Research Article
Hydraulic Jump Properties Downstream a Sluice Gate with Prismatic Sill

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Abstract: Hydraulic Jump is a subjective process affected by substantive conditions; in this phenomenon part of flow kinetic energy changes into potential energy. The effect of substantive condition such as prismatic sill under sluice gate is studied experimentally by changing the sill height four times and compared with a jump without sill. The location of occurrence, total length, relative sequence depth and relative energy loss are the properties of the jump which are investigated. The investigation leads that sill under the gate increases energy loss up to 10%. The energy loss increase as sill slope increase and decreases with the increase of relative sill high to jump location. While the relative sequence depth of jump increases with increase of relative sill high to jump location and decreases with the increase of sill slope. The relative location of jump occurrence to depth of flow after the jump decreases with increase of sill slope. Within the limitations of the present experimental work, four mathematical models of relationship predicting the properties of the jump suggested with adjusted R square more than 0.921.

Keywords: Energy loss, froude number, jump location, sequent depth, sill height

INTRODUCTION
Hydraulic jump is the transition phenomenon from supercritical flow to subcritical flow, which is generally observed in open channel, such as rivers and spillways. The most important applications of the hydraulic jump in open channels are dissipation of energy, mixing of chemicals and aeration of flows. A jump formation in the open channels with smooth bed conditions (classic jump) and roughened beds has been widely investigated by many researches such as Rajaratnam (1961), Alhamid and Negm (1996), Hughes and ErnestFlack (1984) and AboulAtta et al. (2011). Several investigations have been carried out to determine the properties of hydraulic jump including, length of jump, the sequent depth ratio and the energy dissipation. Wu and Rajaratnam (1996), Bessiaih and Rezak (2002) and Afzal and Bushra (2002), have been developed various analytical and empirical models from experimental study for hydraulic jump characteristics. Debabeche and Achour (2007), experimentally investigated the effects of both the thin-crested and the broad-crested sills on the characteristics of the hydraulic jumps in a horizontal symmetric triangular channel. Alikhani et al. (2010), have proposed design criteria for estimation of the stilling basin length without consideration of tail water. Mohamed Ali (1991) carried a series of experiments to study the effect of roughened beds using cube blocks, Ali found that the length of hydraulic jump reduced by around 27 to 67% for a Froude number range of 4 to 10. Elsebaie and Shabayek (2010a) studied the effect of different shapes of corrugated beds on the properties of hydraulic jumps. It was found that, for all shapes of corrugated beds, the tail water depth required to form a jump was appreciably smaller than that for the corresponding jump on smooth beds. Nikmehr and Tabeordbar (2010), studied the properties of hydraulic jump on four adverse slopes, in two cases of rough and smooth beds. Nikmehr and Tabeordbar (2010) results showed that the sequent depth ratio and the length of the jump were greater on smooth beds than rough beds for the same slopes and Froude number and more energy loss occurred on rough beds than smooth beds. Imran and Akib (2013), investigated the potential use of corrugated and roughened beds for reducing the hydraulic jump length and sequent depth, Imran and Akib (2013) found that the corrugated bed always showed better performance than a smooth bed channel in reducing hydraulic jump length and sequent depth.

Bejestan and Neisi (2009), studied the effect of lozenge roughness shape on the hydraulic jump. Bejestan and Neisi (2009) found that this shape reduced the tail water depth by 24% and the hydraulic jump length by 40% compared with the smooth bed. Ead and Rajaratnam (2002), studied the effect of round corrugated bed on hydraulic jump. The results indicated that the sequent depth decreases 20% and the hydraulic jump length decreases 50%. Izadjoo and Shafai-Bejestan (2007), used trapezoidal shape corrugated bed
in their study. Izadjoo and Bejestan (2007) found that the jump length is more dependent on the wave length of corrugations than their amplitude. Esfahani and Bejestan (2012), studied the effect of roughened bed on flow characteristics of hydraulic jump at abrupt drop. Experimental tests were performed for Froude numbers ranging from 3.03 to 11.68. The experimental results showed that roughening bed of stilling basin can reduce the jump length as much as about 40%. Elsebaie and Shabayek (2010b), investigated the effect of different shapes of corrugated beds on the characteristics of hydraulic jumps, experiments were performed for a range of the Froude number from 3 to 7.5. Five shapes of corrugations were tested, it was found that, for all shapes of corrugated beds, the tail water depth required to form a jump was appreciably smaller than that for the corresponding jumps on smooth beds, further, the length of the jump on the different corrugated beds was less than half of that on smooth beds. The objective of the present study is to investigate the effect of prismatic sill on the relative height of hydraulic jump, energy loss and the location of the jump from the sluice gate and then to suggest regression models predicting these characteristics.

