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# Research Article Application of Variable Weight Method to Water Quality Evaluation

<sup>1</sup>Haiping Ren and <sup>2</sup>Hui Zhou

<sup>1</sup>School of Software, Jiangxi University of Science and Technology, Nanchang 330013, <sup>2</sup>School of Mathematics and Computer Science, Yichun University, Yichun 336000, China

**Abstract:** Water quality evaluation is a key step in water resource management. The water quality evaluation problem is a multi-attribute decision making problem because there are many indicators considered in the evaluation process. The aim of this study is to put forward a new water quality evaluation method based on variable weight method. The new evaluation method takes both the relative importance of factors and the horizontal configuration of the value of each indicator into consideration. Finally, a practical example demonstrates that the proposed method is effective and feasible.

Keywords: Attribute measure, multi-attribute decision making, variable weight method, water quality evaluation

### **INTRODUCTION**

Water is indispensable to all life activity substance and it is also the most common food components. With the high-speed development of economy and rapid increasing of population, the supply and demand of water resources has become a serious problem because of the imbalance between the supply and demand of water (Huang and Xia, 2001). Now serious destruction of water environment, in the case of a shortage of water resources, protection of water resources and water environment are of great significance to the research, which have become the most concern in modern society (Li et al., 2014). Water resource safety becomes the key to a sustainable future for human and the evaluation of water quality is one of the main tasks of ensuring water security (Murray et al., 2012). Therefore, effective evaluation methods and concrete evaluation attributes for assessing the quality of water resources must be developed in order to secure water safety for sustainable development and public health (Wong and Hu, 2013). Water quality evaluation problem contains many evaluation indicators and thus this problem is a Multi-Attribute Decision Making (MADM) problem. Based on MADM methods, many water quality evaluation methods are developed, such as fuzzy comprehensive evaluation method (Icaga, 2007), the matter element method (Kou, 2013; Liu and Zou, 2012; Wong and Hu, 2014), comprehensive index method (Chen et al., 2010), attribute recognition Method (Yu et al., 2013), set pair analysis (Wang et al., 2012; Du et al., 2014) and variable fuzzy model (Xie et al.,

2014). In the practical application of these methods for water quality evaluation problem, a key step is to determine the weight (i.e., important degree) of each indicator (Li and Liu, 2014). The importance of every indicator may be the same or different. To avoid subjective opinion, the variable weight method is used to determine the indicators' weights. Variable weight method is firstly proposed by Wang (1985) and it is an objective method (Wang, 1985). The difference between constant weights and variable weights is that variable weights take both the relative importance of factors and the horizontal configuration of the value of each factor (attribute) into consideration and thus it has some advantage over other constant weights method (Li et al., 2004). In this study we induce variable weight method to the determination weights of indicators. And we will propose a new water quality evaluation method for the water quality problem based on variable weight.

Water quality evaluation model: Suppose that there are *m* water samples  $A_1, A_2, ..., A_m$  waited to be evaluated about their water quality grades and each sample belong to some grade of water quality standards denoted by  $C_1, C_2, ..., C_K$ . Each water sample has *n* indicators (indexes, evaluation attributes)  $o_1, o_2, ..., o_n$ . The measurement value of water sample  $A_i$  with respect to indicator  $o_j$  is  $x_{ij}$ . Then the water sample  $A_i$  with respect to indicator  $o_j$  is  $x_{ij}$ . Then the water sample  $A_i$  with respect to indicator  $o_j$  is  $x_{ij}$ . Then the water sample  $A_i$  with respect to indicator  $o_j$  is  $x_{ij}$ . Then the water sample  $A_i$  matrix can be expressed with the following  $n \times m$  matrix:

Corresponding Author: Haiping Ren, School of Software, Jiangxi University of Science and Technology, Nanchang 330013, China

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$$X = (x_{ij})_{m \times n} = A_2 \vdots A_m \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

Let *F* be some attribute space and  $C_1, C_2, ..., C_K$  be an ordered series of grades in the attribute space F. The series satisfies the condition  $C_1 > C_2 > \cdots > C_K$ . Such a space can be established for the standard grades of every evaluation indicator. Let  $a_{ij}$  be the standard value of  $C_j$  with respect to the indicator  $o_i$  and  $a_{ij}$  satisfies  $a_{i1} < a_{i2} < \cdots < a_{iK}$  or  $a_{i1} > a_{i2} > \cdots > a_{iK}$ . Then the standard grade matrix can be expressed with the following matrix:

$$C_{1} \quad C_{2} \quad \cdots \quad C_{K}$$

$$o_{1} \quad \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1K} \\ a_{21} & a_{22} & \cdots & a_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ o_{n} \quad \begin{pmatrix} a_{n1} & a_{n2} & \cdots & a_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nK} \end{pmatrix}$$

Then for establishing the water quality evaluation decision model, the attribute measures are firstly to be calculated as follows (Chen *et al.*, 2008).

