Review Article

Transfer Function Modelling: A Literature Survey

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Abstract: Monitoring and control is very important in maintaining quality outputs, reducing downtimes and improving operation’s efficiency of processes. Transfer function modelling has been the heart of process monitoring and control. This study surveys the past, present and future of transfer function modelling. The survey found that continuous transfer function models used in modelling engineering control systems have the widest applications. Similarly, the discrete transfer function models are applied mostly in forecasting and have wide application in econometrics and the social sciences and to a little extent in the physical and life sciences. The current efforts by researchers in extending the frontiers of discrete transfer function modelling to fault diagnosis, improvement of maintenance and operation’s efficiency, determination of production process capability etc were highlighted. Finally, the future directions of research in transfer function modelling which include but not limited to: leak detection and failure prediction, improved modelling software tools, performance evaluation and improvement in chemical industries, power systems, oil and gas industries, economic and management systems etc was suggested. Our findings show that transfer function would be very important in solving manufacturing dysfunction.

Keywords: Control systems, forecasting, modelling, performance indicators, transfer function

INTRODUCTION

Process monitoring and control is a very active area of research. According to Oakland (2003), a process is the transformation of a set of inputs, which can include materials, actions, methods and operations, into desired outputs, in the form of products, information, services or generally results. All processes, he said, can be monitored and brought under control by gathering and using data. Control refers to measurements of the performance of the process and the feedback required for corrective action, where necessary (Oakland, 2003).

Process control is a statistics and engineering discipline that deals with architectures, mechanisms and algorithms for controlling the output of a specific process. Statistical process control is an effective method of monitoring a process through the use of control charts (Oakland, 2003). Much of its power lies in the ability to monitor both process centre and its variation about that centre.

According to Box et al. (2008), statistical process control which is quite different from engineering process control where various forms of feedback and feed forward adjustment have been used. Statistical process control is employed in industries concerned with the manufacture of discrete parts, while by contrast engineering process control is mostly used in process and chemical industries (Box et al., 2008).

Process monitoring and control makes uses of time series analysis as one of its tools (Box et al., 2008). Engineering process control makes extensive use of transfer function modelling and dynamic system modelling (Coughanowr, 1991; Thomas, 1999). Some of the works done towards improving engineering process control involve time series analysis and its application in transfer function modelling (Box et al., 2008).

Transfer function modelling, an integral part of process monitoring and control, is used to determine the causal relationship between input and output of a process. To buttress this fact, Nwobi-Okoye and Igboanugo (2011, 2012) proposed that in order to better understand the complex relationship between the input and output to the production system, this analytical technique known as transfer function modelling should be used.

Transfer function modelling as has been noted is an integral part of control theory; hence, the history of transfer function modelling is inseparable from the history of automatic control. Automatic control Systems were first developed over two thousand years ago. The first recorded existence of feedback control device is thought to be the ancient Ktesibios’s water...
clock invented by the Greek craftsman, Ktesibios, in Alexandria, Egypt around the third century B.C (Zalewski, 2001). According to Zalewski, Ktesibios’s water clock kept time by regulating the water level in a vessel and, therefore, the water flow from that vessel. This success of this device was evidenced by the fact that water clocks of similar design were still being made in Baghdad when the Mongols captured the city in 1258 A.D. Assorted types of automatic devices have been used over the centuries to accomplish useful tasks or simply used as toys for entertainment. The latter includes the automata, popular in Europe in the 17th and 18th centuries, featuring dancing figures that would repeat the same task over and over again; these automata are examples of open-loop control. Milestones among feedback, or "closed-loop" automatic control devices, include the temperature regulator of a furnace attributed to Drebble, circa 1620 and the centrifugal flyball governor used for regulating the speed of steam engines by James Watt in 1788 (Mayr, 1975). In his 1867 paper "On Governors", J. C. Maxwell (who discovered the Maxwell electromagnetic field equations) was able to explain instabilities exhibited by the flyball governor using differential equations to describe the control system (Maxwell, 1867). This elucidated and demonstrated the importance and utility of mathematical models and methods in understanding complex phenomena and systems and signaled the beginning of mathematical control and systems theory and by extension transfer function modelling (Morari and Zafiriou, 1989). Elements of control theory had appeared earlier but not as dramatically and convincingly as in Maxwell's analysis (Morari and Zafiriou, 1989). Historically, modern continuous transfer function modelling started with advent of modern control systems at the onset of the industrial revolution. The industrial revolution started with design and development of various types of machinery and equipment for industrial and home use (Ashton, 1997; Jensen, 2010). Most of these machineries and equipment contain control and instrumentation systems, of which the engine governor earlier mentioned is one of the earliest examples. Control theory made significant strides in the next 100 years since Maxwell’s paper in 1867. New mathematical techniques made it possible to control, more accurately, significantly more complex dynamical systems than the original flyball governor. These techniques include developments in optimal control in the 1950s and 1960s, followed by progress in stochastic, robust, adaptive and optimal control methods in the 1970s and 1980s (Mayr, 1975; Morari and Zafiriou, 1989). Applications of control methodology have helped make possible space travel and communication satellites, safer and more efficient aircraft, cleaner auto engines, cleaner and more efficient chemical processes, to mention but a few (Albarbar et al., 2008; Gebel and Fettis, 1974; Feron et al., 1998; Coughanowr, 1991).

