# Research Article <br> Kinematics Analysis and Modeling of 6 Degree of Freedom Robotic Arm from DFROBOT on Labview 

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#### Abstract

The aim of this study is to analyze the robot arm kinematics which is very important for the movement of all robotic joints. Also they are very important to obtain the indication for controlling or moving of the robot arm in the workspace. In this study the kinematics of ROB0036 DFROBOT Arm will be accomplished by using LabVIEW. Finding the parameters of Denavit-Hartenberg representation, the kinematic equations of motion can be derived which solve the problems of automatic control of the 6 revolute joints DFROBOT manipulator. The kinematics solution of the LabVIEW program was found to be nearest to the robot arms actual measurements.


Keywords: DFROBOT 6DOF robot arm, forward kinematics, inverse kinematics, LabVIEW, robot manipulator

## INTRODUCTION

The kinematics problem is related to finding the transformation from the Cartesian space to thejoint space and vice versa. The solutions of the kinematics problem of any robot manipulatorhave two types; the forward kinematic and inverse kinematics. When all joints are known the forward kinematic will determine the Cartesian space, or where the manipulator arm will be. In the inverse kinematic the calculations of all joints is done if the desired position and orientation of the end- effectors is determined, that means by the inverse kinematic the robotic arm joint space angles will be calculated as referred to Craig (2005).

A six degree of freedom DFROBOT has five rotational joints with a gripper and operate with their servo motors connected as an intersecting or parallel joint axis, it is a low cost educational robot manipulator, flexible and similar to industrial robot arms. In this study the parameters of the standard Denavit Hardenberg listed in Table 1 for the 6DOF Robot Arm shown in Fig. 1 has been used for modeling and simplifying its associated kinematics.

The kinematic analysis of industrial robots was discussed in many literatures (Craig, 2005; Spong et al., 2005). Koyuncu and Guzel (2007) suggested a method for solving the kinematics of the Lynx 6d of Robot and propose a software package named MSG that used to test the behavior of robot motion. Qassem et al. (2010) proposed a software package to solve the kinematics of the AL5B Robot arm. More analysis have been
achieved for modeling a 6dof robotic manipulators using the MATLAB software for their simulation by Iqbal et al. (2012), Kumar et al. (2013) and Singh et al. (2015).

Forward kinematic is much easier than inverse kinematic and so called direct kinematic, Mohammed and Sunar (2015) began their kinematic analysis by using the product of Exponential Formula (PE) to simplify the analysis.

In this study a simple and direct solution to the mathematical model and kinematical analysis of the DFROBOT equations which relate all joints together as refer to the base is achieved. Applying the robot arm kinematics on LabVIEW, the manipulator motion can be introduced with respect to its mathematical analysis.

In this study the target will be on the Kinematics and how to obtain the joint angles from the inverse kinematics modeling that can be used for the control of a variety of industrial processes. The work takes the benefit of using the numeric values for the position and orientation of the end- effector which is the results of the forward kinematics and find all the joint angles of the robot arm from the inverse kinematic solutions applied in the new developed closed form package of the 6 DOF robot which is the case study of this study.

## DFROBOT MODELING

The DFROBOT is a 6DOF robotic arm delivers fast, accurate and repeatable movement and called articulated because it has a series manipulator having

[^0]Table 1: The DH parameters of the DFROBOT

| Joint | Link | $\mathrm{a}_{\mathrm{i}-1} \mathrm{~min}$ | $\alpha_{\mathrm{i}-1}$ degree | $\mathrm{d}_{\mathrm{i}} \mathrm{mm}$ | $\theta \mathrm{i}$ degree |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $0-1$ | 1 | 0 | 0 | 45 | $\theta_{1}$ |
| $1-2$ | 2 | 0 | 90 | 0 | $\theta_{2}$ |
| $2-3$ | 3 | 90 | 0 | 0 | $\theta_{3}$ |
| $3-4$ | 4 | 90 | $0-90$ | 0 | $\theta_{4}$ |
| $4-5$ | 5 | 0 | -90 | $\theta_{5}$ | 30 |
| $5-6$ | 6 | 0 | 0 | 0 | gripper |



Fig. 1: 6DOF robot manipulator


Fig. 2: Kinematic modeling block diagram


Fig. 3: The robot coordinate frame
all joints as revolute. The main features of this kind: base rotation, single plane shoulder, elbow, wrist motion, functional gripper and optional wrist rotate. The kinematic modeling requires the solutions of the forward and inverse kinematics of the manipulator and the link parameters are needed for the two solutions as shown in the block diagram of Fig. 2.

## FORWARD KINEMATICS

The joint variables of the robot are given to determine the position and orientation of the endeffector. Each joint for each frame has a single degree of freedom and can be represented by a single number, which is the angle of rotation in the case of a revolute joint i.e., $\left(\theta_{0}, \theta_{1}, \ldots ., \theta_{n}, \theta_{n-1}\right)$. Starting from the base which is denoted as link 0 to the n links the cumulative effect of the joint variables can be calculated. $z_{i}$ is a unit vector along the axis in space between links i-1 and i. Next, each link is attached with coordinate frames from 1 to $n$, the frame $i$ is rigidly attached to link i. Figure 3 illustrates the DFROBOT frames and links connections.