The attribute measure  $\mu_{ijk} = \mu(x_{ij} \in C_k)$  of indicator value  $x_{ij}$ , which takes the attribute levels from the set  $C_k$ , is found in this way.

Suppose that  $a_{ij}$  (i = 1, 2, ..., m) satisfies  $a_{i1} < a_{i2} < \cdots < a_{iK}$ , then:

• If 
$$x_{ij} \le a_{i1}$$
  
Then  $\mu_{ij1} = 1, \mu_{ij2} = \dots = \mu_{ijK} = 0$ 

• If  $x_{ij} \ge a_{iK}$ Then

$$\mu_{ijK} = 1, \mu_{ij1} = \dots = \mu_{ijK-1} = 0$$
  
If  $a_{il} \le x_{ij} \le a_{i(l+1)}$ 

Then Then

$$\mu_{ijl} = \frac{|x_{ij} - a_{i(l+1)}|}{|a_{il} - a_{i(l+1)}|}, \mu_{ij(l+1)} = \frac{|x_{ij} - a_{il}|}{|a_{il} - a_{i(l+1)}|}$$
And
$$\mu_{ijk} = 0, k < l \text{ or } k > l+1$$

Then the water quality evaluation can be regarded as a MADM model with the following attribute recognition decision matrix:

$$H_{i} = (\mu_{ijk})_{n \times K} = o_{2}$$

$$\vdots$$

$$O_{1} \begin{pmatrix} \mu_{i11} & \mu_{i12} & \cdots & \mu_{i1K} \\ \mu_{i21} & \mu_{i22} & \cdots & \mu_{i2K} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{in1} & \mu_{in2} & \cdots & \mu_{inK} \end{pmatrix}$$

# VARIABLE WEIGHT METHOD

The importance of each indicator may be the same or different. To avoid subjective opinion, the variable weight method is used to determine the indicators' weights. Variable weight method is firstly proposed by Wang (1985) and it is a objective method and has some advantage over other constant weights method (Li *et al.*, 2004). The detail description of this method is given as follows (Wang, 1985).

For some population with *n* influence factors (attributes)  $o_1, o_2, \dots, o_n$  and  $x_1, x_2, \dots, x_n$  are the single evaluation attributes. Here  $x_j$  is explained to the perfect degree of the factor  $o_i$  relative to the population. Suppose that the higher  $x_i$  is, the better  $o_i$  is. Without loss of generality, assume that  $x_i \in [0, 1]$ . If  $x_i = 1$ , then the factor  $o_i$  is very perfect; If  $x_i = 0$ , then the factor  $o_i$  completely loses its function.

**Definition Li and Liu (2014):** The variable weight of attribute  $o_i$  corresponding to the population is:

$$W_i = W_i(x_1, \cdots, x_n), i = 1, \cdots, n$$

It follows from the definition that the variable weight  $w_i$  depends on the evaluation of the single factor  $x_1, x_2, \dots, x_n$ . Different evaluation values of the single factor correspond to different variable weight. Denote  $w_i^{(m)} = w_i(1,\dots,1)$ ,  $i = 1,\dots,n$ , where  $w_i^{(m)}$  is the weight of factor  $o_i$  when the function of collectivity is very perfected and call it the basic weight. The basic weight can be obtained by many methods, such as the Analytic Hierarchy Process (AHP) method, entropy weight method or coefficient of variation method.

