Research Article

Analysis of Journal Bearing Performance in Two Dimensions

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Abstract: Most of the analysis that has been done on the Reynolds’ equation which forms the basis for journal bearing performance utilized the simplified assumption due to mathematical complexity but this study aims at analyzing the Reynolds’ equation using the full two dimensional form to find out the performance of the journal bearing without the assumption that the pressure gradient in one axis is negligible. This become necessary because machineries in industries rotate at a very high speed, carrying heavy load on the shaft so the shaft, no matter how perfectly aligned they are at assembly, become misaligned when subjected to these heavy loads and the hydrodynamic pressure is skewed towards the position of minimum film thickness. The pressure is distributed in two dimensions. Previous literature often made use of the long bearing approximation with pressure gradient along the axial direction taken as zero. To accurately predict the performance of journal bearings, the axial direction was taken into consideration in this study. Numerical methods were employed to analyze the two dimensional Reynolds’ equation without vertical flow. Finite Element Method (FEM) and the Finite Difference Method (FDM) were used to find the nodal pressure and the nodal load capacity applying the half Sommerfeld’s boundary condition. The maximum pressure obtained for the bearing considered was 0.3891MPa and the maximum load the bearing can support is 8.1507 X 10^3 N/m.

Keywords: Finite difference, finite element method, journal bearing, load capacity, nodal pressure, pressure distribution

INTRODUCTION

Journal bearings are designed based on the given parameters. This implies that bearings are not generally designed to fit into all operating conditions but for specified operating conditions such as speed and load, geometrical requirements (length, diameter) and also based on the viscosity of the lubricant.

All these factors have effect on the performance of the bearings. The performance of journal bearings has been analyzed in different literatures over the years applying various methods and covering various aspects of study by different researchers most of which made use of the long bearing approximation without the full two dimensional Reynolds’ equation for the sake of simplicity.

Nuruzzaman et al. (2010) carried out a study on pressure distribution and load capacity of a journal bearing using Finite Element Method and Analytical method and concluded that the finite element results showed better agreement to published results than analytical results.

Stefani (2011) described a general purpose Finite Element Method approach to the Thermoelastohydrodynamic (TEHD) analysis of hydrodynamic bearings. He focused on analysis of steady-loaded bearings in laminar lubrication regime. He concluded that “The numerical examples show how the quasi-3D approach has enhanced the reliability of the mass and energy-conserving lubrication analysis proposed by Kumar and Booker”.

Panday et al. (2012) carried out a numerical unsteady analysis of thin film lubricated journal bearing using a three dimensional computational fluid dynamics, they included turbulence effects, the journal modeled with an absolute speed of 3000rpm. They concluded that the maximum pressure the bearing can withstand is increasing with the Length/Diameter ratio.

Mishra (2014) studied the performance characteristics of a rough elliptical bore journal bearing using expectancy model of roughness characterization and he found out that a viscosity varies pressure drop when compared to isothermal case; increase in temperature causes decrease in viscosity.

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Chauhan et al. (2014) also carried a CFD Thermo-hydrodynamic analysis of circular journal bearings where the computational fluid dynamics techniques were applied through ANSYS fluent software. The oil flow was assumed to be laminar and the steady state condition was assumed their work. The effect of variation of pressure and temperature on the lubricant film was considered during the study. Thermo-hydrodynamic analysis was carried out at an eccentricity of 0.6 and a rotational speed of 2500 rpm. During the analysis, the authors found out that due to the consideration of viscosity variation fewer rises in temperature was observed in thermo-hydrodynamic analysis as compared to isothermal analysis.

Mane and Soni (2013) published a paper on the analysis of hydrodynamic plain journal bearing using COSMOL multiphysics 4.3a software and they concluded that COSMOL gives approximately identical solution for both short and long bearings though COSMOL predicts a slightly lower maximum pressure and higher values of eccentricity predicts a slightly higher maximum pressure.