Assignments of joints and all parameters used to define the robot frames can be defined by using the DH parameters table explained by Tahseen (2013).

Table 1 shows the related six joints parameters of the robotic arm ROB0036 manipulator in order to find the position and orientation of the rigid body which is useful for obtaining the composition of coordinate transformations between the consecutive frames:
where,
$a_{i}$ : The length distance from $z_{i}$ to $z_{i+1}$ measured along $\mathrm{z}_{\mathrm{i}}$
$\alpha_{i}$ : The twist angle between $\mathrm{z}_{\mathrm{i}}$ and $\mathrm{z}_{\mathrm{i}+1}$ measured about $\mathrm{X}_{\mathrm{i}}$
$d_{i}$ : The offset distance from $x_{i}$ to $x_{i+1}$ measured along $\mathrm{Z}_{\mathrm{i}}$
$\theta_{\mathrm{i}}$ : The angle between $\mathrm{x}_{\mathrm{i}}$ and $\mathrm{x}_{\mathrm{i}+1}$ measured about $\mathrm{z}_{\mathrm{i}}$
Forward kinematics analysis is the process of calculating the position and orientation of the endeffector with given joints angles so by substituting these parameters in the homogenous transformation matrix from joint i to joint i+1 (Craig, 2005):

$$
\mathrm{Ai}=\left[\begin{array}{cccc}
C \theta \mathrm{i} & -\mathrm{S} \theta \mathrm{i} C \alpha \mathrm{i} & \mathrm{~S} \theta \mathrm{i} \alpha \alpha \mathrm{i} & \mathrm{aiC} \theta \mathrm{i} \\
\mathrm{~S} \theta \mathrm{i} & C \theta \mathrm{iC} \alpha \mathrm{i} & -\mathrm{C} \theta \mathrm{i} \alpha \mathrm{i} & \mathrm{aiS} \theta \mathrm{i} \\
0 & \mathrm{~S} \alpha \mathrm{i} & C \alpha \mathrm{i} & \mathrm{di} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation matrices $A_{1}$ and $A_{6}$ for the DFROBOT joints can be obtained as shown:

$$
\mathrm{A}_{1}^{0}=\mathrm{A}_{1}=\left[\begin{array}{cccc}
\mathrm{C}_{1} & -\mathrm{S}_{1} & 0 & 0  \tag{1}\\
\mathrm{~S}_{1} & \mathrm{C}_{1} & 0 & 0 \\
0 & 0 & 1 & \mathrm{~d}_{1} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

where, the matrix $A_{1}$ for example shows the transformation between frames 0 and $1, \mathrm{C}_{\mathrm{i}}=\cos \theta_{\mathrm{i}}$ and $\mathrm{S}_{\mathrm{i}}=\sin \theta_{\mathrm{i}}$ :

$$
\begin{align*}
& A_{2}^{1}=A_{2}=\left[\begin{array}{cccc}
\mathrm{C}_{2} & 0 & \mathrm{~S}_{2} & 0 \\
\mathrm{~S}_{2} & 0 & -\mathrm{C}_{2} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{2}\\
& A_{3}^{2}=A_{3}=\left[\begin{array}{cccc}
\mathrm{C}_{3} & -\mathrm{S}_{3} & 0 & \mathrm{a}_{3} \mathrm{C}_{3} \\
\mathrm{~S}_{3} & \mathrm{C}_{3} & 0 & \mathrm{a}_{3} \mathrm{~S}_{3} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{3}\\
& A_{4}^{3}=A_{4}=\left[\begin{array}{cccc}
\mathrm{C}_{4} & -\mathrm{S}_{4} & 0 & \mathrm{a}_{4} \mathrm{C}_{4} \\
\mathrm{~S}_{4} & \mathrm{C}_{4} & 0 & \mathrm{a}_{4} \mathrm{~S}_{4} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{4}\\
& A_{5}^{4}=A_{5}=\left[\begin{array}{cccc}
\mathrm{C}_{5} & 0 & -\mathrm{S}_{5} & 0 \\
\mathrm{~S}_{5} & 0 & \mathrm{C}_{5} & 0 \\
0 & -1 & 0 & \mathrm{~d}_{5} \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{5}\\
& A_{6}^{5}=A_{6}=\left[\begin{array}{cccc}
\mathrm{C}_{6} & -\mathrm{S}_{6} & 0 & 0 \\
\mathrm{~S}_{6} & \mathrm{C}_{6} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{6}
\end{align*}
$$

