
\end{aligned}
$$

We can obtain: when $0<t<\frac{3 \theta}{16-11 k \theta},,_{2}^{2^{*}}>p_{2}^{3^{*}}$; when $0<t<\frac{3 \theta}{18-8 k \theta}, p_{2}^{1^{*}}>p_{2}^{3^{*}}$.

Therefore, when $0<t<\frac{3 \theta}{18-8 k \theta}, p_{2}^{3^{*}}<p_{2}^{1^{*}}<p_{2}^{2^{*}} ;$
when $\frac{3 \theta}{18-8 k \theta} \leq t \leq \frac{3 \theta}{16-11 k \theta}, \quad p_{2}^{1^{*}}<p_{2}^{3^{*}}<p_{2}^{2^{*}} ;$
when $\frac{3 \theta}{16-11 k \theta}<t<1, p_{2}^{1^{*}}<p_{2}^{2^{*}}<p_{2}^{3^{*}}$.
Theorem 3: when $0<t<\frac{1-\sqrt{1-k \theta}}{k}$, for the manufacturer, buy-back mode II is the best; when $\frac{1-\sqrt{1-k \theta}}{k}<t<1$, for the manufacturer, buy-back mode I is the best.

Proof: For $\pi_{m}^{1^{*}}-\pi_{m}^{2^{*}}=\frac{(1-k \theta)}{3}-\frac{(1-k t)^{2}}{3}$, obtain that when $\frac{1-\sqrt{1-k \theta}}{k}<t<\frac{1+\sqrt{1-k \theta}}{k}$, then $\pi_{m}^{1^{*}}>\pi_{m}^{2^{*}} ; \quad$ When $t \geq \frac{1+\sqrt{1-k \theta}}{k}$ or $t \leq \frac{1-\sqrt{1-k \theta}}{k}, \pi_{m}^{1^{*}} \leq \pi_{m}^{2^{*}}$.

For $\pi_{m}^{2^{*}}-\pi_{m}^{3^{*}}=\frac{(1-k t)^{2}}{3}-\frac{2(1+k t)(1-k \theta)(1-2 k t)}{3(2-k \theta)}$, obtain that
when $\frac{1-\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}<t<\frac{1+\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}, \pi_{m}^{2^{*}}<\pi_{m}^{3^{*}} ;$
 obtain that
when $\frac{-1+\sqrt{1+4 k \theta}}{4 k}<t<1, \pi_{m}^{1^{*}}>\pi_{m}^{3^{*}} ;$
when $\frac{-1-\sqrt{1+4 k \theta}}{4 k}<t<\frac{-1+\sqrt{1+4 k \theta}}{4 k}, \pi_{m}^{l^{*}}<\pi_{m}^{3^{*}}$.
Due to the analysis above, can be obtain:
when $0<t<\frac{-1+\sqrt{1+4 k \theta}}{4 k}$, there is $\pi_{m}^{1^{*}}<\pi_{m}^{3^{*}}<\pi_{m}^{2^{*}}$;
when $\frac{-1+\sqrt{1+4 k \theta}}{4 k}<t<\frac{1-\sqrt{1-k \theta}}{k}$, there is $\pi_{m}^{3^{*}}<\pi_{m}^{1^{*}}<\pi_{m}^{2^{*}}$;
when $\frac{1-\sqrt{1-k \theta}}{k}<t<\frac{1-\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}$
and $1-k \theta(6-k \theta) \geq 0$, there is $\pi_{m}^{3^{*}}<\pi_{m}^{2^{*}}<\pi_{m}^{1^{*}}$;
when $\frac{1-\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}<t<\frac{1+\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}$
and $1-k \theta(6-k \theta) \geq 0$, there is $\pi_{m}^{2^{*}}<\pi_{m}^{3^{*}}<\pi_{m}^{1^{*}}$;


Fig. 5: The wholesale price under three buy-back modes
When $\frac{1+\sqrt{1-k \theta(6-k \theta)}}{k(6-k \theta)}<t<1$ and $1-k \theta(6-k \theta) \geq 0$, there is $\pi_{m}^{3^{*}}<\pi_{m}^{2^{*}}<\pi_{m}^{1^{*}}$.

Then we can obtain the theorem 3 , when $0<t<1-\sqrt{ } 1-$ $\mathrm{k} \theta / \mathrm{k}$, for the manufacture, buy-back mode II is the best; when $1-\sqrt{ } 1-k \theta / k<t<1$, for manufacture, buy-back mode I is the best.

## NUMERICAL ANALYSIS

Here we use the MATLAB software to do numerical analysis for describing the three buy-back mode discussed above and comparing the decision variables and profit changes of supply chain members under different buy-back modes and to further verify to the theorems.