Oghenevwaire and Odi-Owei (2014) carried out an analysis of the performance of journal bearing of a reciprocating compressor using Exxocolub 190 as the cylinder lubricant. They used finite element method to analyse Reynolds’ equation considering the effect of vertical flow for a one film geometry. They concluded that when the operating pressure of the journal bearing is increased up to 5000Psi the maximum pressure and load capacity of the journal bearing reduced by 50%.

They recommended that the study be extended to two dimensional film geometry apply the full Sommerfeld’s conditions to compare with their work.

Mukesh et al. (2012) carried out a research on the Thermohydrodynamic (THD) analysis of a journal bearing using Computational Fluid Dynamics (CFD) as a tool. They concluded that the viscosity if lesser decreases the maximum pressure of the lubricant inside the bearing and recommended THD analysis to measure the performance of journal bearing.

The aim of this study is to analyze journal bearing performance in two dimensional domain using the bearing data from Mukesh et al. (2012) extending it to two dimensional domain applying the half Sommerfeld’s conditions to the full Reynolds’ equation. Finite element method and finite difference method was also used to determine the pressure profile and load capacity of the bearing.

**MATERIALS AND METHODS**

For this research conducted in 2015 at the University of Port Harcourt, Nigeria the bearing under analysis whose parameters are given by Mukesh et al. (2012) was used (Table 1).

### Table 1: Input data for the bearing used in this analysis (Mukesh et al., 2012)

<table>
<thead>
<tr>
<th>Bearing parameters</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of bearing (L)</td>
<td>133 mm</td>
</tr>
<tr>
<td>Radius of shaft (R₁)</td>
<td>50 mm</td>
</tr>
<tr>
<td>Radial clearance (C)</td>
<td>0.145 mm</td>
</tr>
<tr>
<td>Eccentricity ratio (E)</td>
<td>0.61</td>
</tr>
<tr>
<td>Angular velocity (ω)</td>
<td>48.1 Rad/sec</td>
</tr>
<tr>
<td>Lubricant density (ρ)</td>
<td>840 kg/m³</td>
</tr>
<tr>
<td>Viscosity of the lubricant (η)</td>
<td>0.0127 Pa.s [mPa.s]</td>
</tr>
</tbody>
</table>

**Methodology:** The pressure distribution of the journal bearing can be obtained with the aid of the finite element method using the Garlen’s method of weighted residuals and Green’s weak statement formulae to provides a mathematical equivalent of the full two dimensional Reynolds’ equation (Zienkiewicz and Taylor, 2000). The two dimensional Reynolds’ equation would be used by bringing all the terms on the right to the left side as shown below:

\[
R(x,y,P) = \frac{\partial}{\partial x} (h^3 \frac{\partial p}{\partial x}) + \frac{\partial}{\partial y} (h^3 \frac{\partial p}{\partial y}) - 12u_\eta \frac{dh}{dx} [1] \\
\int w_i [h^3 \frac{\partial p}{\partial x} + h^3 \frac{\partial p}{\partial y}] \cdot dxdy [2]
\]

We carry out a term by term integration by parts introducing a scalar weight function w_i with the weighted average set to zero (Kwon and Bang, 1997).

\[P_x = a_i x \text{ (L-x)}\]

Which satisfies the boundary condition of 0<x<L.

\[P_y = a_i y \text{ (B-y)}\]

Which satisfies the boundary condition of 0<y<B.

