Research Journal of Applied Sciences, Engineering and Technology 5(20): 4931-4935, 2013
DOI:10.19026/rjaset.5.4345
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: September 30, 2012 Accepted: December 11, 2012 Published: May 15, 2013

Research Article
Extension VIKOR for Priority Orders Based on Three Parameters Interval Fuzzy Number

[1, 2]Qian Zhang, [1]Guiming Chen, [2]Ning Yan and [2]Baozhong Su
[1]Xi'an Research Institute of High-Tech, Xi'an 710025, China
[2]Beijing Research Institute of High-Tech, Beijing 100085, China

Abstract: In this study, an improved VIKOR method was presented to deal with multi-attribute decision-making based on three parameters interval fuzzy number. The attribute weights were unknown but alternative priority of object preference was given. A new non-linear rewards and punishment method in positive interval was proposed to make the attributes normal, information covered reliability and relative superiority degree two methods were used to compare and sort the Three Parameters Interval Fuzzy Number (TPIFN) and a quadratic programming based on contribution was constructed to get attribute weights, then defined the information entropy distance between TPIFN and the optimum object orders was obtained by VIKOR. The numerical example was provided to demonstrate the feasibility and validity.

Keywords: Multiple attribute decision making, priority orders of preference, rewarding and punishing strategy, three parameters interval fuzzy number, VIKOR

INTRODUCTION

Fuzzy analysis leads an important role in Multi-Attribute Decision Making (MADM) and it has made a great progress. Interval fuzzy number is usually be used to describe the fuzzy value, when it is difficult to describe the attribute value assuredly, but is easy to get the scale. The assumption of average but not fuzzy sets in interval is implicated in common two parameters interval fuzzy number and its lacking of fuzzy degree controlling lead to distortion and departure easily. Therefore some scholars define Three Parameters Interval Fuzzy Number (TPIFN) to overcome the shortcoming above (Bu and Zhang, 2001). The research of for TPIFN has just begun. The information aggregation of TPIFN is investigated (Wang, 2008). The method of three parameters interval grey correlation degree is presented (Luo, 2009). Lan and Fan (2009) define the operation and distance formula of TPIFN and advance the relevant TOPSIS method. But professor Opricovic (1998) indicated that there are bugs in TOPSIS and VIKOR can overcome the shortage and make the decision more in reason as a compromise sort method (Opricovic and Tzeng, 2004). The research of VIKOR based on TPIFN has not been reported for the moment. Besides, the relation research insufficient include: firstly, the existing research all most focus on specific values or relation to describe preference (Zeng, 2005; Xu and Zhao, 2004), but it is less in form of priority orders. Secondly, the normalizing methods are chiefly based on the principles of only award but no punishment, leading to low resolution. Moreover, it brings limitation for data processing because of negative data by rewards and punishment. Thirdly, the distance formulas are most use of Euclidean and other geometrical distance, their discrimination is low. In view of above, improved non-linear rewards and punishment method in [0, 1] is proposed to normalize the TPIFN. With alternative priority of known in advance, information covered reliability and relative superiority degree methods are given and a quadratic programming is constructed to get attribute weights, then TPIFN distance formula based on information entropy is defined and is introduced to VIKOR to make decision.

In this study, an improved VIKOR method was presented to deal with multi-attribute decision-making based on three parameters interval fuzzy number. The attribute weights were unknown but alternative priority of object preference was given. A new non-linear rewards and punishment method in positive interval was proposed to make the attributes normal, information covered reliability and relative superiority degree two methods were used to compare and sort the Three Parameters Interval Fuzzy Number (TPIFN) and a quadratic programming based on contribution was constructed to get attribute weights, then defined the information entropy distance between TPIFN and the optimum object orders was obtained by VIKOR. The numerical example was provided to demonstrate the feasibility and validity.