LITERATURE REVIEW AND METHODOLOGY

In a horizontal channel the super critical flow under sluice gate continues to flow until hydraulic jump happened. The location of the jump, sequent depths and percent of energy loss are affected by many physical and geometrical properties. The definition sketch on Fig. 1 shows the flow phenomena with the geometric parameters. The fundamental basics show that the important factors effecting the pattern of jump are $Y_1$, the depth of flow before the jump and $Y_2$, depth of flow after the jump with the velocities of flow. The distance from the sluice gate to the location of hydraulic jump ($L_T$) is effected by the geometry of the channel as the sill height ($P$) and length ($B_2$). The variables inter the phenomena can be expressed by the following function:

$$f(Y_1, Y_2, L_T, L_p, V_1, V_2, g, \rho, \mu, \sigma, P, B_2) = 0 \tag{1}$$

In which

- $L_T$ = The total length from the gate to the end of the jump
- $V_1$ = The velocity of flow before jump
- $V_2$ = The velocity of flow after jump
- $\rho$ = The density of water
- $g$ = The gravitational acceleration
- $\mu$ = The dynamic viscosity
- $\sigma$ = The surface tension

The height of jump relative to the depth of flow before the jump can be formulated by the following functional relationship:

$$\frac{Y_2 - Y_1}{Y_1} = f(Fr_1, Re_1, We_1, \frac{P}{L_p}, \frac{P}{B_2}, \frac{L_2}{L_p}) \tag{2}$$

where,

- $Fr_1$ = Froude number at $Y_1$
- $Re_1$ = Reynolds number at $Y_1$
- $We_1$ = Weber number at $Y_1$

The relative energy dissipated by the jump can also formulated by functional relationship as in Eq. (3):

$$\frac{E_2 - E_1}{E_1} = f(Fr_1, Re_1, We_1, \frac{P}{L_p}, \frac{P}{B_2}, \frac{L_2}{L_p}) \tag{3}$$

To achieve the main purpose of this study, experimental measures was carried out in a horizontal flume of working length 2.4 m, having a rectangular cross section of 0.25 m height and 0.075 m width. The depth of water at center line was measured by point gauge with venier. The experiments carried out by fixing Mahogany wooden prismatic sill under the sluice gate and one group of runs without sill. The sills have different height. Figure 2 shows photo for the models and the flow performance.

The models were classified into five groups depending on the value of sill height, Table 1 shows the experimental study details for the groups with total number of runs 326.

Fig. 1: Definition sketch
The sill effect can be noticed by parameters in Eq. (2) and (3) was studied by employing jet on the inclined sill and the horizontal bed slope of the flow depth \(Y_c\). The effect of sill is also seen when on unity, which means the flow researches to the critical condition. The increase in the sequent relative depths due to the change of in the bed slope.

The relative energy losses \(E_1\) is also logical increasing relative sequent depth with increase of Froude number also. It can be noted that the sill cause a relatively higher jump depth due the potential energy represented by the sill. The increase in the sequent relative depths due to the increase in sill height can be estimated by 9% for the highest sill of slope \(P/B_2 = 0.1776\), this increases coursed by the change of in the bed slope.

The relative energy losses \(E_1\) is also logical increases with increase of Froude number values as presented in Fig. 2. The sill effect can be noticed by increasing the relative energy loses by 10% for the highest slope sill of \(P/B_2 = 0.1776\), this increase is related to the jump propagation without sill. The difference in energy losses caused by the impact of flow jet on the inclined sill and the horizontal bed slope of the channel for the same properties of the flow.