Let  $w_i^{(0)} = w_i(1, \dots, 1, 0, 1, \dots, 1)$   $(i = 1, \dots, n)$  be the weight of factor  $o_i$  when the function of factor  $o_i$  is completely lost and the functions of other factors are very perfect.  $w_i^{(0)}$  is evaluated by the experts or by the following (Wan, 2011):

$$v_i^{(0)} = \frac{w_i^{(m)}}{\min_{1 \le j \le n} \{w_j^{(m)}\} + \max_{1 \le j \le n} \{w_j^{(m)}\}}$$
(1)

v

In order to simply and intuitively obtain the variable weight  $w_i = w_i(x_1, \dots, x_n)$ , we need induce the function  $\lambda_i(x)$  which satisfies non-negative and bounded in range (0, 1), non-increasing function in (0, 1) and  $\lambda_i(0) = \lambda_i^{(0)}$ ,  $\lambda_i(1) = \lambda_i^{(m)}$ . Here  $\lambda_i^{(0)}$  and  $\lambda_i^{(m)}$  are respectively the maximum and minimum of  $\lambda_i(x)$  in the interval (0, 1).

Now, given a group of single factor evaluation  $x_1, x_2, \dots, x_n$ , if the function  $\lambda_i(x_i)$  is obtained  $(i = 1, \dots, n)$ , then the variable weight is defined as:

$$w_i = w_i(x_1, \cdots, x_n) = \lambda_i(x_i) / \sum_{j=1}^n \lambda_j(x_j)$$
<sup>(2)</sup>

where,  $i = 1, \cdots, n$ .

The calculation method of  $\lambda_i(x_i)$  can follow the below introduction.

Assume that the basic weights  $(w_1^{(m)}, \dots, w_n^{(m)})$  are known about some problem and then  $(w_1^{(0)}, \dots, w_n^{(0)})$  can be obtained by (1). Denote:

$$\begin{aligned} \lambda_i^{(m)} &= w_i^{(m)}, w_i^{(0)} = \lambda_i^{(0)} / (\lambda_i^{(0)} + \sum_{j \neq i} w_j^{(m)}) \\ i &= 1, \cdots, n \end{aligned}$$

Then we can get:

$$\lambda_i^{(0)} = w_i^{(0)} \sum_{j \neq i} w_j^{(m)} / (1 - w_i^{(0)}), \ i = 1, \cdots, n$$
(3)

Let,

$$\lambda_i^* = \sum_{j \neq i} \lambda_j^{(0)} , \lambda^* = \sum_{j=1}^n \lambda_j^{(0)}$$
(4)

Then,  $\lambda_i(x)$  can be solved with the following form:

$$\lambda_i(x_i) = \frac{\lambda_i^* \lambda_i^{(0)}}{\lambda^* \exp(\frac{1}{1-k_i} x_i^{1-k_i})}$$
(5)

where,

$$k_{i} = 1 - 1/\ln \frac{\lambda_{i}^{(0)}(\lambda_{i}^{*} + w_{i}^{(m)})}{\lambda^{*} w_{i}^{(m)}}$$
(6)

# IMPROVED ATTRIBUTE RECOGNITION MODEL FOR WATER QUALITY EVALUATION

In this section, we will develop a new water quality evaluation method, which is an improved attribute recognition model based on variable weight method. The specific calculation steps are given as follows.

**Step 1:** Normalize the sample space matrix  $X = (x_{ij})_{m \times n}$ .

An evaluation index can be classified as a benefit type (the sample is better with an increase in the index, such as Dissolved Oxygen (DO)) or as a cost type (the sample is better with a decrease in the index, such as NH<sub>3</sub>-N) depending upon its attributes. Thus normalization is necessary. We can transform the sample space matrix  $X = (x_{ij})_{m \times n}$  into the normalized decision matrix  $R_{il} = (r_{ijl})_{n \times K}$  with the following normalized method (Hwang and Yoon, 1981).

Among these indicators, if the  $j^{th}$  attribute belongs to benefit type, then:

$$r_{ij} = \frac{x_{ij}}{\max_{i} \{x_{ij}\}} \tag{7}$$

Else if the j<sup>th</sup> attribute belongs to cost type, then:

$$r_{ij} = \frac{\min_{i} \{x_{ij}\}}{x_{ii}}$$
(8)

Obviously,  $r_{ij}$  is the data of the  $j^{\text{th}}$  evaluating object on the indicator and  $r_{ij} \in [0,1]$ .

**Step 2:** Determine the attribute weights based on variable weight method as follows.

Suppose that  $w = (w_1, w_2, ..., w_n)$  is the attribute weight vector. Calculate the basic weights of each attribute  $(w_1^{(m)}, ..., w_n^{(m)})$  are acquired through existing weighting method:

- Calculate  $(w_1^{(0)}, \dots, w_n^{(0)})$  by (1)
- According to Eq. (3)-(5), derive  $(\lambda_1(r_{i1}), \dots, \lambda_n(r_{in}))$

With respect to each water sample  $A_i$ , we can obtain the attribute weight vector  $W_i = (w_1(r_{i1}, \dots, r_{in}), \dots, w_n(r_{i1}, \dots, r_{in}))$  according to the variable weight formula (2).