Corresponding Author: Qian Zhang, Xi' an Research Institute of High-Tech, Xi'an 710025, China

DESCRIPTION OF MODEL

TPIFN is the interval fuzzy number with three parameters, marked as $a \in [\underline{a}, \tilde{a}, \overline{a}], (\underline{a} \le \tilde{a} \le \overline{a})$, endpoint $\underline{a}, \overline{a}$ is the upper and lower bounds and center of gravity $\tilde{a}$ is the most possibility value in the interval. The value range of interval is stabilized and center point is emphasized in TPIFN, it improve the precision defects of interval number. The MADM based on TPIFN, object priority orders are known and attribute weights are unknown, described as:

Object sets, $A = \{A_1, A_2, \cdots, A_n\}$ Attribute sets, $U = \{U_1, U_2, \cdots, U_m\}$ the attribute value of A_i in U_j is $a_{ij} \in [\underline{a}_{ij}, \tilde{a}_{ij}, \overline{a}_{ij}]$. The known preference priority order is $A_{i_1 1} \succ A_{i_2 2} \succ \cdots \succ A_{i_m n}$, $A_{i_k k}$ shows A_{i_k} is in the k-th position. $i_1, i_2, \cdots, i_n$ is the sequence numbers of all object. Decision-maker make determination of object orders based on the information above.

MADM BASED ON EXTENSION VIKOR

Rewards and punishment standardization for TPIFN: The original matrix need to be transformed to eliminate dimension difference and repeated information. The existing "rewarding good and punishing bad" method make the data in [-1, 1], it ensure the justice, but the negative lead to programming data limit. Such as the distance of information entropy after, it demands evaluate data is positive. So improve the non-linear rewarding and punishing transform according to method of "rewarding good and punishing bad", but the normalizing value are still in [0,1]. Based on translation and scaling independence (Peng and Xiao, 2011), the basic thought is: translating and scale [-1, 1] interval, if the objects are better than average, assignment scores for higher than 0.5 and lower contrarily. Suppose:

$$z_j = \frac{1}{3n}\sum_{i=1}^{n}(\underline{a}_{ij}, \tilde{a}_{ij}, \overline{a}_{ij}), j = 1, 2, \cdots, m$$

$$M_1 = \max(\max_{1\le i\le n}(a_{ij}) - z_j, z_j - \min_{1\le i\le n}(a_{ij}))$$

$$M_2 = \max(\max_{1\le i\le n}(a_{ij}) - \alpha_j^*, \alpha_j^* - \min_{1\le i\le n}(a_{ij}))$$

$$M_3 = \max(\max_{1\le i\le n}(a_{ij}) - \beta_j^*, \beta_j^* - \min_{1\le i\le n}(a_{ij}))$$
$$j = 1, 2, \cdots m$$

And I_1 is benefit attribute sets, I_2 is cost attribute sets, I_3 is fixation attribute sets, I_4 is deviation attribute sets:

$$r_{ij} = [(\frac{\underline{a}_{ij} - z_j}{M_1^3})^3 \times \frac{1}{2} + \frac{1}{2}, (\frac{\tilde{a}_{ij} - z_j}{M_1^3})^3 \times \frac{1}{2} + \frac{1}{2}, (\frac{\overline{a}_{ij} - z_j}{M_1^3})^3 \times \frac{1}{2} + \frac{1}{2}]$$
$$u_j \in I_1 \quad (1)$$

$$r_{ij} = [\frac{(z_j - \underline{a}_{ij})^3}{M_1^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(z_j - \tilde{a}_{ij})^3}{M_1^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(z_j - \underline{a}_{ij})^3}{M_1^3} \times \frac{1}{2} + \frac{1}{2}]$$
$$u_j \in I_2 \quad (2)$$

$$r_{ij} = [\frac{(|z_j - \alpha_j^*| - |\underline{a}_{ij} - \alpha_j^*|)^3}{M_2^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(|z_j - \alpha_j^*| - |\tilde{a}_{ij} - \alpha_j^*|)^3}{M_2^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(|z_j - \alpha_j^*| - |\overline{a}_{ij} - \alpha_j^*|)^3}{M_2^3} \times \frac{1}{2} + \frac{1}{2}]$$
$$u_j \in I_3 \quad (3)$$

$$r_{ij} = [\frac{(|\underline{a}_{ij} - \beta_j^*| - |z_j - \beta_j^*|)^3}{M_3^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(|\tilde{a}_{ij} - \beta_j^*| - |z_j - \beta_j^*|)^3}{M_3^3} \times \frac{1}{2} + \frac{1}{2}, \frac{(|\overline{a}_{ij} - \beta_j^*| - |z_j - \beta_j^*|)^3}{M_3^3} \times \frac{1}{2} + \frac{1}{2}]$$
$$u_j \in I_4 \quad (4)$$

$j = 1, 2, \cdots m$, and α_j^* is excellent value, β_j^* is bad value.