Performing the composition from the n - th frame to the base frame we multiply the six matrices from 1 to 6 :

$$
A_{n}^{0}=A_{1}^{0} \cdot A_{2}^{1} \ldots . . A_{n}^{n-1}=\prod_{i=1}^{n} \quad A_{i}^{i-1}=\left[\begin{array}{cc}
\mathrm{R}_{\mathrm{n}}^{0} & \mathrm{P}_{\mathrm{n}}^{0} \\
0 & 1
\end{array}\right]
$$

where, R is a $3 \times 3$ matrix for rotation and P is the position, so the total matrix of transformation:

$$
\begin{align*}
& \mathrm{A}_{6}^{0}=\mathrm{A}_{1}^{0} * \mathrm{~A}_{2}^{1} * \mathrm{~A}_{3}^{2} * \mathrm{~A}_{4}^{3} * \mathrm{~A}_{5}^{4} * \mathrm{~A}_{6}^{5} \\
& =\left[\begin{array}{cccc}
\mathrm{n}_{\mathrm{x}} & \mathrm{o}_{\mathrm{x}} & \mathrm{a}_{\mathrm{x}} & \mathrm{p}_{\mathrm{x}} \\
\mathrm{n}_{\mathrm{y}} & \mathrm{o}_{\mathrm{y}} & \mathrm{a}_{\mathrm{y}} & \mathrm{p}_{\mathrm{y}} \\
\mathrm{n}_{\mathrm{z}} & \mathrm{o}_{\mathrm{z}} & \mathrm{a}_{\mathrm{z}} & \mathrm{p}_{\mathrm{z}} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{7}
\end{align*}
$$

where, $p_{x}, p_{y}, p_{z}$ represent the position and $\left\{\left(\mathrm{n}_{\mathrm{x}}, \mathrm{n}_{\mathrm{y}}\right.\right.$, $\left.\left.\mathrm{n}_{\mathrm{z}}\right),\left(\mathrm{o}_{\mathrm{x}}, \mathrm{o}_{\mathrm{y}}, \mathrm{o}_{\mathrm{z}}\right),\left(\mathrm{a}_{\mathrm{x}}, \mathrm{a}_{\mathrm{y}}, \mathrm{a}_{\mathrm{z}}\right)\right\}$, represent the orientation of the end- effector, they can be calculated in terms of joint angles:

$$
\begin{gathered}
\mathrm{n}_{\mathrm{x}}=\mathrm{C}_{6} \mathrm{C}_{12} \mathrm{C}_{345}-\mathrm{S}_{6} \mathrm{~S}_{12} \\
\mathrm{n}_{\mathrm{y}}=\mathrm{C}_{6} \mathrm{~S}_{12} \mathrm{C}_{345}+\mathrm{C}_{12} \mathrm{~S}_{6} \\
\mathrm{n}_{\mathrm{z}}=\mathrm{C}_{6} \mathrm{~S}_{345} \\
\mathrm{o}_{\mathrm{x}}=-\mathrm{C}_{12} \mathrm{~S}_{6} \mathrm{C}_{345}-\mathrm{S}_{12} \mathrm{C}_{6} \\
\mathrm{o}_{\mathrm{y}}=-\mathrm{S}_{6} \mathrm{~S}_{12} \mathrm{C}_{345}+\mathrm{C}_{12} \mathrm{C}_{6} \\
\mathrm{o}_{\mathrm{z}}=-\mathrm{S}_{6} \mathrm{~S}_{345} \\
\mathrm{a}_{\mathrm{x}}=-\mathrm{C}_{12} \mathrm{~S}_{345} \\
\mathrm{a}_{\mathrm{y}}=-\mathrm{S}_{12} \mathrm{~S}_{345} \\
\mathrm{a}_{\mathrm{z}}=\mathrm{C}_{345}
\end{gathered}
$$

$$
\begin{align*}
& p_{x}=a_{4} C_{12} C_{3} C_{4}-a_{4} C_{12} S_{3} S_{4}+S_{12} d_{5}+a_{3} C_{12} C_{3} \\
& p_{y}=a_{4} S_{12} C_{3} C_{4}-a_{4} S_{12} S_{3} S_{4}-C_{12} d_{5}+a_{3} S_{12} C_{3} \\
& p_{z}=a_{4} S_{3} C_{4}+a_{4} C_{3} S_{4}+a_{3} S_{3}+d_{1}  \tag{8}\\
& \text { ere, } \\
& C_{23}=\cos \left(\theta_{2}+\theta_{3}\right), S_{23}=\sin \left(\theta_{2}+\theta_{3}\right), C_{234}= \\
& \cos \left(\theta_{2}+\theta_{3}+\theta_{4}\right) \text { and } S_{234}=\sin \left(\theta_{2}+\theta_{3}+\theta_{4}\right)
\end{align*}
$$

where,

Making use of some trigonometric equations helps for easy solutions:

$$
\begin{aligned}
& \mathrm{C}_{12}=\mathrm{C}_{1} \mathrm{C}_{2}-\mathrm{S}_{1} \mathrm{~S}_{2} \\
& \mathrm{~S}_{12}=\mathrm{C}_{1} \mathrm{~S}_{2}+\mathrm{S}_{1} \mathrm{C}_{2} \\
& \quad \mathrm{C}_{234}=\mathrm{C}_{2}\left(\mathrm{C}_{3} \mathrm{C}_{4-} \mathrm{S}_{3} \mathrm{~S}_{4}\right)-\mathrm{S}_{2}\left(\mathrm{C}_{4} \mathrm{~S}_{3}+\mathrm{C}_{3} \mathrm{~S}_{4}\right) \\
& \mathrm{S}_{234}=\mathrm{S}_{2}\left(\mathrm{C}_{3} \mathrm{C}_{4}-\mathrm{S}_{3} \mathrm{~S}_{4}\right)+\mathrm{C}_{2}\left(\mathrm{~S}_{3} \mathrm{C}_{4}+\mathrm{C}_{3} \mathrm{~S}_{4}\right)
\end{aligned}
$$

## INVERSE KINEMATICS

The solution of Inverse kinematics is more complex than forward kinematics and there is many solutions approach such as geometric and algebraic analysisused for finding the inverse kinematics considering the system structure of the robotic arm. In case of inverse kinematics the joint angles can be determined for any desired position and orientation in Cartesian space. For simplicity of solutions to find the joint angles of 6 d of articulated robot arm of the DFROBOT the transformation matrix in Eq. (7) can be multiplied by $A_{n}^{-1}$ for $n=1, \ldots, 6$ on both sides of the equation sequentially, then solving the produced equations obtained by equating terms of the both sides of matrices:

$$
\left.\begin{array}{l}
\mathrm{A}_{1}^{-1}=\left[\begin{array}{cccc}
\mathrm{C}_{1} & \mathrm{~S}_{1} & 0 & 0 \\
-\mathrm{S}_{1} & \mathrm{C}_{1} & 0 & 0 \\
0 & 0 & 1 & -\mathrm{d}_{1} \\
0 & 0 & 0 & 1
\end{array}\right] \\
\mathrm{A}_{2}^{-1}=\left[\begin{array}{cccc}
\mathrm{C}_{1} & \mathrm{~S}_{2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
\mathrm{~S}_{2} & -\mathrm{C}_{2} & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \\
\mathrm{A}_{3}^{-1}=\left[\begin{array}{cccc}
\mathrm{C}_{3} & \mathrm{~S}_{3} & 0 & -\mathrm{a}_{3} \\
-\mathrm{S}_{3} & \mathrm{C}_{3} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \\
\mathrm{A}_{4}^{-1}
\end{array}\right]=\left[\begin{array}{cccc}
\mathrm{C}_{4} & \mathrm{~S}_{4} & 0 & -\mathrm{a}_{4} \\
-\mathrm{S}_{4} & \mathrm{C}_{4} & 0 & 0 \\
0 & 0 & 1 & 0  \tag{14}\\
0 & 0 & 0 & 1
\end{array}\right] .
$$

## INVERSE KINEMATIC SOLUTIONS

To solve the matrix in Eq. (7) it is easy to use the algebraic solution technique for:

$$
\begin{equation*}
\mathrm{A}_{6}^{0}=\mathrm{A}_{1}^{0} \mathrm{~A}_{2}^{1} \mathrm{~A}_{3}^{2} \mathrm{~A}_{4}^{3} \mathrm{~A}_{5}^{4} \mathrm{~A}_{6}^{5} \tag{15}
\end{equation*}
$$

To solve for $\theta i$ when $A_{6}^{0}$ is given as numeric values, multiply each side by $A_{1}^{-1}$ :
$A_{1}^{-1} *\left[\left[\begin{array}{cccc}n_{x} & o_{x} & a_{x} & p_{x} \\ n_{y} & o_{y} & a_{y} & p_{y} \\ n_{z} & o_{z} & a_{z} & p_{z} \\ 0 & 0 & 0 & 1\end{array}\right]\right]=A_{1}^{-1} * A_{1}^{0} * A_{2}^{1} * A_{3}^{2} * A_{4}^{3} * A_{5}^{4} * A_{6}^{5}$
The matrix manipulations has resulted the following matrix solutions:

$$
\left[\begin{array}{cccc}
. & . & C_{1} a_{x}+S_{1} a_{y} & C_{1} p_{x}+S_{1} p_{y}  \tag{17}\\
. & . & -S_{1} a_{x}+C_{1} a_{y} & -S_{1} p_{x}+C_{1} p_{y} \\
. & . & a_{z} & p_{z}-d_{1} \\
0 & 0 & 0 & 1
\end{array}\right]=\left[\begin{array}{cccc}
. & . & -C_{2} S_{345} & a_{4} C_{2} C_{34}+a_{3} C_{2} C_{3}+S_{2} d_{5} \\
. & . & -S_{2} S_{345} & a_{4} S_{2} C_{34+} a_{3} S_{2} C_{3}-C_{2} d_{5} \\
. & . & C_{345} & a_{4} S_{34}+a_{3} S_{3} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Both matrix elements in Eq. (17) are equated to each other and the resultant $\theta$ values are extracted. By taking $(1,4)(2,4)$ :