**Step 3:** Establish the attribute recognition decision matrix  $H_i = (\mu_{ijk})_{n \times K}$  (i = 1, 2, ..., m). Then the attribute measure of a sample can be calculated following a formula that includes each attribute measure and variable weight:

$$\mu_{ik} = \mu(A_i \in C_k) = \sum_{j=1}^n w_j \mu_{ijk}$$
(9)

(i = 1, 2, ..., m; k = 1, 2, ..., K)

Step 4: Establishing an attribute recognition model.

Attribute recognition will be done by considering the variable weight and attribute measure of each sample. The confidence criterion, includes the term  $\lambda$ (generally,  $\lambda = 0.7$ ) in the attribute recognition model. If,

$$k_i = \arg\min_{1 \le k \le K} \{\sum_{i=1}^k \mu_{ik} \ge \lambda\}$$
(10)

Then the water sample  $A_i$  belongs to the standard  $C_{k_i}$ .

If one wants to make a comparative analysis of evaluation objects, the assessment criterion score defined as follows can be used to rank them and the assessment criterion score of sample  $A_i$  is defined as follows:

$$S_i = \sum_{l=1}^{K} n_l \mu_{il} \tag{11}$$

where,  $n_l = K + 1 - l$ .

The larger of  $S_i$  is, the better water quality of water sample  $A_i$ .

**Case study:** To illustrate the practicability and feasibility of the proposed water quality evaluation method, an example with the water quality evaluation discussed in Du *et al.* (2014) is given.

Zhundong water source is located in the east of Junggar basin, China. Its geomorphic units are mainly the southern mountain area, the middle alluvial plain and fine soil plain desert area of northern Cara and wheat in the southern slope of Mt. erosion and flood plain area. The study area is a typical continental arid climate region, scarce rainfall, strong evaporation, relative small humidity, large temperature difference variation during the year, the lowest temperature of -20.0°C, the annual precipitation 135.9 mm, annual evaporation capacity 1904.07 mm.

Groundwater is mainly pore phreatic water and confined water. The Quaternary phreatic aquifer is composed of Upper Pleistocene alluvium, lithology and brown gray, grayish yellow sand, sand, locally interbedded with gray sand, small gravel and aquifer thickness is 10-20 m. Static water level depth becomes shallow gradually from south to north. The Quaternary aquifer buried, in lower Pleistocene alluvium, is mainly from the aquifer water source. The water source mainly

Table 1: National	quality	standards	of	ground	waters	(GB/T	14848-
93) of Ch	ina (uni	its of mg/L	)				

Grade	01	02	03	04
Ι	≤150	≤300	≤50	≤50
II	≤300	≤500	≤150	≤150
III	≤450	≤1000	≤250	≤250
IV	≤550	≤2000	≤350	≤350
V	>550	>2000	>350	>350

Table 2:	Water	monitoring	data	of	Zhundong	water	source	(units o	f
	mg/L)	-			-				

	Selected evaluation indicators						
Sample	01	02	03	04			
$A_1$	50.0	310.0	86.5	42.5			
$A_2$	80.1	438.8	115.3	74.4			
$A_3$	310.2	925.8	297.8	234.0			
$A_4$	160.1	622.8	192.1	120.5			
A <sub>5</sub>	90.1	560.0	153.7	106.4			

supplies Huoshaoshan and Cainan Oilfield and in recent years, it also supplies a small amount of water for industrial park in Wucaiwan Zhundong. In order to ensure the water quality, the rational exploitation and utilization of groundwater, it is a need for accurate evaluation of groundwater quality.

According to Chinese Quality of Ground Water (GB/T14848-93), there are five ground water quality grades have been derived as follows: I (Good), II (Fine), III (Ordinary), IV (Poor) and V (Very Poor). The standard of water quality is reported in Table 1. According to the water sources of Zhundong groundwater quality monitoring data in September, 2011, we choose the four main indicators: total hardness ( $o_1$ ), total dissolved solids ( $o_2$ ), sulfates ( $o_3$ ) and chlorides ( $o_4$ ) to evaluate Zhundong's five groups of water sampling data water quality. Here,  $o_1$  is the benefit indicator and others are cost indicators. The monitoring points (samples) are  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$  and  $A_5$ . The indicator measure values of samples are reported in Table 2.