On the other hand, historically, efforts at relating the input to output of a system statistically started with regression analysis (Lai, 1979). Hence, regression analysis formed the basis of the traditional statistical method of modelling the relationship between the input and output to systems (Lai, 1979). Regression analysis has many deficiencies. For example: Regression Analysis is inappropriate in situations where the output lags the input and there exists a transient relationship between the input and output (Lai, 1979). Also, when there is a significant amount of noise in the system regression analysis cannot accommodate noise in the filter (Box et al., 2008).

Box and Jenkins (1970) introduces an improved statistical method of modelling the relationship between input and output to a system. This method was named Box-Jenkins transfer function modelling methodology (Lai, 1979).

Since the introduction of transfer function modelling in 1970, efforts have been made by various researchers to improve and extend its application in various fields of life. For example Lai (1979) applied it extensively in modelling geographical systems. DeLurgio (1998) applied it extensively in econometrics and economic forecasting. This literature review will bring to the fore the efforts of various other researchers, engineers and scientists on transfer function modelling.

In continuation, since parameter estimation is determined stochastically in transfer function modelling and fewer parameters are required in transfer function models than in regression models, this implies that transfer function models are expected to be more accurate than comparative regression models.

Furthermore, one of the major reasons for transfer function modeling is for forecasting the output from a process. According to Box et al. (2008), transfer function model forecasts usually have smaller forecasting errors than the forecasts based on univariate models which are based on the output. This assertion was further corroborated by Lai (1979) who said that since a transfer function model represents stochastic-dynamic model of the transient relationship, it usually gives good forecasts of the future output from a process. In the words of DeLurgio (1998), transfer function models have a general advantage over other multivariate and univariate models and their greatest advantage exists when lagged independent variables affect the dependent variable.

According to Box et al. (2008), estimation of transfer functions in industrial systems is made for the following purposes:

- To achieve better control of existing plants
- To improve the design of new plants
In spite of its popularity, the identification or specification stage of transfer function analysis is not sufficiently developed, in particular for multiple-input models (Liu and Hanssens, 1982).

According to Liu and Hanssens (1982), the best known transfer function identification technique was due to Box and Jenkins (1970), but their method could not be generalized for the multi input models. However, works by other researchers have generally improved the transfer function model identification especially for the multi input models. These works include: Priestly (1971), who used double pre-whitening of both input and output series, Haugh and Box (1977), used the study of the SCCF of the pre-whitened input output series to identify the transfer function models. Others are Fask and Robinson (1977) who generalized the double prewhitening method to multivariate dynamic models, Granger and Newbold (1977a, 1977b) who applied this to identification of two way causal systems.

Liu and Hanssens (1982) stated that although double pre-whitening has been widely used, it is more difficult to apply than Box and Jenkins method, even in the one input situation. The major difficulties according to them are that a model is frequently over-structured due to the pre-whitening factors and the lag structure is often difficult to derive.

Tiao et al. (1979), used sample cross-correlation and partial correlation matrices of a set of multiple series for the identification of multiple ARIMA models of which transfer function model is a special case. However, their method has a major drawback because it imposes an extra structure between the factors on the residuals and the transfer function causing the transfer function model to be over structured.

Box and Jenkins (1976) and Priestly (1971) used the frequency domain approach (spectral method) instead of the time domain approach to multi input transfer function model identification. However, spectral method is difficult to apply in practice.

Liu and Hanssens (1982) adapted the least squares methods developed by Priestly (1969) and Caines et al. (1977) which worked very well for single input situations to multi input systems. Liu and Hanssens method is based on the least squares estimate of the original and filtered series and the corner method is used to identify a parsimonious rational form of the transfer function.

This study surveys the literature on transfer function modelling, highlighting its potential applications in solving industrial problems.

THEORETICAL BACKGROUND

Before reading on it is necessary to have background knowledge of the theory of Transfer function. Transfer function modelling is one of the major areas where time series analysis is applied. A transfer function (also known as the network function) is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a (linear time-invariant) system. Transfer function modelling is used in measuring the transient input-output relationship of non equilibrium systems. According to Lai (1979), transfer function is often used to determine the causal relationship between two variables.