$$
\begin{align*}
& \mathrm{C}_{1} \mathrm{p}_{x}+\mathrm{S}_{1} \mathrm{p}_{\mathrm{y}}=\mathrm{a}_{4} \mathrm{C}_{2} \mathrm{C}_{34}+\mathrm{a}_{3} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{S}_{2} \mathrm{~d}_{5}  \tag{18}\\
& -\mathrm{S}_{1} \mathrm{p}_{\mathrm{x}}+\mathrm{C}_{1} \mathrm{p}_{\mathrm{y}}=\mathrm{a}_{4} \mathrm{~S}_{2} \mathrm{C}_{34}+\mathrm{a}_{3} \mathrm{~S}_{2} \mathrm{C}_{3}-\mathrm{C}_{2} \mathrm{~d}_{5} \tag{19}
\end{align*}
$$

Squaring and adding the two Eq. (18) and (19):

$$
\begin{align*}
& \mathrm{C}_{3}=\operatorname{Cos} \theta_{3}=\frac{\sqrt{\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{p}_{\mathrm{y}}^{2}-\mathrm{d}_{5}^{2}}-a_{4} C_{34}}{a_{3}}=\mathrm{n} \\
& \theta_{3}=\operatorname{Cos}^{-1} \mathrm{n}=\operatorname{Atan} 2\left(\mp \sqrt{1-\mathrm{n}^{2}}, \mathrm{n}\right) \tag{20}
\end{align*}
$$

Eq. (3, 4):

$$
\begin{align*}
& \mathrm{S}_{34}=\frac{\mathrm{a}_{3} \mathrm{~S}_{3}-\mathrm{p}_{\mathrm{z}}+\mathrm{d}_{1}}{a_{4}} \\
& \theta_{34}=\operatorname{Atan} 2\left[\frac{\mathrm{a}_{3} \mathrm{~S}_{3}-\mathrm{p}_{\mathrm{z}}+\mathrm{d}_{1}}{a_{4}}, \mp \sqrt{1-\left(\frac{\mathrm{a}_{3} \mathrm{~S}_{3}-\mathrm{p}_{\mathrm{z}}+\mathrm{d}_{1}}{a_{4}}\right)^{2}}\right] \\
& \theta_{4}=\theta_{34}-\theta_{3} \tag{21}
\end{align*}
$$

Multiplying each side of Eq. (15) with $A_{1}^{-1} \mathrm{~A}_{2}^{-1}$ :

$$
\begin{gather*}
A_{1}^{-1} * A_{2}^{-1} *\left[\begin{array}{cccc}
n_{x} & o_{x} & a_{x} & p_{x} \\
n_{y} & o_{y} & a_{y} & p_{y} \\
n_{z} & o_{z} & a_{z} & p_{z} \\
0 & 0 & 0 & 1
\end{array}\right]=A_{3}^{2} * A_{4}^{3} * A_{5}^{4} * A_{6}^{5}  \tag{23}\\
{\left[\begin{array}{cccc}
\cdot & \cdot & \cdot & C_{1} C_{2} p_{x}+C_{1} S_{2} p_{y}+S_{1} p_{z} \\
\cdot & \cdot & \cdot & -S_{1} C_{2} p_{x}-S_{1} S_{2} p_{y}+C_{1} p_{z} \\
S_{2} n_{x}-C_{2} n_{y} & S_{2} \mathrm{O}_{\mathrm{x}}-\mathrm{C}_{2} \mathrm{o}_{\mathrm{y}} & \cdot & \mathrm{~S}_{2} \mathrm{p}_{\mathrm{x}}-\mathrm{C}_{2} \mathrm{p}_{\mathrm{y}}-\mathrm{d}_{1} \\
0 & 0 & 0 & 1
\end{array}\right]=\left[\begin{array}{cccc}
\mathrm{C}_{6} \mathrm{C}_{345} & -\mathrm{S}_{6} \mathrm{C}_{345} & -\mathrm{S}_{345} & \mathrm{a}_{4} \mathrm{C}_{34}+\mathrm{a}_{3} \mathrm{C}_{3} \\
\mathrm{C}_{6} \mathrm{~S}_{345} & -\mathrm{S}_{6} \mathrm{~S}_{345} & \mathrm{C}_{345} & \mathrm{a}_{4} \mathrm{~S}_{34}+\mathrm{a}_{3} \mathrm{~S}_{3} \\
-\mathrm{S}_{6} & -\mathrm{C}_{6} & 0 & \mathrm{~d}_{5} \\
0 & 0 & 0 & 1
\end{array}\right]} \tag{24}
\end{gather*}
$$