The improved methods enlarge the differences between values and it increases the resolving degree.

Attribute weights based on alternative priority: By principle of Linear assignment method, quadratic programming model is constructed based on contribution and aim to make the difference between subjective and objective factors minimum, then compute the weight by Lagrange method.

First of all, get the single ranking alternative of all object in every attribute and construct the single ranking matrix $D = [d_{ij}]_{n\times m}$, $d_{ij} = (1, 2, \cdots, n)$, d_{ij} describe the A_i position of ranking in attribute U_j. There are two methods to get the orders alternative for TPIFN.

First is based on information covered reliability of TPIFN. There are n information in information sets of TPIFN a and E_i is the cover of information i, $a \in \bigcap_{i=1}^{n} E_i$. Suppose the cover of $a_1 \in [\underline{a}_1, \tilde{a}_1, \overline{a}_1]$, $a_2 = [\underline{a}_2, \tilde{a}_2, \overline{a}_2]$ and $a_1 - a_2$ is E_1, E_2 and E separately. $E_1 = [\underline{a}_1, \overline{a}_1]$, $E_2 = [\underline{a}_2, \overline{a}_2]$, $E = [\underline{a}_1 - \overline{a}_2, \overline{a}_1 - \underline{a}_2]$, when a_1 and a_2 are independent mutually. Construct E' and E'', $e' \le 0$;

$\forall e'' \in E'', e'' \geq 0; \forall e' \in E', E' \cup E'' = E$. And $d(E)$ is the length of cover E.

Definition 1: $\theta_1(a_1, a_2) = (1-\alpha)\dfrac{d(E')}{d(E)} + \alpha k(\tilde{a}_1 \leq \tilde{a}_2)$ is the reliability of $a_1 \leq a_2$; $\theta_2(a_1, a_2) = (1-\alpha)\dfrac{d(E'')}{d(E)} + \alpha k(\tilde{a}_1 \geq \tilde{a}_2)$ is the reliability of $a_1 \geq a_2$. $0 \leq \alpha \leq 1$ as a constant reflect the recognition degree of decision-maker for taking center of gravity as compared criterion. is the reliability rate of center of gravity compared with TPIFN:

$$p(a_1 \leq a_2) = \begin{cases} 1 & \tilde{a}_1 < \tilde{a}_2 \\ 0.5 & \tilde{a}_1 = \tilde{a}_2 \\ 0 & \tilde{a}_1 > \tilde{a}_2 \end{cases}$$

$$p(a_1 \geq a_2) = \begin{cases} 1 & \tilde{a}_1 > \tilde{a}_2 \\ 0.5 & \tilde{a}_1 = \tilde{a}_2 \\ 0 & \tilde{a}_1 < \tilde{a}_2 \end{cases} \tag{5}$$

Second is based on relative superiority degree. Define the relative superiority degree about TPIFN.

Definition 2: $a_1 \in [\underline{a}_1, \tilde{a}_1, \overline{a}_1]$, $a_2 = [\underline{a}_2, \tilde{a}_2, \overline{a}_2]$, $p'(a_1 > a_2) = \int_{\underline{a}_1}^{\overline{a}_1} f_1(s)ds \int_{\underline{a}_2}^{\overline{a}_2} f_2(t)dt$ is the relative superiority degree of a_1 towards a_2 and $p'(a_1 < a_2) = 1 - p'(a_1 > a_2)$. $f_1(s)$ and $f_2(t)$ are distribution functions of corresponding TPIFN. the form of triangular membership linear function can be adopted and $f(x)$ is the distribution functions of a about $[\underline{a}, \overline{a}]$:

$$f(x) = \begin{cases} \dfrac{x-\underline{a}}{2(\tilde{a}-\underline{a})(\overline{a}-\underline{a})} & \underline{a} \leq x \leq \tilde{a} \\ \dfrac{x-\overline{a}}{2(\overline{a}-\tilde{a})(\overline{a}-\underline{a})} & \tilde{a} < x \leq \overline{a} \end{cases} \tag{6}$$

Base on the compared relationship of θ, p' with 0.5, the two methods above can be used to make the order in every attribute to construct single order matrix D.