Mathematically in its simplest form a linear transfer function model is represented by:

\[ Y_\infty = gX \]  

Here

\[ Y_\infty = \text{The steady state output} \]

\[ g = \text{The steady state gain} \]

\[ X = \text{The steady state input} \]

According to Lai (1979), the major objective of the investigation of input-output relationship is to model the filter of the system and if the internal mechanism of the system is completely known, the filter can be represented deterministically by differential equations. Continuous transfer function models assume that transfer function observation are taken at continuous time intervals. What this means is that the period in between observation is so small that the time is assumed to be continuous. Continuous transfer function models are usually represented by differential equations which could be first order equations, second order equations or higher order equations (Box et al., 2008). According Box et al. (2008), the general method for representing continuous dynamic systems is the differential equation:

\[ (1 + \xi_1 D + \ldots + \xi_R D^R) Y_t = g(1 + H_1 D + \ldots + H_R D^R) X_{t-h} \]  

On the other hand, discrete transfer function models assume that the time series of transfer function observations are taken at discrete time intervals. Discrete transfer function models are usually represented by difference equations which could be first order equations, second order equations or higher order equations (Box et al., 2008). As stated by Box et al. (2008), the general method for representing discrete dynamic systems in difference equation form is:

\[ (1 + \xi_1 \nabla + \ldots + \xi_R \nabla^R) Y_t = g(1 + \eta_1 \nabla + \ldots + \eta_R \nabla^R)X_{t-b} \]  

Using the backward shift operator on Eq. (3), (4) is obtained:

\[ (1 - \delta_1 B - \ldots - \delta_R B^R) Y_t = (\omega_0 + \omega_1 B + \ldots + \omega_S BS)X_{t-b} \]  

\[ \delta(B)Y_t = \omega(B)X_{t,b} \]  

Equation (5) can be transformed into the following form:
\[
\delta(B)Y_t = \omega(B)B^bX_t
\]

Let:

\[
\Omega(B) = \omega(B)B^b
\]  

(7)

\[
\delta(B)Y_t = \Omega(B)X_t
\]  

(8)

\[
Y_t = \delta^{-1}(B)\Omega(B)X_t
\]  

(9)

APPLICATIONS OF TRANSFER FUNCTIONS

Transfer function concept has been applied in many fields including: economics, management, engineering, education, computer science, sociology, biology etc (Box et al., 2008; DeLurgio, 1998). The various applications of transfer function will be briefly discussed in this section.

Applications of transfer function in production:

Most applications of time series analysis and transfer functions to production in the surveyed literature have to do with quality control of produced goods. Webb and Hardt (1991) developed a transfer function modelling for quality control of sheet metal forming process. Fearn and Maris (1991) developed a model based on Box-Jenkins methodology for the feedback control of the addition of gluten during the milling of flour. Gluten, an animal protein, is a very essential constituent of flour and determines to a large extent its quality. Bulgrin and Richards (1995) and Tsai and Lu (1998) developed empirical models in the form of time series or Auto-Regressive Moving Average (ARMA) for the quality control of the plastic injection molding process. Most of the models involved using time series to model the output from a process and determining the appropriate feedback or feed forward adjustment in order to bring the process in control (Box and Luceno, 2001).

Hatch and Kazmer (2001) developed a methodology using the Design Of Experiments (DOE) for process characterization of DVD manufacture through transfer functions. In their study, they suggested that transfer function can be used to optimise a process, compare different processing systems, or predict the output quality from input processing conditions. As a result DVD manufacturers can increase productivity through cycle time reduction, improve quality, compare materials, or compare different machine and mould manufacturers. This study represents a first attempt to use transfer functions to estimate process capability, although the work is limited to only DVD manufacturing process and Box-Jenkins transfer function methodology was not used in their work. There is therefore the need to use Box-Jenkins transfer function methodology and extend this to other manufacturing processes. Furthermore, the process capability was not measured quantitatively.

Box et al. (2008) applied transfer function models to the monitoring and control of the production of carbon IV oxide in a gas furnace. Similar applications are used in industry in the monitoring of the production of chemicals. Schwartz and Rivera (2010) applied transfer function modelling to solving problems of production inventory management.

\[
\begin{align*}
Y_t &= \delta^{-1}(B)\omega_1(B)X_{1,t-b} + \cdots + \\
&\quad \delta^{-1}(B)\omega_m(B)X_{m,t-b} + N_t
\end{align*}
\]  

(15)
According to DeLurgio (1998) transfer functions are applied to production to determine the causal relationship between process output and input voltage, process output and material thickness, part dimensions and process adjustments, part dimensions and turning speed etc. In spite of all these novel applications, it has not been applied to maintenance of production facilities or to the determination of production process capability quantitatively.

Applications of transfer function in management, social and economic sciences: There are numerous applications of transfer function models in economics, business, management, finance and the social sciences. Edlund and Karlsson (1993), applied transfer function modelling in predicting unemployment in the Swedish economy. Nogales and Conejo (2006), Zareipour et al. (2006) and Garcia-Martos et al. (2007) successfully developed a transfer function model for predicting electricity prices. Disney and Towill (2002) and Schwartz et al. (2005) applied transfer function modelling to supply chain management problems. Preciado et al. (2006) applied transfer function modeling in agricultural economics, specifically they used the model to obtain a census of fish catches and efforts of their fleets which are considered suitable for the elaboration of representative indices of catch rate (catch per unit effort). Johnson and Turner (1956) used transfer function model to determine the relationship between engineering efforts in an enterprise and sales in the enterprise which is used in company planning. Pardo et al. (2002) in their study developed a transfer function intervention model for forecasting daily electricity load from cooling and heating degree-days in Spain. They proved the influence of weather and seasonality and showed that it is significant even when the autoregressive effects and the dynamic specification of the temperature are taken into account.