Equating elements $(3,4)$ of the right hand side matrix and the left hand side matrix of Eq. (24):

$$
\begin{align*}
& \mathrm{S}_{2} \mathrm{p}_{\mathrm{x}}-\mathrm{C}_{2} \mathrm{p}_{\mathrm{y}}-\mathrm{d}_{1}=\mathrm{d}_{5} \\
& \mathrm{~S}_{2} \mathrm{p}_{\mathrm{x}}-\mathrm{C}_{2} \mathrm{p}_{\mathrm{y}}=\mathrm{d}_{1}+\mathrm{d}_{5} \\
& \theta_{2}=\operatorname{atan} 2\left(\mathrm{p}_{\mathrm{x}},-\mathrm{p}_{\mathrm{y}}\right) \mp \operatorname{atan} 2\left[\sqrt{\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{p}_{y}^{2}-\left(\mathrm{d}_{1}+\mathrm{d}_{5}\right)^{2}},\left(\mathrm{~d}_{1}+\mathrm{d}_{5}\right)\right] \tag{25}
\end{align*}
$$

From Eq. (8) we can obtain:

$$
\mathrm{a}_{\mathrm{x}}=-\mathrm{C}_{12} \mathrm{~S}_{345}
$$

$a_{y}=-S_{12} S_{345}$
Dividing the two equations:

$$
\begin{equation*}
\frac{\mathrm{s}_{12}}{\mathrm{c}_{12}}=\frac{\mathrm{a}_{\mathrm{y}}}{\mathrm{a}_{\mathrm{x}}} \theta_{12}=\operatorname{atan} 2\left(\mathrm{a}_{\mathrm{y}}, \mathrm{a}_{\mathrm{x}}\right) \tag{26}
\end{equation*}
$$

And then we find:

$$
\begin{equation*}
\theta_{1}=\theta_{12}-\theta_{2} \tag{27}
\end{equation*}
$$

Then also equating elements $(3,1)$ and $(3,2)$ of the two sides of the matrices in Eq. $(24)$ :

$$
\begin{align*}
& -S_{6}=S_{2} n_{x}-C_{2} n_{y} \operatorname{OrS}_{6}=C_{2} n_{y}-S_{2} n_{x} \\
& -C_{6}=S_{2} o_{x}-C_{2} o_{y} \operatorname{Or} C_{6}=C_{2} o_{y}-S_{2} o_{x} \theta_{6}=\operatorname{Atan} 2\left[\left(C_{2} n_{y}-S_{2} n_{x}\right),\left(C_{2} o_{y}-S_{2} o_{x}\right)\right] \tag{28}
\end{align*}
$$

Or alternatively:

$$
\begin{equation*}
\theta_{6}=\operatorname{Atan} 2\left[\mp \sqrt{1-\left(\mathrm{C}_{12} \mathrm{o}_{\mathrm{y}}-\mathrm{S}_{12} \mathrm{o}_{\mathrm{x}}\right)^{2}},\left(\mathrm{C}_{12} \mathrm{o}_{\mathrm{y}}-\mathrm{S}_{12} \mathrm{o}_{\mathrm{x}}\right)\right] \tag{29}
\end{equation*}
$$

Now multiply each side of Eq. (15) by:

$$
\begin{align*}
& A_{1}^{-1} * A_{2}^{-1} * A_{3}^{-1} *\left[\left[\begin{array}{cccc}
\mathrm{n}_{\mathrm{x}} & \mathrm{o}_{\mathrm{x}} & \mathrm{a}_{\mathrm{x}} & \mathrm{p}_{\mathrm{x}} \\
\mathrm{n}_{\mathrm{y}} & \mathrm{o}_{\mathrm{y}} & \mathrm{a}_{\mathrm{y}} & \mathrm{p}_{\mathrm{y}} \\
\mathrm{n}_{\mathrm{z}} & \mathrm{o}_{\mathrm{z}} & \mathrm{a}_{\mathrm{z}} & \mathrm{p}_{\mathrm{z}} \\
0 & 0 & 0 & 1
\end{array}\right]\right]=\mathrm{A}_{4}^{3} * A_{5}^{4} * A_{6}^{5}  \tag{30}\\
& {\left[\begin{array}{cccc}
\mathrm{C}_{1} \mathrm{C}_{23} & \mathrm{C}_{1} \mathrm{~S}_{23} & \mathrm{~S}_{1} & -\mathrm{a}_{3} \mathrm{C}_{1} \mathrm{C}_{2} \\
-\mathrm{S}_{1} \mathrm{C}_{23} & -\mathrm{S}_{1} \mathrm{~S}_{23} & \mathrm{C}_{1} & \mathrm{a}_{3} \mathrm{~S}_{1} \mathrm{C}_{2} \\
\mathrm{~S}_{23} & -\mathrm{C}_{23} & 0 & -\mathrm{a}_{3} \mathrm{~S}_{2}-d_{1} \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
\mathrm{n}_{\mathrm{x}} & \mathrm{o}_{\mathrm{x}} & \mathrm{a}_{\mathrm{x}} & \mathrm{p}_{\mathrm{x}} \\
\mathrm{n}_{\mathrm{y}} & \mathrm{o}_{\mathrm{y}} & \mathrm{a}_{\mathrm{y}} & \mathrm{p}_{\mathrm{y}} \\
\mathrm{n}_{\mathrm{z}} & \mathrm{o}_{\mathrm{z}} & \mathrm{a}_{\mathrm{z}} & \mathrm{p}_{\mathrm{z}} \\
0 & 0 & 0 & 1
\end{array}\right]=\text { RHS }\left[\begin{array}{cccc}
\mathrm{C}_{6} \mathrm{C}_{45} & -\mathrm{S}_{6} \mathrm{C}_{45} & -\mathrm{S}_{45} & \mathrm{a}_{4} \mathrm{C}_{4} \\
\mathrm{C}_{6} \mathrm{~S}_{45} & -\mathrm{S}_{6} \mathrm{~S}_{45} & \mathrm{C}_{45} & \mathrm{a}_{4} \mathrm{~S}_{4} \\
-\mathrm{S}_{6} & -\mathrm{C}_{6} & 0 & \mathrm{~d}_{5} \\
0 & 0 & 0 & 1
\end{array}\right]=\text { LHS }} \tag{31}
\end{align*}
$$