The relative sequent depth and relative energy losses are presented in Fig. 5 and 6 with \(Y_c/Y_c\). The decreasing values of both sequent depths and energy losses with increase of \(Y_c/Y_c\) is also evident due to the fact that increasing values of \(Y_c/Y_c\) towards the value on unity, which means the flow researches to the critical condition. The effect of sill in also seen when the flow depth \(Y_i\) decreases relative to the critical depth \(Y_c\), for these lower values of \(Y_i/Y_c\), the relative sequence depth and relative energy loses is also increases.

The correlation analysis of dimensionless parameters in Eq. (2) and (3) was studied by employing the facilities of the IBM-SPSS 20 Package., the correlation shows that the relative energy loses \(E_1\) is significantly correlated with the parameters \(F_{r1}, W_e, R_e, L/L_p\) and \(P/B_2\) at the 0.01 level (2-tailed), the heighest correlation factor is with \(F_{r1}\) and the lowest with \(P/B_2\). The Correlation factor of the relative sequence depth \((Y_2-Y_1)/Y_1\) show a little but less than that with \((E_1-E_2)/E_1\), except \((P/L_p)\) and \((P/B_2)\) shows a hiegher correlation factor than that with \((E_1-E_2)/E_1\). It should be noted that all the independent variables in the correalation matrix show asignificant correlation with both relative energy loses and relative sequent at 2-tailed level.

The linear stepwise regression has been employed to find the mathematical relationship between the dependent variable \((Y_2-Y_1)/Y_1\) and independent variables \(F_{r1}, W_e, R_e, L/L_p\) and \(P/B_2\). Stepwise skim states to include the independent varables to the modle of the heighest correlation with the dependent variable and at the same time exclude those which not effect the modle. The modle includes three indepndent varibles \(F_{r1}, P/L_p\) and \(P/B_2\) and exclude \(W_e, R_e, L/L_p\). The best model has adjusted \(R^2\) is 0.934 at confidence level of 95%. The relative energy loss \((E_1-E_2)/E_1\) has been also modeled by the linear stepwise method with the same indepndent variables, the model includes the three indepndent varibles \(F_{r1}, P/L_p\) and \(P/B_2\) as those induded in the relative sequence depth model \((Y_2-Y_1)/Y_1\).

Nonlinear Regression Analysis of 20 different models is carried on by the same package. The models were defined as power mathematical relations. To show some of the linear and nonlinear models, Table 2 present 12 best models. The simplest forms were the linear ones.

The first two relations in Table 2 shows the positive effect of sill on both relative energy losses and

<table>
<thead>
<tr>
<th>Model</th>
<th>Runs no.</th>
<th>Max. (F_{r1})</th>
<th>Max. (Y_2/Y_1)</th>
<th>Max. (E_1-E_2)/E_1)</th>
<th>Max. (L_p) (m)</th>
<th>Max. (L_c) (m)</th>
<th>Max. (L_i) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 0</td>
<td>66</td>
<td>7.85</td>
<td>8.57</td>
<td>0.69</td>
<td>1.235</td>
<td>1.335</td>
<td>0.100</td>
</tr>
<tr>
<td>P = 2</td>
<td>88</td>
<td>6.81</td>
<td>9.90</td>
<td>0.61</td>
<td>1.370</td>
<td>1.485</td>
<td>0.115</td>
</tr>
<tr>
<td>P = 3</td>
<td>66</td>
<td>7.63</td>
<td>9.87</td>
<td>0.69</td>
<td>1.225</td>
<td>1.355</td>
<td>0.130</td>
</tr>
<tr>
<td>P = 4</td>
<td>49</td>
<td>6.44</td>
<td>7.99</td>
<td>0.58</td>
<td>1.269</td>
<td>1.484</td>
<td>0.215</td>
</tr>
<tr>
<td>P = 5</td>
<td>57</td>
<td>7.40</td>
<td>10.47</td>
<td>0.69</td>
<td>1.270</td>
<td>1.570</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Max.: Maximum

RESULTS AND DISCUSSION

The descriptive analysis shows that Froude number varies between 1.926 and 7.85 with standard deviation 1.201299. That means the types of hydraulic jumps are Weak, Oscillating and Steady Chow (1959). The calculated relative jump height \((Y_2-Y_1)/Y_1\) from the experimental data are presented with Froude number in Fig. 3. It can be notice the logical increasing relative sequent depth with increase of Froude number also. It can be noted that the sill cause a relatively higher jump depth due the potential energy represented by the sill. The increase in the sequent relative depths due to the increase in sill height can be estimated by 9% for the highest sill of slope \(P/B_2 = 0.1776\), this increases coursed by the change of in the bed slope.