Zvirblis (2003), formalized models of automated control schemes of transport services expressing the transfer functions of feedback loops and their characteristic equations.

Sometimes transfer function model could be used to analyse response of the populace to enactment of certain laws. Wilson et al. (2007) investigated whether a law in New Zealand making all indoor workplaces including bars and restaurants smoke free which became operational in New Zealand in December 2004 increased calls to the Quitline Service which is a national free-phone Quitline Service which has been operational since 1999 in New Zealand. They concluded that this would suggest there is an extra opportunity for health agencies to promote quitting at such times.

Transfer function modeling could be applied in environmental management. An important application of transfer function modeling to pollution control and environmental management is exemplified by the work of Issarayangyun and Greaves (2006), who used time series Box-Jenkins transfer function modelling as a tool to assess the relative importance of travel speed (which is known as a proxy for traffic conditions) and meteorological conditions on Fine airborne Particulate Matter (PM) exposure level.

Camargo et al. (2010) applied transfer function modeling to study the Brazilian inflationary process and suggested that Governmental controls of the price level should be avoided since they disrupt expectations and inevitably lead to higher inflation rates. Several applications of transfer functions in economics and business involve determining the causal relationship between business/economic variables such as the relationship between advertising and sales, prices and sales, sales and seasons of the year, earnings and stock prices etc. Cooray (2006), researched on the importance of dynamic transfer function models on analysis and evaluation of econometric time series models. Choudhury (2007) used intervention analysis for the event period using transfer function modeling to observe the direction of the effect of the United States Tax Reform Act and its magnitude on business failures. Berument et al. (2006) used transfer function modeling to analyze the relationship between the Turkish stock market performance and the performance of Turkish teams in the European football competitions. There are numerous literature on the application of transfer functions in economics, finance and business. Such applications could be found in DeLurgio (1998), Enders (1995), Kelejian and Oates (1974), Pindyck and Rubinfeld (1976), Limanond et al. (2011) and Forst (2011) etc.

Application of transfer function in biology and life sciences: Transfer function models have wide applications in the life sciences. Chance et al. (1985) developed the concept of transfer function for organ performance (work output vs. biochemical input) for skeletal and cardiac muscle under steady-state exercise conditions. Zhang et al. (1998) used transfer function analysis to model dynamic cerebral autoregulation in humans.

Loha and Lindtjorn (2010) carried out a study to determine if variations in specific meteorological factors are able to consistently predict P. falciparum malaria incidence at different locations in south Ethiopia. They collected retrospective data from 42 locations including P. falciparum malaria incidence for the period of 1998-2007 and meteorological variables such as monthly rainfall (all locations), temperature (17 locations) and relative humidity (three locations). They subsequently used Time series modeling using Transfer Function (TF) models and univariate Auto-Regressive Integrated Moving Average (ARIMA) when there was no significant predictor meteorological variable. From
their research they concluded that past P. falciparum malaria incidence appeared to be a superior predictor than meteorology.

Yingthawornsuk (2003) and Fagergren et al. (2000) applied it in life sciences to study muscle behaviour in humans. For other applications of transfer function modelling life sciences (Bédard et al., 2010).

It is obvious from the foregoing that transfer function modelling has very useful applications in the life sciences and there exists extensive literature on its applications in the life sciences some of which were reviewed above.

Application of transfer function in engineering and physical sciences: There are numerous applications of transfer function modelling in engineering and physical sciences. As a matter of fact engineering and physical science applications represent the widest application of transfer function modelling. Transfer function modelling in engineering often involves the use of frequency based approach (Box et al., 2008; Shumway and Stoffer, 2006).

Often electrical engineers carry out transfer function analysis based on the frequency response of electrical circuits based on these analysis they could compare different circuit configurations. According to Hernandez (2008), the bandwidth, slew-rate, noise, input impedance and gain, among other characteristics of the operational amplifier, are often the performance limiting factors of photometer circuits. For this reason, he carried out in his study a comparative analysis between two photodiode amplifier circuits using input output transfer function analysis based on the frequency response of a photometer circuit based on operational amplifier (op amp). One circuit is based on a conventional current-to-voltage converter connection and the other circuit is based on a robust current-to-voltage converter connection. His results showed that the photodiode amplifier performance can be improved by using robust control techniques. The work of Hernandez (2008) shows the typical application of transfer function modelling in engineering.

Albarbar et al. (2008) carried out research to compare the performance of MEMS accelerometers for condition monitoring using input-output transfer function analysis based on frequency responses, as well as other performance indicators. They investigated the performances of three of the MEMS accelerometers from different manufacturers and compared them to a well calibrated commercial accelerometer which was used as a reference for MEMS sensors performance evaluation. Tests were performed on a real CNC machine in a typical industrial environmental workshop and the achieved results showed that MEMS sensors could be a good alternative to standard sensors mainly for wireless implementation as there is no need to carry heavy charge amplifiers, but the choice has to be made according to specifications and through validation tests. This study equally represented another important application of transfer function modelling in engineering.