Equating elements $(3,4)$ from the two sides of Eq. (31):

$$
\begin{align*}
& S_{23} p_{x}-C_{23} p_{y}-a_{3} S_{2}-d_{1}=d_{5} \\
& S_{23} p_{x}-C_{23} p_{y}=a_{3} S_{2}+d_{1}+d_{5} \\
& \theta_{23}=\operatorname{atan} 2\left(p_{x^{\prime}}-p_{y}\right) \mp \operatorname{atan} 2\left[\sqrt{p_{x}^{2}+p_{y}^{2}-\left(a_{3} S_{2}+d_{1}+d_{5}\right)^{2}},\left(a_{3} S_{2}+d_{1}+d_{5}\right)\right]  \tag{32}\\
& \theta_{3}=\theta_{23}-\theta_{2} \tag{33}
\end{align*}
$$

From the Eq. in (8) we can also obtain:

$$
\begin{align*}
& \mathrm{C}_{345}=\mathrm{a}_{\mathrm{z}} \theta_{345}=\operatorname{atan} 2\left(\mp \sqrt{1-\mathrm{a}_{\mathrm{z}}^{2}}, \mathrm{a}_{\mathrm{z}}\right) \ldots  \tag{34}\\
& \theta_{5}=\theta_{345}-\theta_{3}-\theta_{4} \tag{35}
\end{align*}
$$



Fig. 4: The robot angles


Fig. 5: Forward kinematics simulation with the matrices $\mathrm{A}_{6}$ and $\mathrm{A}_{\mathrm{n}}$ not shown in diagram


Fig. 6: Inverse kinematic simulation
The developed software will calculate the required angles for target orientation and target positioning, the angle values are calculated by using the equations as follows:

$$
\begin{aligned}
& \theta_{2}=\operatorname{atan} 2\left(\mathrm{p}_{\mathrm{x}^{\prime}}-\mathrm{p}_{\mathrm{y}}\right) \mp \operatorname{atan} 2 \\
& \quad\left[\sqrt{\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{p}_{\mathrm{y}}^{2}-\left(\mathrm{d}_{1}+\mathrm{d}_{5}\right)^{2}},\left(\mathrm{~d}_{1}+\mathrm{d}_{5}\right)\right] \\
& \theta_{12}=\operatorname{atan} 2\left(\mathrm{a}_{\mathrm{y}}, \mathrm{a}_{\mathrm{x}}\right) \\
& \theta_{l}=\theta_{12}-\theta_{2} \\
& \theta_{23}=\operatorname{atan} 2\left(\mathrm{p}_{\mathrm{x}},-\mathrm{p}_{\mathrm{y}}\right) \mp \operatorname{atan} 2 \\
& {\left[\sqrt{\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{p}_{\mathrm{y}}^{2}-\left(\mathrm{a}_{3} \mathrm{~S}_{2}+\mathrm{d}_{1}+\mathrm{d}_{5}\right)^{2}},\left(\mathrm{a}_{3} \mathrm{~S}_{2}+\mathrm{d}_{1}+\mathrm{d}_{5}\right)\right]} \\
& \theta_{3}=\theta_{23}-\theta_{2} \\
& \theta_{3}=\operatorname{Cos}^{-1} \mathrm{n}=\operatorname{Atan} 2\left(\mp \sqrt{1-\mathrm{n}^{2}}, \mathrm{n}\right)
\end{aligned}
$$