If the object priority orders is $A_1 \succ A_2 \succ \cdots \succ A_n$, it can construct the preference matrix $C = [c_{ik}]_{n \times n}$ and $c_{ik} = 1$ show A_i is at k-th position in priority orders, otherwise $c_{ik} = 0$. $[\pi_{ik}]_{n \times n}$ is the matrix of total contribution for all objects, $\pi_{ik} = \sum_{j=1}^{m} \omega_j \delta_{ijk}$, $i,k = 1,2,\cdots,n$ show the contribution of A_i lie in k-th position in total orders. $\delta_{ijk} = 1$ show A_i is at k-th position of single orders in attribute i, otherwise $\delta_{ijk} = 0$. π_{ik} Describe the objective evaluation about A_i in k position and it has a increasing relationship with possibility about object lie in k-th. Suppose $\eta_{ik} = \mu_i c_{ik}$, it shows the preference degree about A_i is at k-th position and it is subjective preference. μ_i is the weights of A_i and $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$ because of object priority orders.

The total difference between objective evaluation and subjective preference can be described by σ and when it is the minimum, the result achieve optimum. Quadratic programming model is constructed based above:

$$\begin{aligned} &\min \sigma^2 = \sum_{i=1}^{n}\sum_{k=1}^{n}\Big(\sum_{j=1}^{m} \omega_j \delta_{ijk}, -\mu_i c_{ik}\Big)^2 \\ &\text{s.t} \quad \sum_{j=1}^{m} \omega_j = 1, \ \omega_j \geq 0 \\ &\qquad \sum_{i=1}^{n} \mu_i = 1, \ \ \mu_i \geq \mu_{i+1}, \mu_i \geq 0 \end{aligned} \tag{7}$$

Define

$$\Delta_i = \begin{cases} \mu_i - \mu_{i+1} & i = 1,2,\cdots,n-1 \\ \mu_m & i = n \end{cases}$$

(7) can be transform to (8):

$$\begin{aligned} &\min \sigma^2 = \sum_{i=1}^{n}\sum_{k=1}^{n}\Big(\sum_{j=1}^{m} \omega_j \delta_{ijk}, -c_{ik}\sum_{h=i}^{n}\Delta_h\Big)^2 \\ &\text{s.t} \quad \sum_{j=1}^{m} \omega_j = 1, \ \omega_j \geq 0, \ j = 1,2,\cdots,m \\ &\qquad \sum_{i=1}^{n} i\Delta_i = 1, \Delta_i \geq 0, i = 1,2,\cdots,n \end{aligned} \tag{8}$$

ω_j^* can be achieved by solve the model。

VIKOR based on distance of information entropy: The basic thoughts of VIKOR are definition of positive and negative ideal solution and give the orders of object according to the close degree between objects and ideal ones. The distance between TPIFN is the foundation of MADM. The existing researches adopt geometrical distances mostly, but they are difficult to rank the point lie in perpendicular bisector between two objects accurately, it is low in discrimination. Based on information entropy theory, measurement the difference between X and Y by K-L distance, give the definition of the relative entropy distance.

Definition 3: The relative entropy of TPIFN between $a \in [a^1, a^2, a^3]$ and $b = [b^1, b^2, b^3]$ is:

$$H(a,b) = \sum_{i=1}^{3} a^i \log_2 \frac{a^i}{0.5(a^i + b^i)} + \sum_{i=1}^{3} (1-a^i) \log_2 \frac{1-a^i}{1-0.5(a^i+b^i)}.$$

When, $a^i = 0$; $a^i \log_2 \frac{a^i}{0.5(a^i + b^i)} = 0$

when $1-a^i = 0$, $(1-a^i)\log_2 \frac{1-a^i}{1-0.5(a^i+b^i)} = 0$.