For applications of transfer functions in chemical engineering and chemical process control see Chang et al. (1992) and Huang and Yu (1989). Olason and Watt (1985) used multivariate transfer function model to forecast the average daily flow of a river in Canada. Otok (2009) successfully applied transfer function models in forecasting rainfall in various locations in Indonesia. The inputs to his transfer function models were Dipole Mode Index (DMI) and Sea Surface Temperature (SST) which were used to forecast rainfall.

Talon et al. (2009) applied transfer function modeling in speech recognition application and speech signal processing. Gebel and Fettis (1974) applied transfer function modelling in image processing applications. Feron et al. (1998) successfully applied transfer function modeling to aeronautics in an attempt to better identify the aeroelastic behavior of NASA Dryden’s F-18 systems research aircraft and to predict its flutter boundaries using in-flight experimental data.

Lai (1979) wrote on the applications of transfer function models in geography. In addition to giving an extensive write-up on the methodology of its application, he surveyed the literature of some authors/researchers that have applied transfer function models in geography and found out that transfer function models is very useful in geographical modelling. For other applications of transfer function modelling in physical sciences and engineering (Cluckie et al., 1982; Cluckie, 1993; Toth et al., 2000; Lees, 2000; Young, 2004; Hong et al., 2010; Sample et al., 2011; Rodriguez et al., 2011).

Transfer functions in education: Outside the traditional application of transfer functions in science and engineering, transfer functions could be applied in education to model the dynamic relationship between the impartation of knowledge from teachers and learning/acquisition of knowledge by students. Some authors like Losenický (2010a, 2010b) modelled the educational process as linear control system. His model assumed the controller to be the teachers and the controlled object to be students. His work as he reported proved that the agreement between results of his mathematical modelling and practical knowledge is surprisingly identical. For another application of transfer function modelling to education see Stepanov et al. (2010).

Transfer functions and game theory: There is a rich literature on game theory. Several authors such as Binmore (2007), Von Stengel and Turocy (2008), Von Stengel (2008), Gibbons (1992), Osborne (1998),

In the rich literature on game theory, attempts to formally relate transfer functions to game theory has been scanty. Although some authors such as Nwobi-Okokeye and Igboanugo (2011) modelled some Bayesian games where the transfer function of a system was assumed to be its type.

Transfer functions and maintenance: The fact that the transfer function is an integral part of control systems and could be used as a performance indicator and condition monitor, means that it could be used in planning maintenance and replacement. Transfer function modelling is a forecasting technique and as pointed out by Razak et al. (2009) forecasting using time series analysis could be used in maintenance scheduling. There is a rich and extensive literature on maintenance. Several authors which include Dhillon (2002), Dhillon and Liu (2006), Zhao et al. (2006), Zhou (2006), Leou (2006), Chen and Liao (2005), Petcu and Faltings (2006), Garg and Deshmukh (2006), Koomsap et al. (2005), Mytakidis and Vlachos (2008), Al- Anzi and Al-Zamel (2005) and Limbourg and Kochs (2005) etc have researched and written on maintenance. The works of these authors has helped shape and define the direction of modern maintenance practices. An issue related to maintenance effectiveness is plant performance. Recently attempt has been made to formally link transfer function to maintenance scheduling and plant performance evaluation (Nwobi-Okoey and Igboanugo, 2012; Nwobi-Okoey and Igboanugo, 2015; Nwobi-Okoey et al., 2015).

Transfer functions and process capability measurement: There are many literatures on quality control and metrics of measuring the capability of a process. Several authors have researched and written about the process capability measurement indices. These researchers include: Oakland (2003), Kotz and Lovelace (1989), Grant and Leavenworth (1996), Owen (1993), Pyzdek (1990), Aquilana and Chase (1999), Wheeler and Chambers (1992), Kazmer and Hatch (2002) and Kazmer (2008), Czarski (2008), Abdolshah et al. (2009a, 2009b), Jeang and Chung (2009), Flaig (2009), Jeang et al. (2008) and Sadigh et al. (2009) etc. It is noteworthy that in none of the surveyed literature was transfer function ever used as a metric for accessing the capability of a process, but in a recent work, Igboanugo and Nwobi-Okokeye (2011a) used transfer function in process capability measurements.

Transfer functions and artificial neural network: Many papers have been written on the application of ANNs to time series analysis. Kuligowski and Ana (1998) in their study described a simple precipitation forecasting model based on artificial neural networks. Their model used the radiosonde-based 700-hPa wind direction and antecedent precipitation data from a rain gauge network to generate short-term (0-6 h) precipitation forecasts for a target location. Guh and Hsieh (1999) in their study proposed an artificial neural network based model, which contains several back propagation networks, to both recognize the abnormal control chart patterns and estimate the parameters of abnormal patterns such as shift magnitude, trend slope, cycle amplitude and cycle length, so that the manufacturing process can be improved. Their numerical results showed that the proposed model also has a good recognition performance for mixed abnormal control chart patterns.

Maier and Dandy (2000) reviewed 43 papers dealing with the use of neural network models for the prediction and forecasting of water resources variables in terms of the modelling process adopted. They identified inadequate model building as the obstacle mitigating against accurate predictions using artificial networks. They suggested that ANN models must be properly evaluated before its application in time series analysis.