The relative energy losses \(E_1\) is also logical increases with increase of Froude number values as presented in Fig. 4. The sill effect can be noticed by increasing the relative energy loses by 10% for the highest slope sill of \(P/B_2 = 0.1776\), this increase is related to the jump propagation without sill. The difference in energy losses caused by the impact of flow jet on the inclined sill and the horizontal bed slope of the channel for the same properties of the flow.

The relative sequent depth and relative energy losses are presented in Fig. 5 and 6 with \(Y_c/Y_c\). The decreasing values of both sequent depths and energy losses with increase of \(Y_c/Y_c\) is also evident due to the fact that increasing values of \(Y_c/Y_c\) towards the value on unity, which means the flow researches to the critical condition. The effect of sill in also seen when the flow depth \(Y_i\) decreases relative to the critical depth \(Y_c\), for these lower values of \(Y_i/Y_c\), the relative sequence depth and relative energy loses is also increases.

The correlation analysis of dimensionless parameters in Eq. (2) and (3) was studied by employing
relative jump height, as the linear relation are simplest forms which can be selected, an automatic linear modelling developed in SPSS since version 19 is used. The mentioned method is carried on to improve the linear models predicted by conventional linear regression in Table 3. The new linear modelling accelerates the data analysis and improves the model by automatic variable selection and automatic data preparation to fit the contributed predictors, this skim carried on by using Machine learning to give best predictive model (Yang, 2013). To find the model of the relative sequent \((\frac{Y_2 - Y_1}{Y_1})\), automatic data preparation and forward stepwise model is employed. The output criterion information of the best fit is give in Table 3 with adjusted \(R^2 = 0.937\). Equation (4) present the relationship:

\[
\frac{Y_2 - Y_1}{Y_1} = 1.540F_{r1} - 1.367 \\
R^2 = 0.836
\]

\[
\frac{Y_2 - Y_1}{Y_1} = 1.498F_{r1} - 1.275 \\
R^2 = 0.888
\]

\[
\frac{Y_2 - Y_1}{Y_1} = 1.199F_{r1} - 0.204 \\
R^2 = 0.804
\]

\[
\frac{Y_2 - Y_1}{Y_1} = 1.275F_{r1} - 0.752 \\
R^2 = 0.979
\]

\[
\frac{E_1 - E_2}{E_1} = 0.134F_{r1} - 0.157 \\
R^2 = 0.896
\]

\[
\frac{E_1 - E_2}{E_1} = 0.126F_{r1} - 0.157 \\
R^2 = 0.909
\]

\[
\frac{E_1 - E_2}{E_1} = 0.121F_{r1} - 0.752 \\
R^2 = 0.979
\]

\[
\frac{E_1 - E_2}{E_1} = 0.105F_{r1} - 0.063 \\
R^2 = 0.949
\]
Table 2: The regression models analysis