Definition 4: The relative entropy distance of TPIFN between $a \in [a^1, a^2, a^3]$ and $b = [b^1, b^2, b^3]$ is: $d(a,b) = H(a,b) + H(b,a)$. And the character of $d(a,b)$ include:

- $d(a,b) \geq 0$; if and only if $a = b$, $d(a,b) = 0$
- $d(a,b) = d(b,a)$

Improve traditional VIKOR based above and the process of extending method is:

- Transform TPIFN matrix to eliminate dimension difference based on rewards and punishment rules, R is achieved:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ & & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix}$$

$$r_{ij} = (\underline{r}_{ij}, \tilde{r}_{ij}, \overline{r}_{ij})$$

- The thought of object priority orders is adopted to construct quadratic programming model based on covered reliability or relative superiority degree two kinds of methods and weights ω_j^* are got
- Get positive and negative ideal solution
- Compute group avail $S_i = [\underline{S}_i, \tilde{S}_i, \overline{S}_i]$ and individual regret value $R_i = [\underline{R}_i, \tilde{R}_i, \overline{R}_i]$ based on relative entropy distance:

$$S_i = \sum_{j=1}^{n} \omega_j \frac{d(r_j^+ - r_{ij})}{d(r_j^+ - r_j^-)}, \forall i \quad R_i = \max_j \left\{ \omega_j \frac{d(r_j^+ - r_{ij})}{d(r_j^+ - r_j^-)} \right\}, \forall i$$

- Achieved compromise value Q_i:

$Q_i = \lambda(\frac{S_i - S^+}{S^- - S^+}) + (1-\lambda)(\frac{R_i - R^+}{R^- - R^+})$, $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$, $R^- = \max_i R_i$, $\lambda \in [0,1]$ is coefficient of decision-making, $\lambda > 0.5$ show decision aim to make the most of S_i; $\lambda = 0.5$ show decision according to make the balanced of S_i and R_i. $\lambda < 0.5$ show decision aim to make the minimum of R_i

- Get the order by Q_i. The value of Q_i is smaller, the corresponding object is more excellent.

CASE STUDY

A repair support shop need to stock a group of spare parts. The influential parameter attribute include: repair of reach ability (U_1), life of spare parts (U_2), failure rate (U_3) and comprehensive cost (U_4). There are 5 types of spare parts can be selected ($A = \{A_1, A_2, A_3, A_4, A_5\}$). The experts group evaluate every spare parts in each attributes and give the scoring matrix by experience:

$$R = \begin{bmatrix} (1.5\ \ 2\ \ 3) & (25\ 26\ \ 28) & (20\ 21\ 22) & (8\ \ 9\ \ 9.5) \\ (1\ \ 1.5\ \ 2) & (9\ \ 9.5\ \ 12) & (7\ \ 7.9\ \ 9) & (6\ \ 6.3\ \ 8) \\ (2\ \ 3\ \ 3.5) & (20\ 21\ 24.5) & (32\ 33\ 34) & (10\ 11\ 12) \\ (4\ \ 4.2\ \ 5) & (9\ \ 11\ \ 15) & (6\ \ 7\ \ 8) & (3\ \ 6\ \ 9) \\ (3\ \ 3.6\ \ 4.5) & (30\ 31\ \ 34) & (9\ \ 10\ 11) & (9\ \ 9.5\ \ 10) \end{bmatrix}$$

Normalized result by (1) , (2), (3) ,(4):

$$\begin{bmatrix} (0.3409\ \ 0.4567\ \ 0.5000) & (0.5199\ \ 0.5356\ \ 0.5882) & (0.4802\ \ 0.4883\ \ 0.4939) & (0.4960\ \ 0.4994\ \ 0.5002) \\ (0.1068\ \ 0.3409\ \ 0.4567) & (0.2148\ \ 0.2510\ \ 0.3866) & (0.5260\ \ 0.5407\ \ 0.5564) & (0.5002\ \ 0.5299\ \ 0.5445) \\ (0.4567\ \ 0.5000\ \ 0.5109) & (0.5000\ \ 0.5000\ \ 0.5142) & (\ 0\ \ \ 0.0780\ \ 0.1473) & (0.3559\ \ 0.4461\ \ 0.4876) \\ (0.5700\ \ 0.6165\ \ 1.0000) & (0.2148\ \ 0.3408\ \ 0.4703) & (0.5392\ \ 0.5564\ \ 0.5778) & (0.4994\ \ 0.5445\ \ 1.0000) \\ (0.5000\ \ 0.5174\ \ 0.7191) & (0.6769\ \ 0.7377\ \ 1.0000) & (0.5091\ \ 0.5161\ \ 0.5260) & (0.4876\ \ 0.4960\ \ 0.4994) \end{bmatrix}$$