Their assertion is corroborated by Chatfield (1993) when commenting on the suitability of ANNs for time series analysis and forecasting, who commented thus: “when the dust has settled, it is usually found that the new technique is neither a miraculous cure all nor a complete disaster, but rather an addition to the analyst’s toolkit which works well in some situations and not in others”.

Optimising time series and transfer function models: He et al. (1996) presented a novel approach to assist the user in exploring appropriate transfer functions for the visualization of volumetric datasets. They treated the search for a transfer function as a parameter optimization problem and addressed it with stochastic search techniques. Starting from an initial population of (random or pre-defined) transfer functions, the evolution of the stochastic algorithms is controlled by either direct user selection of intermediate images or automatic fitness evaluation using user-specified objective functions. Their approach essentially shields the user from the complex and tedious "trial and error" approach and demonstrates effective and convenient generation of transfer functions. Igboanugo and Nwobi-Okoey (2011b) used genetic algorithm to optimize transfer function model developed for a production process. Other researchers such as Sheta and De Jong (2001), Wu and Chang (2002), Mani (1996), Ong et al. (2005), Gaetan (2000), Versace et al. (2004), Min et al. (2006), Vossa and Feng (2002) and Cordon et al. (1999) applied stochastic optimisation in time series modelling. The work of He et al. (1996) and other researcher mentioned above corroborated to the fact that optimisation of time series models could be done
by any of the metaheuristics methods discussed above, but according to the “No Free Launch Theorem” by Wolpert and Macready (1997), the averaged performance of all search algorithms over all problems is equal. In other words, no search algorithm performs better in all problems, it is left for the modeller to find the algorithm that best suites a particular problem at hand (Haupt and Haupt, 2004).

**Evaluation of previous researches:** Box et al. (2008) did a very significant work in transfer function modelling by introducing ARIMA and noise models into transfer function modelling. This significantly improved the efficiency and reliability of transfer function models. In addition to this, transfer function model forecasts usually have smaller forecasting errors than the forecasts based on univariate models which are based on the output and gives good forecasts of the future output from a process which is very significant (Kinney, 1978; Nwobi-Okoye and Igboanugo, 2015).

The use of artificial neural network in the modelling brought more tools which improved transfer function models when properly utilized.

Meta-heuristic equally improved the efficiency of transfer function modelling through natural optimisation algorithms.

The application of transfer function models in quality control of industrial processes is very significant indeed and it is invaluable in quite a lot of industries ranging from food, chemical, metal industries etc.

Transfer function modelling is at the heart of process control and control engineering. Hence, it is invaluable in aeronautics, communications, power engineering, automobile engineering, systems engineering etc.

Generally, most application of transfer function modelling appears to be centred on forecasting and system dynamic response analysis.

**Merits of previous studies:** From the literature, the merits of previous studies on transfer function modelling are numerous. First, in comparison to regression analysis, transfer function modelling has many characteristics which make it superior to regression analysis in modelling the relationship between system input and output. First it can model appropriately and accurately the input-output relationship in transient systems where the output lags the output. Secondly, it can accommodate effectively systems with noisy filters (Box et al., 2008). Thirdly, parameter estimation is determined stochastically in transfer function modelling. Also, fewer parameters are required in transfer function models than in regression models, hence, models have better parsimony than regression models (Lai, 1979).

Again, as has been mentioned earlier, continuous transfer function models are at the heart of engineering process control. Thus controlling complex engineering systems such as rockets, airplanes, helicopters, CNC machine tools, automobiles, robot, chemical reactors etc depends largely on transfer function modelling (Dunn, 2005; Albarbar et al., 2008; Gebel and Fettis, 1974; Feron et al., 1998; Coughanour, 1991).

Discrete transfer function modelling has been applied to very successfully in econometric modelling and in other fields primarily in forecasting outputs from inputs (Box et al., 2008; Lai, 1979; DeLurgio, 1998; Cluckie et al., 1982; Cluckie, 1993; Toth et al., 2000; Lees, 2000; Young, 2004). From the literature, forecasts from transfer function models are more accurate than corresponding forecasts from univariate statistical models.

In general, transfer function models are extensively applied in studying the dynamic response of systems subjected to various forms of input such as: step input, impulse, ramp input etc. This dynamic response analysis is found in various fields such as: economics, engineering, biology and many other fields.

**Nature of existing research:** From the literature, existing research on continuous transfer function modelling is largely centred on developing and improving continuous transfer function models for engineering process control.

The nature of existing research in discrete transfer function modelling exists largely in the field of econometrics and physical sciences. It is usually applied to forecasting economic variables in econometrics. Currently there are applications of discrete transfer function models in other fields such as management, education, production, business, social sciences etc. But most of these applications are centered on forecasting.

Some researchers have been trying to direct efforts at transfer function modelling research away from forecasting. For example Nwobi-Okoye and Igboanugo in their work in 2011, applied transfer function modelling to manufacturing process capability calculations (Igboanugo and Nwobi-Okoye, 2011a). They took input-output data from a real life production process over a given period and developed a transfer function model for the process which they used in determining its process capability. After the analysis, 99.1269% of the output was found to conform to specification. The process capability calculated through their new method was found to be 0.75. The result of this study they opined is expected to open new ways to process optimisation. In addition to this, effective determination of the capability of a process will lead to better comparison of alternative materials, equipment or process and production of better quality goods.