The weak statement equivalent of the differential equation (Green’s formula) will yield:

\[
\int_{\Gamma} [\frac{\partial}{\partial x} (h^3 \frac{\partial u}{\partial x})] \cdot d\Gamma \equiv - \int_{\Gamma} \frac{\partial u}{\partial x} (h^3 \frac{\partial p}{\partial x}) \cdot d\Gamma + \int w_i h^3 \frac{\partial p}{\partial x} n_x \cdot d\Gamma [3] \\
\int_{\Gamma} [\frac{\partial}{\partial y} (h^3 \frac{\partial u}{\partial y})] \cdot d\Gamma \equiv - \int_{\Gamma} \frac{\partial u}{\partial y} (h^3 \frac{\partial p}{\partial y}) \cdot d\Gamma + \int w_i h^3 \frac{\partial p}{\partial y} n_y \cdot d\Gamma [4]
\]

where,

Ω : The domain
\[Γ : \text{The boundary}\]

Since we are analyzing without vertical flow we thus have:
To achieve effective computation, the element nodal sequence has to be the same direction for all elements in the domain:

\[ p = [1 \ x \ y] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \] (6)

Substitute the x and y values into each of the nodal points we have:

\[ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \] (7)

\[ x_i \text{ and } y_i \text{ are the coordinate values at the } i^{th} \text{ node and } p_i \text{ is the nodal variable} \]

Inverting the matrix Eq. (7) yields:

\[ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} x_2 y_3 - x_3 y_2 & x_3 y_1 - x_1 y_3 & x_1 y_2 - x_2 y_1 \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & y_3 - y_2 & x_2 - x_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \] (8)

where \( A \) is equal to the area of the linear triangular element chosen for the discretization:

\[ A = \frac{1}{2} \det \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \] (9)

\[ p = H_1(x, y)p_1 + H_2(x, y)p_2 + H_3(x, y)p_3 \] (10)

\( H_1(x, y) \) is the shape function for the linear triangular element and is given as:

\[ H_1 = \frac{1}{2A} \left[(x_2 y_3 - x_3 y_2) + (y_2 - y_3)x + (x_3 - x_2)y\right] \] (11)

\[ H_2 = \frac{1}{2A} \left[(x_3 y_1 - x_1 y_3) + (y_3 - y_1)x + (x_1 - x_3)y\right] \] (12)

\[ H_3 = \frac{1}{2A} \left[(x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y\right] \] (13)

These shape function satisfies the condition:

\[ H_i(x_j, y_j) = \delta_{ij} \] (14)

The sum of all the shape function is unity:

\[ \sum_{i=1}^{3} H_i = 1 \]

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \]

The elemental matrix is computed to give:

\[ [K^e] = h^3 \int_{\Omega^e} \left[ \begin{array}{ccc} \frac{\partial H_1}{\partial x} & \frac{\partial H_1}{\partial y} & 0 \\ \frac{\partial H_2}{\partial x} & \frac{\partial H_2}{\partial y} & 0 \\ 0 & 0 & \frac{\partial H_3}{\partial x} \end{array} \right] d\Omega = \int_{\Omega^e} \left[ \begin{array}{ccc} \frac{\partial H_1}{\partial x} & \frac{\partial H_1}{\partial y} & 0 \\ \frac{\partial H_2}{\partial x} & \frac{\partial H_2}{\partial y} & 0 \\ 0 & 0 & \frac{\partial H_3}{\partial x} \end{array} \right] d\Omega \] (15)

where, \( \Omega^e \) is the element domain

Equation 11 to 13 substituted into 15 yields:

\[ [K^e] = h^3 \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \] (16)

where, \( h \) is the film thickness, then for each of the element of the matrix we have:

\[ k_{11} = \frac{1}{4A} \left[ (x_3 - x_2)^2 + (y_2 - y_3)^2 \right] \] (17)

\[ k_{12} = -\frac{1}{4A} \left[ (x_3 - x_2)(x_1 - x_3) + (y_2 - y_3)(y_3 - y_1) \right] \] (18)

\[ k_{13} = -\frac{1}{4A} \left[ (x_3 - x_2)(x_2 - x_1) + (y_2 - y_3)(y_1 - y_2) \right] \] (19)

\[ k_{22} = \frac{1}{4A} \left[ (x_1 - x_3)^2 + (y_3 - y_1)^2 \right] \] (20)

\[ k_{23} = -\frac{1}{4A} \left[ (x_1 - x_3)(x_2 - x_1) + (y_3 - y_1)(y_1 - y_2) \right] \] (21)

\[ k_{33} = \frac{1}{4A} \left[ (x_2 - x_1)^2 + (y_1 - y_2)^2 \right] \] (22)