<table>
<thead>
<tr>
<th>No.</th>
<th>Equation</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{E_1 - E_2}{E_1} = -0.119 + 0.118Fr_i + 0.021 \frac{P}{B_i} )</td>
<td>0.913</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{Y_2 - Y_1}{Y_1} = -1.125 + 1.335Fr_i + 3.246 \frac{P}{L_p} )</td>
<td>0.856</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{E_1 - E_2}{E_1} = -0.133 + 0.122Fr_i - 0.103 \frac{P}{L_p} + 0.141 \frac{P}{B_i} )</td>
<td>0.918</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{Y_1 - Y_1}{Y_1} = -0.471 + 1.166Fr_i + 4.726 \frac{P}{L_p} - 2.222 \frac{P}{B_i} )</td>
<td>0.935</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{E_1 - E_2}{E_1} = 0.073 (Fr_i)^{1.277} \cdot (Re_e)^{0.020} + 0.14 \left( \frac{P}{L_p} \right)^{-0.309} \cdot \left( \frac{P}{B_i} \right)^{1.141} )</td>
<td>0.899</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{Y_1 - Y_1}{Y_1} = 1.153 (Fr_i)^{1.174} \cdot (Re_e)^{-0.017} + 2.153 \left( \frac{P}{L_p} \right)^{-0.309} \cdot \left( \frac{P}{B_i} \right)^{1.019} )</td>
<td>0.929</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{E_1 - E_2}{E_1} = 0.064 (Fr_i)^{1.241} \cdot (Wc_e)^{0.004} - 0.001 \left( \frac{P}{L_p} \right)^{0.309} \cdot \left( \frac{P}{B_i} \right)^{-1.141} )</td>
<td>0.905</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{Y_1 - Y_1}{Y_1} = 0.875 (Fr_i)^{1.186} \cdot (Wc_e)^{-0.051} + 2.153 \left( \frac{P}{L_p} \right)^{-0.309} \cdot \left( \frac{P}{B_i} \right)^{1.019} )</td>
<td>0.929</td>
</tr>
<tr>
<td>9</td>
<td>( \frac{E_1 - E_2}{E_1} = 0.064 Fr_i^{0.116} - 2.835 Wc_e^{0.809} - 149.849 \left( \frac{P}{L_p} \right)^{-0.002} + 0.0002 \left( \frac{P}{B_i} \right)^{1.362} )</td>
<td>0.941</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{Y_1 - Y_1}{Y_1} = 22.075 Fr_i^{0.414} - 25.639 Wc_e^{0.020} + 4.606 \left( \frac{P}{L_p} \right)^{-0.407} - 3.424 \left( \frac{P}{B_i} \right)^{-0.002} )</td>
<td>0.948</td>
</tr>
<tr>
<td>11</td>
<td>( \frac{E_1 - E_2}{E_1} = -1.186 + Fr_i^{0.119} + \left( \frac{P}{L_p} \right)^{0.014} + \frac{P}{B_i}^{-1.744} )</td>
<td>0.930</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{Y_1 - Y_1}{Y_1} = -0.481 + Fr_i^{1.102} + \left( \frac{P}{L_p} \right)^{0.102} + \frac{P}{B_i}^{-0.002} )</td>
<td>0.891</td>
</tr>
</tbody>
</table>

Fig. 5: Relation between sequent jump depths and \( \frac{Y_1}{Y_c} \)

\[
\frac{Y_i - Y_c}{Y_c} = -0.535 + 1.176Fr_i + 5.577 \frac{P}{L_p} - 2.645 \frac{P}{B_i}
\]  
(4)

The relative energy loss (\( E_1 - E_2 \) ) model is also predicted by linear automatic model employing the...
The relation between the Locations of hydraulic jump from the sluice gate relative to its sequence depth ($Y_2$) is analyzed by the regression with approaching Froude number $Fr_1$ and the slope of the sill ($P/B_2$). The total experimental data inters automatic linear modelling, relationships have been found as the out of regression with adjusted $R^2 = 0.921$, for the Location ($L_p$) Eq. (6) and adjusted $R^2 = 0.936$, for the total length from the gate to the end of the jump ($L_T$) Eq. (7):

$$\frac{L_p}{Y_2} = -0.505 + 0.951 \frac{L_T}{Y_2} - 10.72 \frac{P}{B_2} - 0.271Fr_1$$  \hspace{1cm} (6)$$

$$\frac{L_T}{Y_2} = -6.102 + 0.842 \frac{L_T}{Y_2} + 29.276Fr_1 + 17.983 \frac{P}{B_2}$$  \hspace{1cm} (7)$$

**CONCLUSION**

The properties of hydraulic jump of relative sequant depth, relative energy loss and Location of jump are studied from the experimantal data, the following conclusions may forwarded:

- The sill under the gate increase energy loss up to 10%.
- The energy loss increase with increase of $P/B_2$ and decreases with increase of $P/L_p$.
- Relative sequent depth of jump decreases with increase of $P/B_2$ and increases with increase of $P/L_p$.
- The relative location of jump occurrence to depth of flow after the jump decreases with increase of sill slope $P/B_2$.
Within the limitations of the present experimental work, four different Eq. (4) to (7) for predicting jump properties is suggested with adjusted $R^2>0.921$.

REFERENCES