Nwobi-Okoye and Igboanugo (2012) developed a new and better way of evaluating the performance of hydropower generation facilities in order to improve their performance using transfer function modelling. It
involves taking input-output data from a hydropower generation process over a 10-year period and developing transfer function models of the process for the ten years, which were used as performance indicators. Based on the performance indicators they obtained from the models, the results show that the efficiency of the power generation facility was worst in the year 2006 and best in 2003. Generally the indicators (coefficient of performance) were undulating over the ten-year period. Finally based on the fact the power plant has underperformed over the years, a value of coefficient of performance which must be above the highest coefficient of performance of 7.523 obtained was suggested as a benchmark below which a hydropower plant is assumed to have underperformed.

Nwobi-Okoye and Igboanugo (2015) equally developed an improved way of determining the efficiency of gas power generation facilities using transfer function modelling. It involved taking input-output data from a gas power generation process over a 10-year period and developing transfer function models of the process for the ten years, which are used as performance indicators. Based on the performance indicators they obtained from the models, the results show that the efficiency of the gas power generation facility was worst in the years 2002-2006 with a coefficient of performance of 0.002073617. Similarly, with a coefficient of performance of 0.002343345, plant performance/efficiency was best in the years 2007-2011. The result is remarkable because given the state of the facilities, it correctly predicted the period of expected high system performance i.e. 2007-2011 period.

The result of their research described above is expected to open new ways to improving maintenance effectiveness and efficiency of gas power generation facilities. The study would be of immense help in improving power generation in Nigeria and elsewhere.

**Challenges of transfer function modelling:** Although most transfer function models are linear, quite a few are nonlinear. Modelling nonlinear transfer function models is quite challenging and constitutes one of the most difficult challenges faced in modelling transfer functions. Also, identifying the appropriate noise models for transfer function is quite tasking and constitutes a difficult challenge in developing appropriate transfer function models. Similarly, modelling Multi-Input-Single-Output (MISO), Single-Input-Multi-Output (SIMO) and Multi-Input-Multi-Output (MIMO) systems is a daunting task and considerable research efforts are being directed towards this area.

Again, the complexity of transfer function modelling requires the development of complex and sophisticated software to aid in the modelling. Developing such software is quite challenging. Similarly, optimization of transfer function models using metaheuristics is quite challenging.

**Gaps in past works and future direction for research:** Approaches to maintenance as has been identified in the literature rely heavily on the use of charts and tables for predictive maintenance. Transfer function characteristics could act as predictive and diagnostic tools. Hence, applying transfer function to maintenance scheduling would be a very attractive area of research.

In the literature on past works, no attempt has been made to determine process capability using Box-Jenkins transfer function modelling methodology. Similarly, there has not been any formal application of transfer function modelling to performance evaluation of production systems and the development of methodology for application of transfer function modelling to maintenance and assessment of production operations efficiency.

Also there appears to be a dearth of software dedicated to transfer function model identification, estimation and diagnostics. Development of highly sophisticated software dedicated to transfer function modelling would be a very attractive area of research considering the versatility of transfer functions as a processing tool.

Game theory has been in at heart of economic theory and is increasingly being used in diverse fields of life. Applying transfer functions to game theoretic analysis would be very novel. This powerful theory and method of modelling strategic interaction is expected to help optimise and improve decisions taken by competing industrial organizations.

Increased improvement and use of modern artificial neural network models is expected to define the future direction of transfer function modelling. It is expected that transfer function modelling will increasingly be used in maintenance scheduling and management. It is equally expected that transfer function modelling will be increasingly used in performance evaluation and assessment of production operations efficiency. In addition to these, better optimisation techniques currently being developed would be of great benefit to transfer function modelling and time series analysis.

According to DeLurgio (1998), many more potential applications for transfer function modelling exist and these applications will grow as more powerful and user friendly software becomes available to more users. Hence, a potential future direction for research is in the area of development of more powerful and user friendly transfer function modelling software, especially those based on artificial intelligence.

In continuation, it is expected that it would be increasingly used in game theoretic modelling. It is expected that the modelling of multi-input production
processes should be a focus of future research. It is
equally expected that the concept of Coefficient of
Performance (COP) introduced by Nwobi-Okoye and
Igboanugo (2012, 2015), which was used in analysis of
two types of electric power generation systems be
further applied to other types of power generation
systems such as: wind power generation systems, steam
power generation systems etc. Also this should be
extended to Multi Input Single Output (MISO) systems,
Multi Input Multi Output (MIMO) systems, Multi
Output Single Input (MOSI) systems etc. Currently,
Nwobi-Okoye et al. (2015) are introducing the concept
of COP to MISO systems.
Furthermore, the concept of Coefficient of
Performance (COP) should be applied extensively to oil
gas industries, power transmission and distribution
systems, chemical and communication industries, as
well as every system in general.
Also, the concept of transfer function modelling
could be applied to the prediction of pipeline
failure/fracture/rupture as well as failure of critical parts
of machineries or equipment. Even pipeline leaks could
be detected by discrete transfer function modelling.
These should be the subject of future investigation. In
addition application of transfer function modelling to
Material Requirement Planning (MRP) in the process
industry should be a subject for future research.