The element vector is computed using term:

\[ \int_{\Omega^e} W_i \frac{\partial h}{\partial x} \ dxdy \] (23)

Then Eq. (23) is integrated over each linear triangular element it produces:

\[ \int_{\Omega^e} \left[ \begin{array}{c} H_1 \\ H_2 \\ H_3 \end{array} \right] \cdot \frac{\partial w}{\partial x}(x, y) \ d\Omega \] (24)
For the load capacity of the bearing we have:

\[ W = \int_0^B \int_0^L P \, dx \, dy \]

where, \( dx \, dy \) represent the area of the bearing given the boundaries of the bearing.

The long bearing approximation was selected since it is a long bearing using the following formulas to obtain the maximum pressure and its location (Stachowiak and Batchelor, 2006):

\[ P = \frac{6 \omega \eta R^2 (2 + \varepsilon \cos \theta) \sin \theta}{\varepsilon^2 (2 + \varepsilon^2) (1 + \varepsilon \cos \theta)^2} \quad (25) \]

\[ \theta_m = \cos^{-1} \left[ \frac{3 \varepsilon}{2 + \varepsilon^2} \right] \quad (26) \]

Maximum pressure location

The journal bearing is discretized into 50 element having coordinates on both domains as shown in Fig. 1.

**RESULTS AND DISCUSSION**

The pressure profile in both domains is shown in Fig. 2 and 3. As the oil moves from the inlet to the outlet of the bearing, the pressure drops with a maximum pressure value of \( p_{\text{max}} = 0.3891 \text{MPa} \) which fall within the same range produced by the fluent 6.3 software used by Mukesh *et al.* (2012). The pressure drop is as a result of reduction in the hydrodynamic oil film thickness as shown in Fig. 4 and it is also observed that the profile flatten as we move from inlet to the outlet. The maximum load capacity of the bearing occurs at the maximum pressure location with a value of \( 8.1507 \times 10^3 \text{N/m} \).

From the results it shows in Fig. 5 that the pressure decreases across the nodes in the axial direction from inlet of the bearing to the outlet. This shows significantly that the pressure in the axial direction cannot be assumed to the negligible because it has effects on the overall performance of the journal bearing.

**CONCLUSION**

From the results of this analysis of Journal bearing performance in two dimension it can be reached that:

- The hydrodynamic film thickness decreases uniformly from inlet to outlet in both the one dimensional work of Oghenevwaire and Odi-Owei (2014) and this study as shown in Fig. 4.
Fig. 3: Load distribution in the circumferential direction of the bearing

Fig. 4: Pressure profile in the axial direction

Fig. 5: Three dimension plot of the bearing
As shown in Fig. 3, the pressure varies, from inlet to outlet across each node in both directions the load capacity of the bearing also varies.

Practically, the pressure in the axial direction cannot be negligible because of the presence of side flow (side leakage). So the Engineer has been informed that the pressure at various points of the bearing differs over the bearing face from inlet to outlet due to film thickness drop from maximum at the inlet to minimum at the outlet.

There is a very significant discovery which is, the position of the maximum pressure location at node 5, as shown in Fig. 2 shifted from the initial position of 144° at the boundary to 108° (which is closest to the maximum pressure location for a short bearing) which is 36° before its theoretically calculated position and maintain that across the nodes of the bearing to the outlet.

ACKNOWLEDGMENT

The Authors would love to acknowledge the support of Mr and Mrs M.O Mabilogho for their unremitting financial and emotional support all through this study.

REFERENCES