Set $\mathrm{k}=0.5, \theta=0.7$, Fig. 5 show the manufacturer's wholesale price under the three buy-back modes, in which the repurchase price that point $A$ represented is $\theta / 4-3 \mathrm{k} \theta$ point $B$ represented is $\theta$. Figure 5 shows that, when the repurchase price is higher than point $B$, the wholesale prices is $\mathrm{w}_{3}{ }_{3}>\mathrm{w}^{*}{ }_{1}>\mathrm{w}_{2}$ under these three modes; when the repurchase price is lower than point $A$, we can get $\mathrm{w}^{*}>{ }_{3}>\mathrm{w}_{1}>\mathrm{w}^{*}{ }_{2}$ from the figure; when the repurchase price is higher than point $A$, but lower than point $B$, by the relationship between the curves we can see that $\mathrm{w}_{3}{ }_{3}>\mathrm{W}^{*}{ }_{2}>\mathrm{W}_{1}$. Figure 5 further verifies Theorem 1.

Figure $6 \mathrm{a}, \mathrm{b}$ represented the retailer 1 and retailer 2's retail price under the three buy-back modes. Figure 6 a shows that for retailer 1, the retail price under these three buy-back modes is $\mathrm{p}^{1^{*}}{ }_{1}<\mathrm{p}^{2^{*}}{ }_{1}<\mathrm{p}^{3^{*}}{ }_{1}$; In Fig. 6b, where point $A$ represents the repurchase price is $3 \theta / 16-11 \mathrm{k} \theta$, while point $B$ represents the repurchase price is $3 \theta / 18-8 \mathrm{k} \theta$; thus to the retailer 2 , when the repurchase price is lower than the point $B$, there is $\mathrm{p}^{3^{*}}{ }_{2}<\mathrm{p}^{1^{*}}{ }_{2}<\mathrm{p}^{2 *}{ }_{2}$; when the repurchase price is higher than the point $A$ but lower than point $B$, there is $\mathrm{p}^{1^{*}}{ }_{2}<\mathrm{p}^{3^{*}}{ }_{2}<\mathrm{p}^{2^{*}}{ }_{2}$; when the repurchase price is higher than the point $B$, there is $\mathrm{p}^{1^{*}}{ }_{2}<\mathrm{p}^{2^{*}}{ }_{2}<\mathrm{p}^{3 *}{ }_{2}$. Figure 6 effectively verifies the conclusions in Theorem 2.

(b)

Fig. 6a-b: The retail price of competitive retailer under the three buy-back modes


Fig. 7: The manufacture buy-back mode choice
Figure 7 is the selection of the manufacturer's buyback modes, where the repurchase price represents by point $A$ is $1-\sqrt{ } 1-\mathrm{k} \theta / \mathrm{k}$, point $B$ represents is $-1=\sqrt{ } 1+$ $4 \mathrm{k} \theta 4 \mathrm{k}$. Since $1-\mathrm{k} \theta(6-\mathrm{k} \theta)<0$, so to the manufacturers, buy-back mode II is always better than buy-back mode I. Figure 7 shows that, when the repurchase price is lower than point $A$, the buy-back mode II is the best; when the repurchase price is higher than point $A$, the manufacturer choose the buy-back mode I as the optimal mode. Figure 7 effectively verifies the conclusions in Theorem 3.

## CONCLUSION

This study focus on the customer strategic returns which is growing in retail market now, abstracted three supply chain buy-back modes from the reality to solve the customer strategic returns, which are the cooperative buy-back mode, independence buy-back mode and asymmetric buy-back mode. The results showed that: there are the differences in the optimal decisions and profits of the manufacturer and retailers under different buy-back modes; the mode choice of the manufacturer depends on the manufacturer's repurchase price, the customer return rate and the retailer buy-back subsidies rate, but the asymmetric mode is never the best mode for the manufacturer.

However, which is worth noticing that, although the manufacturer can improve their profits by choosing the appropriate buy-back mode, but the returns will cause some losses no matter for the consumers, retailers or manufacturers. Therefore, in the long-term view, the manufacturers and retailers should conduct a detailed market survey, to clear the consumer's demand, for meeting the customer expectations and reducing the inconsistency of the value between before and after customer purchase, thus to reduce customer strategic returns and improve the performance of supply chain members.

## ACKNOWLEDGMENT

The study is funded by National Nature and Science Funding (70902044), Tianjin City High School Science and Technology Fund Planning Project (20102127) and Research Development Funding Project of Tianjin University of Finance and Economics (Y1109).

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