**DISCUSSION**

The use of the transfer function tools as an aid in
the manufacturing sector would likely engender
economic growth and development in the industrial
sector.

The transfer function model as we have from the
literature could be used to compare the performance of
different materials or equipment used in a production
process. As different materials or equipment are likely
to have different transfer functions poor quality
materials or equipment could easily be detected

With transfer function modelling, the behaviour and
process capability of existing plant, equipment or
process will be better monitored and accessed which
will lead to improved plant design and product quality.
In general, the analysis here has shown that transfer
function modelling is a very good quality control and
production management tool which compares very
favourably with conventional quality control tools. In
addition to this process capability of non discrete items
cannot be measured by the conventional methods but
this new method can do it very effectively.

As one of the triumphs of game and contract
theories, governments and organizations are
increasingly resorting to competitive bidding and
auctions before contract awards and by so doing
generating billions of dollars in revenue in addition to
improved efficiency in the execution of public works
(Binmore, 2007; von Stengel and Turocy, 2008).
Consequently, firms, especially contracting firms must
be able to determine to a very high degree of accuracy
the process capability of their facilities in terms of the
quantity they are capable of processing within a given
period of time. Hence, the model relating transfer
functions to Bayesian games and mechanism design is
extremely useful

Furthermore, from the literature initial transfer
function models developed by Box-Jenkins method are
tentative and require more efficient estimation. Hence,
opimisation is needed. Igboanugo and Nwobi-Okoye
(2011b) demonstrated this assertion by developing
software based on genetic algorithm which was used to
optimise a developed transfer function model.

The success of genetic algorithm in transfer
function model optimisation necessitates the need to try
other meta-heuristics methods such as Tabu Search and
Ant Colony Optimisation. A comparison of the various
meta-heuristics methods would reveal the best method
for transfer function model optimisation (Winker and
Gilli, 2004).

Furthermore, performance evaluation is very
important in every system. From this survey it is
apparent that transfer function modelling would be
invaluable in systems performance evaluation. The
works of Nwobi-Okoye and Igboanugo (2012, 2015)
and Nwobi-Okoye et al. (2015) are very strong
evidences to support this. In view of the foregoing
facts, it is obvious that transfer function could
determine operations and maintenance effectiveness.

**CONCLUSION**

The importance of production process transfer
functions and its relationship to quality control and
production management as elucidated in the literature
will help engineers and managers have a better
understanding of transfer functions and its relationship
to achieving effective and optimum utilization of raw
materials and production of high quality goods and
effective management of production processes.

Furthermore, several factors work conjointly to
affect manufacturing dysfunction. The transfer function
modelling approach is effective in dealing with the
problem of process variability and degraded output.

In continuation, effective maintenance and efficient
performance of plants and facilities is highly desirable
(Ghedamsi and Aouzellag, 2010; Alves de Sousa et al.,
2010; Zhang et al., 2011). This is indeed applicable to
all manufacturing and production facilities. From the
literature a very sound and statistically robust method
of evaluating the performance of hydropower and gas
power plants using transfer function modelling has been
successfully developed. This research could be
extended to other production and manufacturing systems. Transfer function models are extremely useful and have wide application in industry. They are widely used in forecasting, plant design and redesign etc., hence excellent models result to excellent forecasts and designs.

**NOMENCLATURE, SYMBOLS AND NOTATIONS**

\[ \begin{align*}
\gamma & = \text{Cross correlation function} \\
\delta & = \text{Difference equation variable for output} \\
\varphi & = \text{Difference operator} \\
\zeta & = \text{Difference equation variable for output} \\
\eta & = \text{Difference equation variable for input} \\
\Theta & = \text{Moving average operator} \\
\Xi & = \text{Output variable of differential equation} \\
\rho & = \text{Auto correlation function} \\
\sigma & = \text{Population standard deviation} \\
\varphi & = \text{Autoregressive operator} \\
\omega & = \text{Difference equation variable for input} \\
\Omega & = \text{Difference equation variable for input} \\
a_t & = \text{Error term/white noise} \\
b & = \text{Transfer function lag} \\
B & = \text{Backshift operator} \\
D & = \text{Differential operator} \\
d & = \text{Number of differencing} \\
H & = \text{Input variable of differential equation} \\
k & = \text{Lag variable} \\
r & = \text{Order of the output series} \\
RSE & = \text{Residual standard error} \\
S & = \text{Sample standard deviation} \\
s & = \text{Order of the input series} \\
t & = \text{Time} \\
v(B) & = \text{Transfer function} \\
X_t & = \text{Process input} \\
x_t & = \text{Differenced input series} \\
\hat{X}_t & = \text{Input forecast} \\
Y_t & = \text{Process output} \\
y_t & = \text{Differenced output series} \\
\hat{Y}_t & = \text{Output forecast}
\end{align*} \]

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