The steps of the proposed method are given as follows.

# **Step 1:** According to Table 1, the sample space matrix is obtained as follows:

		$o_1$	$o_2$	03	$o_4$
$X = (x_{ij})_{5\times 5} =$	$A_1$	( 50.0	310.0	86.5	42.5
	$A_2$	80.1	438.8	115.3	74.4
	$A_3$	310.2	925.8	297.8	234.0
	$A_4$	160.1	622.8	192.1	120.5
	$A_5$	90.1	560.0	153.7	106.4

Take monitoring point (sample)  $A_1$  as the example, the steps of the proposed method are given as follows.

**Step 2:** The attribute recognition decision matrix  $H_1 = (\mu_{1ik})_{4\times 5}$  are obtained as follows:

Weight	01	02		03		04			
$W_1$	0.1680	0.25	569	0.32	88	0.2462			
$W_2$	0.1742	0.24	488	0.30	62	0.2709			
$W_3$	0.1706	0.23	393	0.32	.99	0.2602			
$W_4$	0.1772	0.23	383	0.32	48	0.2597			
$W_5$	0.1596	0.24	483	0.31	86	0.2735			
T-1-1- 4.	C-111	-1f							
Table 4:	Calculated v	II		easure	IV	IV			
	1		III	000					
$A_1$	0.8671	0.1329		000	0.0000	0			
$A_2$	0.5613	0.4387	0.0	000	0.0000	0			
$A_3$	0.0000	0.2362	0.6	062	0.1577	0			
$A_4$	0.2419	0.5628	0.1	952	0.0000	0			
$A_5$	0.2789	0.6795	0.0	416	0.0000	0			
Table 5:	Comparison	of different	method	S					
Method		$A_1$	$A_2$	$A_3$	$A_4$	A			
This stuc	dy	Ι	Π	III	II	II			
SPA (Du	i et al.,	Ι	Ι	III	II	Π			
2014)									
Official		П	П	Ш	Ш	III			

Table 3: Indicators' weights using variable weight method Selected evaluation indicators

cial result	п	11		un	III	
	Ι	II	III	IV	V	
$egin{array}{c} o_1 \\ H_1 = o_2 \\ o_3 \\ o_4 \end{array}$	(1.0000	0	0	0	0)	
$H_1 = o_2$	0.9500	0.0500	0	0	0	
03	0.6350	0.3650	0	0	0	
$O_4$	(1.0000	0	0	0	0)	

Step 3: Now, the steps of the methodology, proposed in Section above, are carried out as described below.

Normalize the original attribute values according to Eq. (7) and (8) and the normalization decision matrix is given as follows:

	$o_1$	$o_2$	03	$o_4$
$A_{1}$	(1.0000	1.0000	1.0000	1.0000
$A_2$	0.6242	0.7065	0.7502	0.5712
$A_3$	0.1612	0.3348	0.2905	0.1816
$A_4$	0.3123	0.4978	0.4503	0.3527
$A_5$	$\begin{pmatrix} 1.0000\\ 0.6242\\ 0.1612\\ 0.3123\\ 0.5549 \end{pmatrix}$	0.5536	0.5628	0.3994)

The basic weights of each attribute used here are obtained by as follows (Du *et al.*, 2014):

$$(w_1^{(m)}, \dots, w_4^{(m)}) = (0.158, 0.252, 0.350, 0.240)$$

According to Eq. (3) to (5), the Indicators' weights are obtained and reported in Table 3 and 4.

The distinction of the stations is determined using the confidence criterion (11) with  $\lambda = 0.7$ . The sorted list of the grade for each location is shown in Table 5.

The results show that new water quality evaluation method based on variable weight makes the results closer to the objective reality. The most of the water quality of Zhundong belongs to grade I and II and then the overall situation is good and it is basically not contaminated.

### CONCLUSION

This study is focus on water quality evaluation problem, which contains many evaluation indicators and thus this problem can be solved by using MADM method. Thus this study put forward a new water quality evaluation method based on a MADM method named variable weight method. The proposed method is easy to calculation and can be easily solve by computer software, such as MATLAB, excel software. Finally, an example of water quality evaluation of Zhundong water resource demonstrates that the proposed method is effective and feasible. The proposed method can also be applied to other similar problems, such as ecological environment quality evaluation, natural disaster risk degree assessment.

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