Computer based on covered reliability method:

Single order matrix $D = \begin{bmatrix} 4 & 2 & 4 & 3 \\ 5 & 5 & 2 & 2 \\ 3 & 3 & 5 & 5 \\ 1 & 4 & 1 & 1 \\ 2 & 1 & 3 & 4 \end{bmatrix}$

Preference matrix

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Construct quadratic programming model based on (7), (8) and $\omega^* = (0.0625\ ,\ 0.3750,\ 0.2500,\ 0.3125)$ are achieved.

Positive and negative ideal solution:

$$r_j^+ = \begin{bmatrix} (0.5700 & 0.6165 & 1.0000) \\ (0.6769 & 0.7377 & 1.0000) \\ (0.5392 & 0.5564 & 0.5778) \\ (0.4994 & 0.5445 & 1.0000) \end{bmatrix}$$

$$r_j^- = \begin{bmatrix} (0.1068 & 0.3409 & 0.4570) \\ (0.2148 & 0.2510 & 0.3866) \\ (0.5260 & 0.5407 & 0.5564) \\ (0.5002 & 0.5299 & 0.5445) \end{bmatrix}$$

Computer result:
S = (1.1684, 1.1722 , 0.9177, 1.1728 , 1.1713) *0.001, R = (1.1636,1.1677, 0.9131, 1.1679, 1.1664) *0.001, Q = (0.9829 , 0.9982, 0 , 1.0000 , 0.9941)。

Get the order by Q: $A_3 \succ A_1 \succ A_5 \succ A_2 \succ A_4$ scheme 3 is the optimized choice.

CONCLUSION

An improved VIKOR method is given to research MADM with preference alternative priority based on TFIFN. The common normalizing method and distance formula are improved to advance discrimination. The result of example satisfies the fact, it is feasible and effective. And the principle is simple and easy to operate. The method can be applied to many varieties of decision-making.

ACKNOWLEDGMENT

The authors wish to thank the helpful comments and suggestions from my tutors and colleagues in Xi'an research institute of High-tech and Beijing research institute of High-tech.

REFERENCES

Bu, G. and Y. Zhang, 2001. Grey fuzzy comprehensive evaluation method based on interval numbers of three parameters. Syst. Eng. Electr., 23(9): 43-45.

Lan, R. and J. Fan, 2009. TOPSIS decision-making method on three parameters interval-valued fuzzy sets. Syst. Eng. Theory Pract., 29(5): 129-136.

Luo, D., 2009. Decision-making methods with three-parameter interval grey numbers. Syst. Eng. Theory Pract., 29(1): 124-130.

Opricovic, S., 1998. Multi-criteria optimization of civil engineering systems. Ph.D. Thesis, Faculty of Iviil, Yugoslavia, Belgrade.

Opricovic, S. and G. Tzeng, 2004. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. Eur. J. Oper. Res., 156(2): 445-455.

Peng, A. and X. Xiao, 2011. Normalization methods for attribute values in fuzzy multi-attribute decision making with interval number. Machine Des. Res., 27(6): 5-8.

Wang, X., 2008. Information aggregation operators over a continuous three parameters interval argument and their application to decision making. Syst. Eng. Electr., 30(8): 1468-1473.

Xu, Z. and H. Zhao, 2004. A method for fuzzy multi-attribute decision making with preference information in the form of fuzzy reciprocal judgment matrix. Fuzzy Syst. Math., 18(4): 115-121.

Zeng, L., 2005. Fuzzy multiple attribute decision making method with preference values on alternatives. J. Guilin Univ. Electr. Technol., 25(1): 55-58.