where,

$$
\left.\left.\begin{array}{l}
\mathrm{n}=\operatorname{Cos} \theta_{3}=\frac{\sqrt{\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{p}_{\mathrm{y}}^{2}-\mathrm{d}_{5}^{2}}-a_{4} C_{34}}{a_{3}} \\
\theta_{34} \\
=\operatorname{Atan} 2\left[\frac{\mathrm{a}_{3} \mathrm{~S}_{3}-\mathrm{p}_{\mathrm{z}}+\mathrm{d}_{1}}{a_{4}}, \mp \sqrt{\left.1-\left(\frac{\mathrm{a}_{3} \mathrm{~S}_{3}-\mathrm{p}_{\mathrm{z}}+\mathrm{d}_{1}}{a_{4}}\right)^{2}\right]}\right. \\
\theta_{4}=\theta_{34}-\theta_{3}
\end{array}\right] \begin{array}{c}
\theta_{345}=\operatorname{atan} 2\left(\mp \sqrt{1-\mathrm{a}_{\mathrm{z}}^{2}}, \mathrm{a}_{\mathrm{z}}\right) \\
\theta_{5}=\theta_{345}-\theta_{3}-\theta_{4} \\
\theta_{6}=\operatorname{Atan} 2
\end{array}\right] \begin{gathered}
{\left[\left(\mathrm{C}_{2} \mathrm{n}_{\mathrm{y}}-\mathrm{S}_{2} \mathrm{n}_{\mathrm{x}}\right),\left(\mathrm{C}_{2} \mathrm{o}_{\mathrm{y}}-\mathrm{S}_{2} \mathrm{o}_{\mathrm{x}}\right)\right]}
\end{gathered}
$$

## RESULTS AND DISCUSSION

Kinematic analysis with mathematical solutions of the 6dof DFROBOT was done in this study. With the Denavit-Hartenberg method and for a given joint angles to be applied as the desired angles for an example (45.92, 75.3, 45.92, 20.20, 90, 31.23) in degrees shown in Fig. 4, forward and inverse kinematic solutions are generated by the developed software with LabVIEW. Figure 5 shows the implementations of these angles to find the total homogenous transformation matrix which contains position and orientation parameters related to the motion kinematic of the robot arm. The other simulator isthe inverse kinematic part with anew analysis technique for the user that use the result parameters of equations (8) being solved to obtain the new joint angles $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}, \theta_{5}$ and $\theta_{6}$ for a given task of the Robot Arm.

For any desired joint frame with the position and orientation shown in total matrix the inverse kinematic nonlinear system solver will be run as shown in Fig. 6.

## CONCLUSION

An analytical solution with a newly developed system solver for the Kinematics of a 6dof Robot Arm from DFROBOT is derived and developed in this study. This model gives correct joint angles so that the robot arm with its end- effector can easily moved to any reachable positions and orientations for performing a pick and place task. Less difference is found between measured and calculated valued which give an exactdesired points in the kinematics simulation process. Also this method can be used to solve kinematics for other robotics arm.

## REFERENCES

Craig, J.J., 2005. Introduction to Robotics: Mechanics and Control. 3rd Edn., Pearson, Prentice Hall, Upper Saddle River, NJ, USA.
Iqbal, J., M.R.U. Islam and H. Khan, 2012. Modeling and analysis of a 6 DOF robotic arm manipulator. Can. J. Electr. Electron. Eng., 3(6): 300-306.
Koyuncu, B. and M.S. Guzel, 2007. Software development for the kinematic analysis of a Lynx 6 robot arm. Int. J. Comput. Electr. Automat. Control Inform. Eng., 1(6): 1575-1580.
Kumar, K.K., A. Srinath, G. Jugalanvesh, R. Premsai and M. Suresh, 2013. Kinematic analysis andsimulation of 6 Dof KukaKr 5 robot for welding application. Int. J. Eng. Res. Appl., 3(2): 820-827.
Mohammed, A.A. and M. Sunar, 2015. Kinematics modeling of a 4-DOF robotic arm. Proceeding of the IEEE International Conference on Control Automation and Robotics. Singapore, pp: 87-91.
Qassem, M.A., I. Abuhadrous and H. Elaydi, 2010. Modeling and simulation of 5 DOF educational robot arm. Proceeding of the 2nd International Conference on Advanced Computer Control (ICACC, 2010). Shenyang, pp: 569-574.
Singh, E.H., N. Dhillon and E.I. Ansari, 2015. Forward and inverse kinematics solution for six DOF with the help of robotics tool box in matlab. Int. J. Appl. Innov. Eng. Manage., 4(3): 17-22.
Spong, M.W., S. Hutchinson and M. Vidyasagar, 2005. Robot Modeling and Control. 1st Edn., John Wiley and Sons Inc., New York.
Tahseen, F.A., 2013. Forward kinematics modeling of 5DOF stationary articulated robots. Eng. Tech. J., 31(3): 500-513.


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